BRADBURY'S EATON'S PRACTICAL ARITHMETIC

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PRACTICAL

ARITHMETIC,

COMBINING

ORAL AND WRITTEN EXERCISES.

BY

WILLIAM F. BRADBURY, A. M.,

HEAD MASTER OF THE CAMBRIDGE LATIN SCHOOL; AUTHOR OF A SERIES OF MATHEMATICAL TEXT-BOOKS.

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PREFACE.

In the preparation of this work the author has endeavored to make a text-book that shall aid in teaching the *art* of arithmetic without any unnecessary discussion of the subject as a science. All the subjects have been made as practical as possible; and all obsolete and useless matter and puzzling examples have been omitted.

The number of rules has been reduced to a minimum. For example, but *one* rule is given for multiplication of fractions, *one* for division of fractions, and *two* for *all* cases of reduction of compound numbers, whether integral or fractional.

It combines oral and written exercises; and special pains has been taken to give examples that are practical, and that conform to the usual experiences of daily life. On pages 281 to 331 are given seven hundred additional examples for written work, to be drawn from at the discretion of the teacher, for use in connection with the regular work, and for a general review of the subject.

Decimals as far only as thousandths (the place of mills in United States currency) are introduced at the beginning with integral numbers. The Decimal, or Metric, system of weights and measures has been treated as a part of decimals, and independently of any other system of weights and measures. Nu-

merous illustrations are given showing the exact size of the metric weights and measures and the relation of their units to each other.

In the operations the process of simplifying by cancellation, whereby much time and labor is saved, is kept constantly in view. In the processes of Commission, Interest, Banking, Equation of Payments, etc., business men and bankers have been consulted, and those methods adopted that experience has proved to be the simplest and the best. Only such tables of compound numbers have been given in the body of the work as are used in ordinary transactions.

Circulating Decimals, Annual Interest, Foreign Exchange, Custom House Business, Arithmetical and Geometrical Progression, and Alligation, not being required in the grammar school grade, have been placed in the Appendix. Additional tables, monetary equivalents of different countries, rates of interest in the several States, and other matter of this nature used mainly for reference, are also given in the Appendix.

The author desires to return his thanks to those teachers who have so freely aided him with suggestions and examples. His obligations are specially due to Mr. L. A. Wheelock, Master of the Rice School, Boston, Mr. James A. Page, Master of the Dwight School, Boston, and Mr. L. M. Chase, Master of the Dudley School, Boston, for valuable assistance.

W. F. B.

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ARITHMETIC.

1. A **Unit** is a single thing of any kind; as, one day, one book.

2. A **Number** is a unit, or a collection of units; as, six days, ten books.

3. A **Concrete Number** is a number that is applied to a particular object; as, six books, ten men, four days.

4. An **Abstract Number** is a number that is not applied to any particular object; as, six, ten, seventeen.

5. **Arithmetic** is the *science* of numbers, and the *art* of computation.

NOTATION AND NUMERATION.

6. **Notation** is the writing of numbers.

7. **Numeration** is the reading of numbers.

8. To express numbers ten figures are used, viz.:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Zero,	**One,**	**Two,**	**Three,**	**Four,**	**Five,**	**Six,**	**Seven,**	**Eight,**	**Nine.**

9. The first figure is called *zero,* a *cipher*, or *naught;* standing alone, it signifies *nothing.* The remaining nine figures represent in order the numbers one, two, three, four, five, six, seven, eight, nine.

10. No number greater than *nine* can be expressed by a single figure, but by repeating the figures, and arranging them differently, all numbers may be represented.

11. *Ten* is expressed by writing the figure 1 at the left of the cipher; thus, 10. In like manner, twenty, thirty, forty, etc., are expressed by placing 2, 3, 4, etc., at the left of 0; thus,

20,	30,	40,	50,	60,	70,	80,	90.
Twenty,	Thirty,	Forty,	Fifty,	Sixty,	Seventy,	Eighty,	Ninety.

12. The numbers from 10 to 20 are expressed by placing the figure 1 at the left of each of the ten figures except zero; thus,

11,	12,	13,	14,	15,	16,	17,	etc.
Eleven,	Twelve,	Thirteen,	Fourteen,	Fifteen,	Sixteen,	Seventeen,	etc.

In a similar manner all the numbers up to one hundred may be written; thus,

21,	36,	66,	98,	etc.
Twenty-one,	Thirty-six,	Sixty-six,	Ninety-eight,	etc.

13. One hundred is expressed by placing the figure 1 at the left of *two ciphers;* thus, 100. In like manner two hundred, three hundred, etc., are written; thus,

200,	300,	600,	800,	etc.
Two hundred,	Three hundred,	Six hundred,	Eight hundred,	etc.

14. The other numbers up to one thousand may be expressed by putting another figure in the place of one, or in the place of each, of the two ciphers; thus,

Two hundred three, expressed in figures, is 203,
Six hundred eighty, expressed in figures, is 680,
Nine hundred ninety-eight, expressed in figures, is 998.

15. The **place** of a figure in a number is the position it occupies with reference to other figures; thus, in 436, the 6, counting from the right, is in the *first* place, 3 is in the *second* place, and 4 in the *third* place.

The figure in the *first place* represents *simple units;* the *second* figure represents *tens;* the *third, hundreds;* the *fourth, thousands,* etc.; thus, in the number 3576, the 6 represents 6 units; the 7, 7 tens; the 5, 5 hundreds, etc.

16. Therefore, to determine the value of a number expressed in figures, two things must be considered: first, *how many* units each figure represents; and, second, the *place* of each figure. Thus, in each of the numbers 2, 20, 200, the left-hand figure is two, but in the first it represents two units, in the second, two tens, and in the third, two hundreds.

17. *It is also evident that a figure is made to represent units of tenfold value by removing it one place towards the left; a hundred fold by removing it two places; a thousand fold by removing it three places, etc.;* that is, ten units make one ten, ten tens make one hundred, ten hundreds make one thousand, and so on.

18. The *cipher,* when used with other figures, fills a place that would otherwise be vacant; thus, in 206 the cipher occupies the place of *tens,* because there are *no tens* in the given number.

19. As figures are written on the left of the unit figure, so they are written on the right and are subject to the same law; that is, *ten units in any place make one unit in the next place to the left.*

20. Figures on the right of units are called *decimals*, and are separated from the unit figure by a period, which is called the decimal point. The first place at the right of the point is *tenths*, the second, *hundredths*, the third, *thousandths*, and so on. Thus, 1.1 is one, and one tenth; 1.01 is one, and one hundredth; 1.001 is one, and one thousandth; and so on.

21. The following table shows the method of writing numbers and the names of the places.

NUMERATION TABLE.

Hundred Billions,	Ten Billions,	Billions,	Hundred Millions,	Ten Millions,	Millions,	Hundred Thousands,	Ten Thousands,	Thousands,	Hundreds,	Tens,	Units.		Tenths,	Hundredths,	Thousandths.
7	0	6,	4	7	6,	0	0	1,	8	4	3	.	5	4	8
4th period, Billions.			3d period, Millions.			2d period, Thousands.			1st period, Units.				Decimals, Thousandths.		

22. For convenience in reading, figures are often separated into periods of three figures, as shown in the table. The first period on the right of the decimal point contains tenths, hundredths, and thousandths, and is called the *thousandths'* period; the first period at the left of the decimal point contains units, tens, and hundreds, and is called the *units'* period; the second period contains thousands, tens of thousands, and hundreds of thousands, and is called the *thousands'* period; and so on, as shown in the table.

NOTE. This table can be extended to any number of periods, as shown in the Appendix.

23. The value of the figures in the table, expressed in words, is seven hundred six billion four hundred seventy-six million one thousand eight hundred forty-three, and five hundred forty-eight thousandths.

24. To read figures at the left of the decimal point,

Rule.

1. Beginning at units, numerate and point off the number into periods of three figures each.

2. Beginning at the left, read each period separately, giving the name of each period except that of units.

To read figures at the right of the decimal point,

Rule.

Read the figures at the right of the point the same as figures at the left, and then add the name of the place of the right-hand figure.

Thus, 0.6 is six *tenths;* 0.65 is sixty-five *hundredths;* 0.659 is six hundred fifty-nine *thousandths.*

NOTE. To avoid ambiguity in writing in words and in reading a decimal we shall use the conjunction *and* in such cases only between the unit and the decimal. Thus, 0.203 is not two hundred *and* three thousandths, but two hundred three thousandths; and 200.003 is two hundred, and three thousandths.

Or, the word *decimal* may be written before the decimal. Thus, 306.205 is three hundred six, and decimal two hundred five thousandths (or decimal two, naught, five).

Exercises in Numeration.

25. Read the following numbers:

1.	3.	7.	400306.
2.	35.	8.	9400500.
3.	467.	9.	2367842.
4.	8783.	10.	27654984.
5.	19347.	11.	451632758.
6.	876251.	12.	6325478953.

13.	0.7	17.	2.8	21.	373.
14.	0.18	18.	34.73	22.	37.3
15.	0.756	19.	376.854	23.	3.73
16.	3.333	20.	909.909	24.	0.373

26. To write numbers,

Rule.

Beginning at the left, write the figures of each successive period in their order, filling each vacant place with a cipher and placing the decimal point between units and tenths.

NOTE. Where there are no decimal figures the decimal point is usually omitted.

Exercises in Notation and Numeration.

27. Write the following numbers in figures:

25. Two hundred five. Ans. 205.

NOTE. Since no figure of the second order is given, a *cipher* is written in the second place.

26. Six hundred thirty-eight.

27. Three hundred fifty-six.

28. Six hundred fifty-three.

29. Five hundred sixty-three.

30. Three hundred sixty-five.

31. Six hundred fifty-one.

32. One thousand six hundred fifty-one.

33. Forty-two thousand five hundred fifty-four.

34. Eight hundred sixteen thousand two hundred.

35. Six million one hundred four thousand two hundred seventy-six.

36. Three hundred six thousand five hundred two.

37. Nine hundred forty-six million five hundred fourteen thousand nine hundred twenty-five.

38. Six billion fifteen million seven thousand four hundred.

39. Five million six hundred fifty-one thousand four hundred six.

40. Seventy-four million.

41. Sixty-three million fourteen thousand seven hundred.

42 Two, and five tenths.

43. Fifty-two hundredths.

44. Sixty, and four hundredths.

45. Two hundred four thousandths.

46. Eight hundred, and fourteen thousandths.

47. Four million sixteen thousand seven, and four hundredths.

48. Seventeen million seventeen thousand seventeen, and seventeen thousandths.

49. One billion one million one thousand one hundred, and one hundredth.

50. Eleven billion eleven thousand, and eleven thousandths.

51. Sixteen million six thousand six hundred, and six hundredths.

52. One billion one million one thousand one, and one hundredth.

53. Ten billions ten millions ten thousand ten, and one tenth.

54. Five hundred fifty-five thousand five hundred fifty-five, and fifty-five thousandths.

55. Sixteen hundred and sixteen hundredths.

56. Two thousand twenty, and two hundredths.

28. The Roman Numerals (so called because used by the Romans),

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
1,	2,	3,	4,	5,	6,	7,	8,	9,	10,

and so on, are used on the faces of clocks and watches, and in numbering chapters, sections, and the like. For a full explanation of the Roman notation, see the Appendix, page 333.

ADDITION.

29. 1. If one man gives me 5 pears and another gives me 4 pears, how many pears do I have?

2. John gives to William 7 marbles, to James 4, and to Edward 5; how many marbles does John give away?

30. The *sum* or *amount* of two or more numbers is a number which contains the same number of units as the two or more numbers together; thus, 7 is the sum of 3 and 4, because there are just as many units in 7 as in 3 and 4 together; for a like reason 11 *days* is the sum of 2 days, 4 days, and 5 days.

31. *Only numbers of the same kind can be added;* thus, 3 books and 4 books are 7 books; but 3 hats and 4 books are neither 7 hats nor 7 books.

32. **Addition** is the process of finding the sum of two or more numbers of the same kind.

33. A **Sign** is a mark which indicates an operation to be performed, or which is used to shorten some expression.

34. The *sign of dollars* is written thus, $; thus, $2 represents *two dollars;* $10, *ten dollars,* etc.

35. The *sign of equality,* =, signifies that the quantities between which it stands are equal to each other; thus, $1 = 100 cents, or, one dollar equals one hundred cents.

36. The *sign of addition,* +, called *plus,* denotes that the quantities between which it stands are to be added together; thus, $3 + 2 = 5$, that is, three plus two equals five, or, three and two are five.

37. Oral Exercises.

3. James paid $4 for a pair of skates, $3 for a sled, and $1 for a knife; what did he pay for all?

4. What is the sum of \$6 and \$3? $\$5 + \$3 + \$7 = ?$

5. What is the sum of $4 + 6 + 2 + 3$? $3 + 5 + 8 + 2 = ?$

6. I bought 7 quarts of strawberries of one man, 4 quarts of another, and 5 of another. How many quarts did I buy?

7. John had 8 cents, Peter 9, and Henry 7. How many cents have the three boys?

8. $6 + 5 = ?$	$7 + 8 = ?$	$4 + 7 = ?$
$3 + 4 = ?$	$2 + 9 = ?$	$8 + 6 = ?$
$5 + 3 = ?$	$9 + 5 = ?$	$6 + 8 = ?$

9. $1 + 2 + 3 = ?$	$2 + 3 + 4 = ?$	$3 + 4 + 5 = ?$
$4 + 5 + 6 = ?$	$5 + 6 + 7 = ?$	$6 + 7 + 8 = ?$
$7 + 8 + 9 = ?$	$2 + 4 + 6 = ?$	$3 + 5 + 7 = ?$

10. $1 + 2 + 3 + 4 = ?$	$2 + 3 + 4 + 5 = ?$
$3 + 4 + 5 + 6 = ?$	$4 + 5 + 6 + 7 = ?$
$5 + 6 + 7 + 8 = ?$	$6 + 7 + 8 + 9 = ?$
$1 + 3 + 4 + 7 = ?$	$3 + 5 + 7 + 9 = ?$

11. Add to fifty by twos, beginning with 0; with 1.

12. Add to fifty by threes, beginning with 0; with 1; with 2.

13. Add to sixty by fours, beginning with 0; with 1; with 2, with 3.

14. Add to sixty by fives, beginning with 0; with 1; with 2; with 3; with 4.

15. Add to sixty by sixes, beginning with 0; with 1; with 2; with 3; with 4; with 5.

16. Add to seventy by sevens, beginning with 0; with 1; with 2; with 3; with 4; with 5; with 6.

17. Add to eighty by eights, beginning with 0; with 1; with 2; with 3; with 4; with 5; with 6; with 7.

18. Add to ninety by nines, beginning with 0; with 1; with 2; with 3; with 4; with 5; with 6; with 7; with 8.

19. I paid 12 cents for butts for a screen door, 4 cents for

screws, 6 cents for a knob, and 3 cents for a hasp; what did the trimmings cost me?

20. For my breakfast I bought 5 cents' worth of crackers, 3 cents' worth of berries, and 4 cents' worth of milk; what did my breakfast cost me?

21. Bought 2 quarts vinegar for 24 cents, 1 box ginger for 12 cents, a pound of cream tartar for 35 cents, 6 eggs for 12 cents; how many cents must be paid to settle the bill?

22. Find the sum of 24, 17, 23, and 13.

Add by 10's; thus, 24 and $17 = 24 + 10 + 7$; say 24, 34, 41. 41 and $23 = 41 + 10 + 10 + 3$; say 41, 51, 61, 64; or, say 41, 61, 64, adding the 20 at once. 64 and $13 = 64 + 10 + 3$; say 64, 74, 77. To add the numbers given, then, say 24, 34, 41 (looking at 17), 61, 64 (looking at 23), 74, 77 (looking at 13). Ans. 77.

23. Find the sum of 43, 18, 27, and 35.

24. On my bill I find 6 lemons 15 cents, 7 pounds of beef 98 cents, a cabbage 10 cents, turnips 5 cents, a half peck of beans 15 cents; for these how much do I owe?

Begin with the 98, the largest number; thus, 98, 108, 113, etc.

25. $83 + 13 + 17 + 24 = ?$

26. $77 + 16 + 12 + 15 = ?$

27. $125 + 11 + 19 + 21 = ?$

38. Exercises for Written Work.

28. A manufacturer sold 125 yards of cloth to one merchant, 342 to another, and 231 to another; how many yards did he sell in all?

$$\begin{array}{r} 125 \\ 342 \\ 231 \\ \hline \text{Sum, } 698 \end{array}$$

As we add units to units, tens to tens, etc., we write the units under the units, the tens under the tens, and the hundreds under the hundreds.

Adding the units, thus, 1 (and 2 are) 3, (and 5 are) 8, we write the 8 units under the units' column; then adding the tens, thus, 3 (and 4 are) 7, (and 2 are) 9,

we write the 9 tens under the tens' column, and so proceed, till all the columns are added.

In adding it is better not to name the numbers we are adding; thus, in Ex. 28, omit the words enclosed in parentheses in the explanation above.

(29.)	(30.)	(31.)	(32.)
134	712	610	3112
312	201	211	4123
542	314	134	2044

(33.)	(34.)	(35.)	(36.)
1345	2111	10120	1000
2231	4001	34210	1010
2423	2403	13234	2423
3000	3121	42321	5341

37. What is the sum of 4123, 1331, and 2211?

38. What is the sum of 1123, 2112, 3201, and 2213?

39. A gentleman paid $ 125 for a horse, $ 231 for a carriage, and $ 32 for a harness; what did he pay for all?

40. Add together 27, 93, and 145.

	27
	93
	145
Ans.	265

Arranging the numbers as before, we add the units, thus, 5, 8, 15 units, or, 1 ten 5 units. The 5 units we write under the units' column, and add the 1 ten to the tens in the second column, thus, 1, 5, 14, 16 tens, or 1 hundred and 6 tens. The 6 tens we write under the tens' column, and add the 1 hundred to the hundreds in the third column, thus, 1, 2 hundreds, which we write under the hundreds' column.

41. What is the sum of 37.43, 567.8, 8.148, 917.767, and 0.09?

	37.43
	567.8
	8.148
	917.767
	0.09
Ans.	1531.235

Writing the numbers with units of the same order in the same column, we add the units of the right-hand column, thus, 7, 15; writing the 5 of the 15 in its place, we add the 1 to the next column, thus, 1, 10, 16, 20, 23; writing the 3 of 23 in its place, we add the 2 to the next column, thus, 2, 9, 10, 18, 22; and so on, precisely as in Ex. 40, remembering also to place the decimal point directly under the points in the numbers added.

Hence,

39. To add numbers,

Rule.

Write the numbers in order, units under units, tenths under tenths, etc., and draw a line beneath. Add together the figures in the right-hand column, and write the units of this sum directly under this column, and the tens of this sum, if there are any, add to the units of the next column. Thus proceed till all the columns are added, writing the whole sum of the last column, and place the decimal point directly under the points in the numbers added.

40. Proof. *Add the columns in the opposite direction, and if the work is right, the two sums will be alike.*

Note 1. By this process, we *combine* the numbers *differently*, and hence are likely to detect any mistake which may have been made in the first addition.

	(42.)
	37684
	48297
	68746
	94852
Sum,	249579
Proof,	249579

In adding the first column *upward* we say, 2, 8, 15, 19; but in adding *downward*, we say, 4, 11, 17, 19; thus obtaining the *same result*, but by *different combinations*.

If we do not obtain the same result by the two methods, one operation or the other is wrong, perhaps both, and the work must be carefully performed again.

(43.)	(44.)	(45.)	(46.)
627	4.87	6841	10.246
438	4.29	7623	76.348
784	8.83	4288	82.403

(47.)	(48.)	(49.)	(50.)
432	426	219.5	27
147	924	138.4	5814
217	247	738.4	48237
189	538	482.8	61829
684	196	800.6	76048

(51.)	(52.)	(53.)	(54.)
64	0.28	273	7
4352	64.04	7648	843
43627	394.24	69458	64398

55. Add 6.24, 2.8, 7.364, 4.69, and 31.485.
56. Add 23, 432, 4869, 87246, and 42867.
57. Add 3, 8.642, 7.9, 268.4, 894.72, and 8030.
58. Add 43, 78, 438, 7432, 2964, and 94216.
59. Add 435.075, 21.007, 0.035, and 2000.02.
60. Add 7.06, 18.2, 4.17, 0.006, and 13.
61. Add 134.008, 2.1, 144.35, and 147.

NOTE 2. In United States money the dollar is the unit, and as 100 cents make a dollar, and 10 mills a cent, cents occupy two decimal places, tenths and hundredths, and mills one place, thousandths. Thus, $42.255 is 42 dollars and twenty-five hundredths of a dollar and five thousandths of a dollar, or 42 dollars 25 cents and 5 mills.

62. Add $ 26.37, $ 137.55, $ 215.445, and $ 0.155.

63. What is the sum of 8432, 42698, 34, 1892, 70068, 5142, and 68742?

64. What is the sum of $24.68, $135.79, $3.76, and $42.55?

65. What is the sum of 34.06, 87.2, 6.541, 2, and 1.7?

66. What is the sum of 3910, 4, 876, 27, and 89462?

(67.)	(68.)	(69.)	(70.)
10500	42040	53211	$13.110
61081	34446	50002	62.222
28003	30409	10050	66.626
13294	90000	55555	55.555
82183	90005	45544	33.333

71. How many are 687 + 2987 + 42698 + 7070?

72. How many are 936.04 + 61.2 + 49.82 + 7.4?

73. How many are 18 + 4 + 76984 + 327 + 14?

74. 927 + 468 + 300 + 48 = ?

75. 846.2 + 7.679 + 7.28 + 28.437 = ?

76. 3004 + 4906 + 76842 + 36 = ?

77. 4682 + 19735 + 100 + 6402 + 178 + 19 = ?

78. Add four hundred sixty-two; three thousand two hundred fourteen; seventy-nine thousand six hundred fifty-nine; and two hundred eighty-four.

79. Add five hundred and decimal six thousandths; forty-five thousandths; eighty-four million and decimal twelve hundredths; and decimal thirty-five thousandths.

80. What is the sum of one thousand two hundred, and one hundred forty-four thousandths; two hundred five, and sixteen thousandths; and eight hundred forty-nine, and six hundredths?

81. What is the sum of fifty, and five thousandths; four hundred, and four hundredths; and seventy, and seventeen thousandths?

(82.)

$ 876.43
785.32
176.78
987.63
857.16
———
22
21
31
35
33
———
$ 3683.32

In Ex. 82 we write the sum of the units of each column in its proper place beneath, and then add these sums. In adding long columns this is an excellent way. It often saves adding a column a second or third time after interruption, or to find how much belongs to the next left-hand column.

83. Add 4670, fourteen thousand five, six million seven hundred four, 87637, 76504, fifteen thousand ten, seven hundred thousand four hundred one, and 237.

84. A man bought a horse for $ 250, a carriage for $ 175, a harness for $ 74.50, a whip for $ 1.25, a carriage blanket for $ 3.45. What did he pay for all?

85. If I pay $ 33.50 for a coat, $ 8.50 for a pair of pantaloons, $ 5.35 for a vest, $ 3.75 for a hat, $ 4 for a pair of boots, and $ 6.85 for other articles, what do I pay for all?

86. In building a house, the digging of the cellar cost $ 67; the stone and the laying of it for the cellar, $ 403; the granite trimming at the base of the house, $ 172; the brick and masons' work, $ 2257; the stone trimmings, $ 306; the window-frames, $ 157; the windows, $ 196; the door-frames, $ 98; the doors, $ 175; the lumber for frame, etc., $ 603; the carpenters' work, $ 1757; sundry other expenses, plumbing, furnace, gas-pipes, slating, etc., $ 1417: find the cost of the house.

87. If the population of North America is 89250000; of South America, 36420000; of Europe, 380200000; of Asia, 850000000; of Africa, 163953000; and of Australasia, 4730000: what is the population of the globe?

88. The cost of the American army for five successive years, beginning in 1812, was \$ 12187046, \$ 19906362, \$ 20608366, \$ 15394700, and \$ 16475412; what was the cost for five years?

89. B owes to C \$ 150, to D \$ 4682, to E \$ 267, to F \$ 54, and to G \$ 1353; how much does he owe?

90. Add four hundred, and fifty-six hundredths; eight thousand four hundred, and seventy-two thousandths; fifteen thousand seven hundred, and twenty-one hundredths; forty-three million seven hundred thirty-three thousand eight hundred, and fifty-nine thousandths; and ten.

91. A merchant beginning business had in cash, \$ 4376.75; goods worth \$ 3780.37; bank stock worth \$ 2700; and other property valued at \$ 5496.55. In a year he gained \$ 2475.17; what was he worth at the end of the year?

92. In one year a farmer sold a pair of oxen for \$ 125.83, two cows for \$ 75.47, three swine for \$ 96.75, twenty sheep for \$ 120, and a horse for \$ 156; what did he receive for all?

93. On Monday a merchant sold goods for \$ 357.15, on Tuesday for \$ 463.87, on Wednesday for \$ 279.19, on Thursday for \$ 318.67, on Friday for \$ 687.27, and on Saturday for \$ 348.48; what was the value of the goods sold during the week?

94. A man dying left by his will, to his widow \$ 3784, to his only son \$ 2578, to the elder of his two daughters \$ 2100, to the younger \$ 1700, and for charitable objects the balance of his estate, \$ 315. Find the value of the man's property.

95. The area of Maine is 33040 square miles; New Hampshire, 9305; Vermont, 9565; Massachusetts, 8315; Rhode Island, 1250; Connecticut, 4990. What is the area of New England?

96. England and Wales contain about 58000 square miles; Scotland, 30000; and Ireland, 33000. What is the area of the British Islands?

97. In 1890 the population of Maine was 661086; of New

Hampshire, 376530; of Vermont, 332422; of Massachusetts, 2238943; of Connecticut, 746258; of Rhode Island, 345506. What was the population of these six States in 1890?

98. According to the latest authorities, the population of the following cities in even thousands is, London, 4231000; Paris, 2447000; Pekin, 1000000; Canton, 1600000; Vienna, 1364000; New York, 1891000; Tokio, 1389000. What is the population of these seven cities together?

(99.)	(100.)	(101.)	(102.)
65990	91998	95905	78382
86895	65859	17989	35902
98121	82981	85182	57575
55555	37129	73997	97572
26717	39333	55004	11111
47624	47048	56766	88888
42914	70486	47689	55764
59002	79864	52512	90632

(103.)	(104.)	(105.)	(106.)
82784	69588	74242	24165
61984	90954	62855	40050
17442	69598	28679	82827
99225	46569	71212	40025
20872	89025	86246	27286
50216	67922	68212	62272
78217	26792	42779	62651
42257	72615	72524	46552
52124	26797	65927	72482
14724	30177	17215	10475

107. In January there are 31 days, in February 28, in March 31, in April 30, in May 31, in June 30, in July 31, in August 31, in September 30, in October 31, in November 30, and in December 31; how many days are there in a year?

108. A gardener has 3476 apple-trees, 8476 pear-trees, 5684 peach-trees, 1845 plum-trees, 4680 quince-trees, and 9487 ornamental trees; how many trees are there in his nursery?

109. The first of three numbers is 4768, the second 8942, and the third is as much as the other two; what is the sum of the three numbers?

110. I have $ 376.25 in one bank, $ 4678.85 in another, and in another as much as in both of these; how much money have I in the three banks?

111. An army consists of 276450 infantry, 14875 cavalry, and 27846 artillery men; what is the number of men in the army?

112. A carpenter engaged to build 4 houses, the first for $ 3462, the second for $ 6875, the third for $ 8963, and the fourth for $ 12462; what shall he receive for the four houses?

113. During the year ending June 30, 1878, the value of the issue of ordinary postage stamps by the Post-Office Department amounted to $ 19468618; of newspaper stamps, to $ 1093845; of stamped envelopes and wrappers, to $ 4905774; and of postal cards, to $ 2000630. Find the total value of these items.

114. During the year ending June 30, 1894, the value of the issue of ordinary postage stamps by the Post-Office Department amounted to $31557223; of newspaper stamps, to $2613920; of stamped envelopes and wrappers, to $4322328; and of postal cards, to $4728275. Find the total value of these items.

115. I deposit in a savings bank in January, $317.14; in February, $223.85; in April, $408.25; in June, $143.19; in September, $217.18; and in November, $518.79. What is the sum of these deposits?

116. A locomotive ran on Sunday 325 miles; on Monday, 223 miles; on Tuesday, 372 miles; on Wednesday, 318 miles; on Thursday, 217 miles; on Friday, 225 miles; and on Saturday, 315 miles. How many miles did the locomotive run during the week?

SUBTRACTION.

42. 1. Arthur had 7 peaches, but he has given 3 of them to John; how many peaches has he left?

2. James had 9 cents, and spent 5 cents; how many cents did he have left?

43. *Only numbers of the same kind can be subtracted from each other;* thus, we can take 5 books from 8 books; but we cannot take 5 books from 8 marbles.

44. **Subtraction** is the process of finding the *difference* between two numbers of the same kind.

45. The greater number is called the **Minuend**; the less number is called the **Subtrahend**; and the result is called the **Difference** or **Remainder.**

46. The *sign of subtraction*, $-$, called *minus*, signifies that the number after it is to be taken from the number before it; thus, $7 - 4 = 3$, that is, seven minus four, or seven diminished by four, equals three.

47. Oral Exercises.

3. If I give away 5 oranges from 9 which I have, how many shall I have left?

4. If my father gives me 10 cents and I lose 6 of them, how many shall I have left?

5. If Mr. Russell buys 7 liters of peas and sells from these 4 liters, how many liters will he have left?

6. A merchant has 19 meters of cloth and sold 8 meters; how many meters had he left?

7. John had 27 cents, but gave 8 to James, and then 6 to Arthur; how many cents did he have left?

8. Subtract by twos from 50 to 0; from 49 to 1.

9. Subtract by threes from 50 to 2; from 49 to 1; from 48 to 0.

10. Subtract by fours from 50 to 2; from 49 to 1; from 48 to 0; from 47 to 3.

11. Subtract by fives from 50 to 0; from 49 to 4; from 48 to 3; from 47 to 2; from 46 to 1.

12. Subtract by sixes from 50 to 2; from 49 to 1; from 48 to 0; from 47 to 5; from 46 to 4; from 45 to 3.

13. Subtract by sevens from 50 to 1; from 49 to 0; from 48 to 6; from 47 to 5; from 46 to 4; from 45 to 3; from 44 to 2.

14. Subtract by eights from 50 to 2; from 49 to 1; from 48 to 0; from 47 to 7; from 46 to 6; from 45 to 5; from 44 to 4; from 43 to 3.

15. Subtract by nines from 50 to 5; from 49 to 4; from 48 to 3; from 47 to 2; from 46 to 1; from 45 to 0; from 44 to 8; from 43 to 7; from 42 to 6.

16. If I buy 17 cents' worth of sugar, 33 cents' worth of tea, and 42 cents' worth of flour, and hand to the vender two fifty-cent pieces, how much change ought he to give me?

17. Mr. B. owes me 47 cents and I owe him 73 cents; how shall we settle?

18. If a horse is bought for $ 250, and sold for $ 325, what is the gain?

19. If of 76 chickens 27 are caught by foxes, how many are left?

20. From a piece of sheeting containing 44 yards, there are

sold to one man 8, to another 9, to another 7; how many yards are left?

21. Mr. Ames owed his neighbor \$ 57; Mr. Ames brought to pay the bill \$ 4 worth of eggs, \$ 6 worth of butter, \$ 8 worth of potatoes, \$ 21 worth of wood, and a twenty-dollar bill; how did the account stand then?

48. Exercises for Written Work.

22. From 796 take 582.

Minuend,	796
Subtrahend,	582
Remainder,	214

Write units under units, tens under tens, etc., as in addition. Then 2 units from 6 units leave 4 units, which we write under the units' column; 8 tens from 9 tens leave 1 ten, which we write under the tens' column; 5 hundreds from 7 hundreds leave 2 hundreds, which we write under the hundreds' column. The remainder then is 2 hundreds, 1 ten, 4 units, or 214.

	(23.)	(24.)	(25.)	(26.)
Minuend,	469	5642	9884	8072
Subtrahend,	327	4130	3623	3051
Remainder,	142			

	(27.)	(28.)	(29.)	(30.)
From	2741	5462	6408	8420
Take	1301	1350	3207	3110

31. A farmer bought a farm for \$ 4875 and sold it again for \$ 3463; how much did he lose by the transactions?

32. By the census of 1890 the population of Maine was 661086, and that of New Hampshire was 376530. How many more people were there in Maine than in New Hampshire?

33. If I borrow \$4687 and afterwards pay \$2423, how much do I still owe?

34. From 483 take 257.

Minuend,	483
Subtrahend,	257
Remainder,	226

As we cannot take 7 units from 3 units, *one* of the 8 *tens* is put with the 3 units, making 13 units, and then, 7 units from 13 units leave 6 units. Now as *one* of the 8 *tens* has been put with the 3 *units*, only 7 tens remain in the minuend, and 5 tens from 7 tens leave 2 tens, and, finally, 2 hundreds from 4 hundreds leave 2 hundreds; hence, the entire remainder is 2 hundreds, 2 tens, 6 units, or 226.

35. From 256.17 take 187.245.

	256.17
	187.245
Ans.	68.925
Proof,	256.17

Writing units of the same order in the same column, as there are no thousandths in the minuend, we take of the 7 hundredths 1 hundredth, = 10 thousandths; then, 5 thousandths from 10 thousandths leave 5 thousandths; 4 from (7 — 1, or) 6 leave 2; and so on, exactly as in Ex. 34, remembering also to place the decimal point directly under the points in the minuend and subtrahend.

49. Hence, to perform subtraction,

Rule.

1. Write the less number under the greater, units under units, tenths under tenths, etc., and draw a line beneath.

2. Beginning at the right hand, take each figure of the subtrahend from the figure above it, and place the remainder under the line, with the decimal point directly under the decimal points in the minuend and subtrahend.

3. If any figure in the subtrahend is greater than the figure above it, add ten to the upper figure and take the lower figure from the sum; write down the remainder and, considering the next figure in the minuend **one less,** *proceed as before.*

50. PROOF. *Add the subtrahend and the remainder together, and the sum should be the minuend.*

NOTE. This proof rests upon the self-evident truth, that *the whole of a thing is equal to the sum of all its parts;* thus, the *minuend* is separated into the *two parts, subtrahend and remainder; hence* the *sum* of those parts *must be* the *minuend.*

	(36.)	(37.)	(38.)
Minuend,	9875	732.659	8574826
Subtrahend,	265	378.492	4326169
Remainder,	9610		
Proof,	9875		

	(39.)	(40.)	(41.)
From	618.724	8540672	68492.37
Take	529.728	7230498	48187.64

	(42.)	(43.)	(44.)
From	347682	8387086	7777.777
Take	264327	4206040	6665.696

(45.)

	(5)	(9)	(12)
Minuend,	6	0	2
Subtrahend,	4	3	8
Remainder,	1	6	4

Here we cannot take 8 from 2, nor can we take from the tens' place, as there is 0 there; but we can take *one* of the 6 *hundreds* and separate the one hundred into 9 *tens* and 10 *units*; then, putting the 9 tens in the place of tens and adding the 10 units to the 2 units, we can subtract 8 from 12, 3 from 9, and 4 from 5.

	(46.)	(47.)	(48.)
From	7082	3004	800700
Take	4265	1876	284723

	(49.)	(50.)	(51.)
From	4.692	97.424	69.04
Take	2.968	81.621	18.274

52. From 704 take 586.

53. From 876.4 take 32.89.

54. From 72068 take 36819.

55. Subtract 3247 from 8604.

56. Subtract 610.847 from 9476.728.

57. Subtract 2376948 from 9000000.

58. Take 78.604 from 754.29.

59. Take 2879046 from 9287406.

60. How many are 4236 less 1684 ?

61. How many are 98647 less 52046 ?

62. $74.98 - 43.68 = ?$

63. $8274 - 4293 = ?$

64. The difference between two numbers is 536, and the greater number is 768; what is the less ?

65. What number added to 8763 will give 9764 ?

66. What number taken from 7284 leaves 4378 ?

67. From 875.8 take 87.886.

68. From 8948.71 take 2981.7814.

69. From $ 257.57 take $ 175.89.

70. From $ 7788.40 take $ 5999.99.

71. From 8654.78 take 334.986.

72. From 4168.946 take 683.78.

73. From 8469.762 take 2754.892.

74. From 4021.876 take 2174.941.

75. From one thousand eight hundred seventy-six, and decimal three hundred sixty-four thousands, take eight hundred sixteen, and decimal three hundred three thousandths.

76. From ten take six thousandths.

77. A man owned eighty-seven hundredths of a railroad and sold forty-eight hundredths of it; what part of the road did he still own ?

78. If a man owes $ 87.55 and pays $ 69.78, how much does he still owe?

79. If a man sells his farm for $ 2763, and pays $ 1975.55 for a house and garden in the village, how much of this money has he left?

80. Of 1368 soldiers in a regiment, 919 are under 30 years of age. How many are over 30 years of age?

81. Above the junction of the Missouri and Mississippi Rivers, the Missouri drains 518000 square miles and the Mississippi 169000 square miles. How many more square miles, above their junction, does the Missouri drain than the Mississippi?

82. From five million eight thousand four, and four thousandths, take three hundred fifteen thousand eight hundred six, and seven hundredths.

83. How many years from the discovery of America, 1492, to the birth of Washington, 1732?

84. If a merchant commences business with $ 3575.44 and retires with $ 10847.13, how much does he make?

85. The number of inhabitants in Massachusetts in 1890 was 2238943, and in Vermont, 332422. How many more in Massachusetts than in Vermont?

86. If a man sells a factory which cost him $9784.85 for $13378.25, what does he gain?

87. In 1890 the population of the United States was 62622250, and in 1840 it was 17063353. What was the increase in 50 years?

88. A borrowed of B $ 876.55, and paid $ 598.78; how much remained due?

89. Subtract 4987.54 from 18745.6.

90. Subtract $ 253.97 from $ 431.23.

91. Take 217.657 from 413.2.

92. Take $ 874.23 from $ 1000.

93. How many are 8764 — 7985?

94. How many are 234.56 — 87.678?

95. The distance of the sun from the earth is about 93000000 miles, and of the moon, 240000. How much farther from the earth is the sun than the moon?

96. The area of Texas is 265780 square miles, and of the six New England States, 63465 square miles. Find by subtraction how many sections as large as New England can be cut out of Texas.

51. Miscellaneous Examples.

97. In the Auburn (N. Y.) State Prison there were at one time 1368 prisoners. Of these 51 had had an academical education, 16 a collegiate, 12 a high-school, 646 a common-school, 270 could only read and write, 187 could read only, and the rest had had no education. How many had had no education?

98. From their junction to their sources the Missouri River has a length of 3047 miles, and the Mississippi 1330 miles, and from their junction to the Gulf of Mexico it is 1656 miles. How much longer is the Missouri from its source to the Gulf than the Mississippi?

99. Bought a barrel of sugar for \$9.50, four yards of cloth for \$2.75, 40 pounds of sugar for \$2.15, a gallon of syrup for \$0.96, and 2 dozen of eggs for \$0.48, and gave in payment a ten and two five dollar bills. How much change ought I to receive back?

100. The area of Maine is 33040 square miles; New Hampshire, 9305; Vermont, 9565; Massachusetts, 8315; Rhode Island, 1250; Connecticut, 4990. How much less is Maine than the other five States together?

101. If I am 783 miles from home Monday morning, and travel toward home 137 miles Monday, 143 Tuesday, 153 Wednesday, 148 Thursday, how many miles from home shall I be Thursday night?

102. If I have on deposit in one savings-bank \$984.43, in another \$876.84, and in another \$698.17, how much more must I borrow to enable me to build a house worth \$4375?

103. If 87654 is the sum of three numbers of which two are 17896 and 25683, what is the other number?

104. Mr. Roe has cash to the amount of \$276.87, but owes Mr. Doe \$36.44, Mr. Lowe \$137.15, and Mr. Poe \$87.18. After paying these debts, how much will Mr. Roe have left?

105. Four men, A, B, C, and D, beginning business together, furnished money as follows: A, \$2475; B, \$3475; C, \$2850; and D, \$4500. At the end of a year they closed business, having lost \$3225; how much money had they to divide between them?

106. During the year ending June 30, 1877, the value of the issue of ordinary postage stamps by the Post-Office Department was \$18181676; of newspaper stamps, \$1000605; of stamped envelopes and wrappers, \$4616932; and of postal cards, \$1700155; while the total value of the issue of the following year was \$27468867. Find the excess of the value of the issue of the year ending June 30, 1878, over that of the year ending June 30, 1877.

107. During the year ending June 30, 1893, the value of the issue of ordinary postage stamps by the Post-Office Department amounted to \$54379936; of newspaper stamps, to \$2850324; of stamped envelopes and wrappers, to \$12765676; of postal cards, to \$5444288. Find the excess of the value of the issue for the year ending June 30, 1893, over that of the year ending June 30, 1894, as given in Example 114, page 18.

108. Find the excess of the value of the issue of stamps, wrappers, etc., by the Post-Office Department for the year ending June 30, 1893, as given in Example 107, over that of the year ending June 30, 1877, as given in Example 106.

MULTIPLICATION.

52. 1. There are 7 days in 1 week; how many days in 4 weeks?

This example may be solved by addition; thus, $7+7+7+7=28$; or, more briefly, by multiplication; thus, 4 times 7 are 28, Ans.

53. **Multiplication** is the process of finding how many units there are in any number of times a given number.

54. The **Multiplicand** is the number to be repeated.

The **Multiplier** is the number which shows how many times the multiplicand is to be taken.

The **Product** is the *result* of the multiplication.

The *Multiplicand* and *Multiplier* are called **Factors.**

55. The *sign of multiplication,* $\times$, signifies that the two numbers between which it stands are to be multiplied together; thus, $6 \times 5 = 30$, that is, six multiplied by five equals thirty; or, five times six are thirty.

A Parenthesis, ( ), or **Vinculum,** ——, indicates that all the quantities included are to be considered as a single quantity. Thus, $27-(8+4) \times 2$, or $27-\overline{8+4} \times 2$, means that two times the sum of 8 and 4, or 24, is to be subtracted from 27.

$$27-(8+4) \times 2 = 3.$$

The effect of a sign of multiplication (or division) does not pass over the signs + or −. Thus, $18+4 \times 3 = 18 + 12$. It is not 22×3; but $(18+4) \times 3 = 22 \times 3$.

56. *The multiplier is always an abstract number.*

The product is of the same kind as the multiplicand.

In the example in Art. 52 the multiplier is 4, not 4 weeks. We cannot take 7 days 4 *weeks* times; but we take 7 days 4 times, that is, as many times as there are units in the number of weeks, and the answer is *days*, 4 times 7 days = 28 days.

57. The pupil before advancing further must learn the following

Multiplication Table.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

58. Oral Exercises.

2. How many are 8 times 3? 3 times 8? 6 times 4? 4 times 6? 7 times 7? 5 times 9?

3. How many are 9 times 7? 9 times 11? 8 times 6; 6 times 12? 12 times 6? 9 times 8?

4. If I deposit $10 a month in a savings bank, how many dollars shall I deposit in 4 months? In 7 months? In 5 months? In 12 months?

5. When wood is worth $6 a cord, what shall I pay for 3 cords? 5 cords? 8 cords? 11 cords?

6. In one year there are 12 months, how many months in 2 years? 4 years? 7 years? 12 years?

7. If I study 11 hours in a day, how many hours shall I study in 3 days? 5 days? 8 days? 11 days?

8. What are 7 tons of coal worth at $6 a ton?

9. If salt beef is worth 12 cents a pound, what must I pay for 9 pounds?

10. What will 7 melons cost at 8 cents each?

11. At 9 cents a yard, what will 6 yards of ribbon cost?

12. At $7 a cord, what will 11 cords of wood cost?

13. If flour is worth $9 a barrel, what must I pay for 12 barrels?

59. Exercises for Written Work.

14. In one bushel there are 32 quarts; how many quarts in 6 bushels?

BY ADDITION.	BY MULTIPLICATION.
32	32
32	6
32	Product, 192
32	
32	
32	
Sum, 192	

In 6 bushels there are 6 times as many quarts as in 1 bushel, and the number of quarts in 6 bushels may be obtained by *adding*, as in the margin; or, more briefly, by *multiplying;* thus, 6 times 2 units are 12 units = 1 ten and 2 units; write the 2 units in the units' place, and then say 6 times 3 tens are 18 tens, which, increased by the 1 ten previously obtained, make 19 tens = 1 hundred and 9 tens, and these, written in the place of hundreds and tens respectively, give the true product.

	(15.)	(16.)	(17.)
Multiplicand,	327	7134	4064
Multiplier,	2	5	8
Product,	654	35670	32512

(18.)	(19.)	(20.)	(21.)
8423	6453	62934	47968
7	4	3	9
58961			

(22.)	(23.)	(24.)
36042	8747324	4204237
6	8	7
216252		29429659

25. How many quarts in 46 bushels?

Multiplicand,	32
Multiplier,	46
	192
	128
Product,	1472

First multiply by 6, as though 6 were the only figure in the multiplier; then multiply by 4, and place the first figure of this product in the place of *tens;* for, multiplying by the 4 *tens* is the same as multiplying by 40, and 40 times 2 units are 80 *units* = 8 *tens;* that is, the product of *units* by *tens* is *tens.* Having multiplied by each figure in the multiplier, find the *sum* of the *partial products* and this will be the *true product.*

NOTE. So much of the product as is obtained by multiplying the whole multiplicand by *one figure* of the multiplier is called *a partial product;* thus, in the 25th example, 192 is the first partial product and 128 *tens* is the second.

60. Multiplication with Decimals in the Multiplicand.

26. Multiply 783.24 by 7.

	783.24
	7
Ans.	5482.68

7 times 4 hundredths are 28 hundredths = 2 tenths and 8 hundredths; we write the 8 hundredths below the 7; then 7 times 2 tenths are 14 tenths, and the 2 tenths from the 28 hundredths added to it give 16 tenths = 1 unit and

6 tenths; we write the 6 tenths in its place and add the 1 unit to the product of 7 and 3 units, and so on, exactly as in Ex. 25, remembering also to place the decimal point between the units and tenths. This explanation shows that there will be the same number of decimal places in the product as in the multiplicand.

27. Multiply 18.783 by 75.

$$\begin{array}{rr} & 18.783 \\ & 75 \\ \hline & 93915 \\ & 131481 \\ \hline \text{Ans.} & 1408.725 \end{array}$$

61. From the preceding examples we derive for multiplication the following

Rule.

1. Write the multiplier under the multiplicand and draw a line beneath.

2. Beginning at the right hand of the multiplicand, multiply the multiplicand by each figure in the multiplier, writing the first figure of each partial product directly under the figure of the multiplier which produces it.

3. Add these partial products and point off as many decimal places in the product as there are in the multiplicand, and the result will be the true product.

62. PROOF. *Multiply the multiplier by the multiplicand, and, if correct, the product will be the same as the first product.*

NOTE 1. This proof rests on the principle, that the *order* of the factors is immaterial; thus, $3 \times 4 = 4 \times 3$; $5 \times 2 \times 7 = 2 \times 7 \times 5$.

NOTE 2. For multiplication with decimals in the multiplier, see Art. 161.

28. Multiply 5236 by 2413.

			PROOF.
Multiplicand,	5236		2413
Multiplier,	2413		5236
	15708		14478
	5236		7239
	20944		4826
	10472		12065
Product,	12634468	=	12634468

	(29.)	(30.)
Multiplicand,	2870364	91284743
Multiplier,	2462	97

(31.)	(32.)	(33.)	(34.)
83674	34275	6845	9573
45	432	643	426

35. How many are 726 × 27 ? Ans. 19602.

36. How many are 46.28 × 554 ? Ans. 25639.12

37. 3648 × 36 = how many ? Ans. 131328.

38. 4275 × 54 = how many ? Ans. 230850.

39. 3759 × 8463 = ? Ans. 31812417.

40. 346.52 × 36 = ?

41. 6402 × 452 = ?

42. 8627 × 364 = ?

43. 9.672 × 635 = ?

44. 4621 × 791 = ?

45. 42637 × 75 = ?

46. 93742 × 638 = ?

47. 40327 × 485 = ?

48. 86.003 × 124 = ?

49. 86501 × 978 = ?

50. If 32 men do a piece of work in 73 days, in how many days will 1 man do the same work ?

51. What is the value of 34 acres of land, at $ 37 per acre ?

52. If a horse can travel 43 miles per day, how far can he travel in 71 days?

53. How many yards of cloth in 67 pieces, if each piece contains 42 yards?

63. As removing a figure one place to the left multiplies it by ten, two places multiplies it by a hundred, etc., to multiply by 10, 100, 1000, or 1 with any number of ciphers annexed,

Rule.

Move the decimal point as many places to the right as there are ciphers in the multiplier.

NOTE. If there are not as many decimal places in the multiplicand as ciphers in the multiplier, in order that the point may be moved as required, ciphers must be annexed to the multiplicand.

54. Multiply 8.756 by 10. Ans. 87.56.
55. Multiply 356.4 by 100. Ans. 35640.
56. Multiply 89.657 by 1000.
57. Multiply 843.96 by 100000.
58. Multiply 362.748 by 100000.
59. Multiply 437.82 by 100000.
60. Multiply 47 by 10.
61. Multiply 689 by 10000.
62. Multiply 9846 by 1000.
63. $84697 \times 100000 = ?$
64. $87.954 \times 1000000000 = ?$

65. Multiply 17.57 by 50.

$$\begin{array}{r} 17.57 \\ 50 \\ \hline 878.5 \end{array}\ \text{Ans.}$$

After multiplying by 5, the product 87.85 is multiplied by 10 by moving the decimal point one place to the right which gives 878.5, Ans.

66. Multiply 743 by 3500.

$$\begin{array}{r} 743 \\ 3500 \\ \hline 371500 \\ 2229 \\ \hline 2600500 \end{array} \text{ Ans.}$$

67. Multiply 84693 by 480000. Ans. 40652640000.

68. $9432876 \times 2700 = ?$

69. $84294635 \times 9400 = ?$

70. Multiply 63.24 by 60.

71. Multiply 847.2 by 800.

72. Multiply 63.47 by 300.

73. Multiply 26.45 by 600400.

74. Multiply 8000 by 900.

$$\begin{array}{r} 8000 \\ 900 \\ \hline 7200000 \end{array} \text{ Ans.}$$

75. Multiply 730000 by 2900.

$$\begin{array}{r} 730000 \\ 2900 \\ \hline 657000000 \\ 1460000 \\ \hline 2117000000 \end{array} \text{ Ans.}$$

76. Multiply 408 by 7200000.

77. $9367000 \times 965000 = ?$

78. Multiply 5723 by 2004.

	5723
	2004
	22892
	11446
Product,	11468892

The 2 of the multiplier is 2000, and 2000 times 3 are 6000, hence, the 6 of the partial product should be written in the thousands' place, that is, directly under the 2 of the multiplier.

79. Multiply 3724 by 4008. Ans. 14925792.

80. $986274 \times 62400 = ?$

81. $70098 \times 80200 = ?$

82. Multiply 754.18 by 325.

83. Multiply 16.74 by 204.

84. Multiply 7849.93 by 400.

85. What will 7 horses cost at \$ 175.35 each?

86. At \$ 6.50 a ton, what will 16 tons of coal cost?

87. At \$8.17 a barrel, what will 58 barrels of sugar cost?

88. What will 14 dormer window-frames cost, at \$ 19 each?

89. If hay is \$18.75 a ton, what will 76 tons cost?

90. How many yards of cloth in 10 bales, if each bale contains 25 pieces and each piece 37 yards?

91. Multiply seven million six hundred four thousand forty-eight, and sixteen thousandths, by seventy-eight thousand sixteen.

92. What will be received for 17 city lots, if sold for \$ 1265.75 each?

93. In its annual revolution the earth moves 19 miles a second; how far does it move in an hour, there being 60 seconds in a minute and 60 minutes in an hour?

94. How far will light move in an hour, if it moves 186380 miles a second?

95. Multiplicand $= 76.78$; multiplier $= 4008$; product $= ?$

64. Miscellaneous Examples.

96. Multiplier = 3333; multiplicand = 44.44; product = ?

97. If horses are worth $117 each, and oxen $85.50 a pair, what must I pay for 18 horses and 5 pairs of oxen?

98. If hay is $15.50 a ton, and oats $0.55 a bushel, what must I pay for 28 tons of hay and 100 bushels of oats?

99. If each horse eats a bushel of oats a week, and oats are $0.60 a bushel, how much will it cost for oats for 27 horses for 52 weeks?

100. If it costs $38754 to build a mile of railroad, what will it cost to build 27 miles?

101. If sound travels 1090.6 feet a second, how far will it travel in 37 seconds?

102. Massachusetts contains 8315 square miles. What is its population if there are 269 persons to a square mile?

103. What do I save a year if my income is $2500, and my expenses $37 a week, 52 weeks making a year?

104. If I pay $5.50 a ton for coal and $0.25 a ton for putting it in, what will my year's supply of 16 tons cost me?

105. If I purchase 56 barrels of cranberries at $11 a barrel, and to pay for it give 48 barrels of apples at $2 a barrel, and the rest in money, how much money do I pay?

106. A cattle train is made up of 17 cars, and each car contains 53 sheep. If the sheep average 115 pounds each in weight, what do all the sheep weigh?

107. A certain hall will seat 1543 persons. If the seats are occupied by persons who weigh on the average 125 pounds each, what will be the weight of the persons supported by the floor?

108. If Mr. Brown owns 3 houses, the first worth $2783, the second 3 times, and the third 7 times as much as the first, what are the three houses together worth?

109. How many yards in 48 bales of goods, each bale containing 34 pieces, and each piece 37 yards?

110. How many miles will a locomotive run in 7 days if it runs 20 miles an hour and 15 hours each day?

111. A farmer had 23 acres of corn that yielded 49 bushels to the acre. How much is his corn worth at $0.75 a bushel?

112. Miss F., a teacher, pays $6 a week for her board for the 40 weeks of school, $10 a week for the 12 weeks of vacation, and for other expenses for the year $153. How much can she save in 7 years if her salary is $650 for the first three years and $750 for the last four years?

113. A farmer sold 56 loads of hay at $5 a load, and received 40 yards of cloth at $2.50 a yard. What is the balance still due him?

114. If I buy 18 tons of iron at $39.25 a ton, and 27 tons at $41.75 a ton, what shall I gain by selling the whole at $43.25 a ton?

115. A drover bought a drove of 33 oxen, paying as many dollars for each ox as there were oxen in the drove. He paid $514.85, and gave his note for the balance. For how much does he give his note?

116. $117 + 19 \times 8 + 74 = ?$ Ans. 343.

117. $108 - (16 + 3) \times 4 = ?$ Ans. 32.

118. $219 + 75 \times \overline{15 + 13} = ?$

119. $2135 - \overline{1218 - 413} \times 2 = ?$

120. $8102 - 5 \times (240 - 128) = ?$

121. $(164 + 151) \times 2 - 3 \times (57 - 28) = ?$

122. $1824 - 184 + 375 \times 3 = ?$

123. $1824 - (184 + 375) \times 3 = ?$

124. $(1824 - 184) + 375 \times 3 = ?$

125. $(1824 - 184 + 375) \times 3 = ?$

DIVISION.

65. 1. How many oranges, at 4 cents each, can be bought for 12 cents?

Solution. As many oranges as there are times 4 cents in 12 cents; 4 cents are contained in 12 cents, 3 times; hence, 3 oranges, at 4 cents each, can be bought for 12 cents.

2. If I buy 5 apples for 10 cents, how much do I pay for each?

66. **Division** is the process of finding how many times one number is contained in another, or of finding one of the equal parts of a number.

67. The **Dividend** is the number *to be divided.*

The **Divisor** is the number *by which to divide.*

The **Quotient** is the *number of times* the dividend contains the divisor.

If the dividend does not contain the divisor an *exact number of times,* the part of the dividend which is left is called the **Remainder.**

68. The *sign of division,* $\div$, indicates that the number before it is to be divided by the number after it; or division may be indicated by writing the divisor under the dividend, with a line between the dividend and the divisor; thus, $8 \div 2 = \frac{8}{2} = 4$, that is, 8 divided by 2 equals 4, or 2 in 8, 4 times.

3. How many are $6 \div 2$? Ans. 2 in 6, 3 times.

69. *Of the divisor and quotient one must always be an*

abstract number, while the other is of the same kind as the dividend.

Thus, in Ex. 1, 4 cents is contained in 12 cents, 3 times, *not* 3 oranges times. The divisor and dividend are of the same kind, and the quotient abstract.

In Ex. 2 we divide 10 cents into 5 equal parts of which one part is 2 cents; we do not divide 10 cents by 5 apples. The dividend and quotient are of the same kind, and the divisor abstract.

70. Oral Exercises.

4. 32 are how many times 4? 8? 2? 16?
5. 48 are how many times 4? 6? 12? 8? 3? 16?
6. 36 are how many times 12? 6? 9? 3? 4? 2?
7. 40 are how many times 8? 4? 2? 10? 5? 20?
8. 72 are how many times 2? 3? 4? 6? 8? 9? 12?
9. How many are $24 \div 6$, or $\frac{24}{6}$?
10. How many are $48 \div 8$, or $\frac{48}{8}$?
11. How many are $66 \div 11$, or $\frac{66}{11}$?
12. How many are $84 \div 12$, or $\frac{84}{12}$?
13. How many are $63 \div 9$, or $\frac{63}{9}$?
14. How many are $48 \div 6$, or $\frac{48}{6}$?
15. How many are $77 \div 11$, or $\frac{77}{11}$?
16. How many are $72 \div 8$, or $\frac{72}{8}$?
17. How many are $96 \div 12$, or $\frac{96}{12}$?
18. How many are $88 \div 8$, or $\frac{88}{8}$?
19. How many are $72 \div 12$, or $\frac{72}{12}$?
20. How many yards of broadcloth, at $3 a yard, can be bought for $69?

21. How many miles do I ride in one hour, if I ride 55 miles in 11 hours?

22. If 1 man can build a certain wall in 48 days, how long will it take 4 men?

In this example is the divisor abstract or concrete?

23. If you buy 12 pounds of soap for 96 cents, how much do you pay a pound?

71. Exercises for Written Work.

24. Divide 1384 by 4.

$$\begin{array}{rl} \text{Divisor, } 4) & \underline{1384} \text{ Dividend.} \\ \text{Quotient, } & 346 \end{array}$$

Having written the divisor and dividend as in the margin, as 4 is not contained in the 1 that stands in thousands' place, we say 4 in 13 hundreds, 3 hundreds times and 1 hundred remainder; writing the 3 in hundreds' place in the quotient, we reduce the 1 hundred remainder to tens, that is, call it 10 tens, and adding it to the 8 tens, we have 18 tens; 4 in 18 tens, 4 tens times and 2 tens remainder; writing the 4 in tens' place in the quotient, we reduce the 2 tens remainder to units, that is, call it 20 units, and adding it to the 4 units we have 24 units; 24 units divided by 4 gives 6 units, which we write in units' place in the quotient, thus completing the division.

Division with Decimals in the Dividend.

25. Divide 872.37 by 9.

$$\begin{array}{rl} 9) & \underline{872.37} \\ \text{Ans.} & 96.93 \end{array}$$

Proceeding exactly as in Ex. 24, at units we have 8 units remainder, which is equal to 80 tenths; 80 tenths and 3 tenths = 83 tenths; 83 tenths divided by 9 give 9 tenths and 2 tenths remainder; 2 tenths = 20 hundredths; 20 hundredths and 7 hundredths = 27 hundredths; 27 hundredths divided by 9 give 3 hundredths; place the decimal point between units and tenths, and the division is complete.

This operation is called *Short Division,* and is usually adopted when the divisor is so small that the process may be readily carried in the mind. Hence,

72. To perform Short Division:

Rule.

Write the divisor at the left of the dividend, and draw a line between divisor and dividend, and a line under the dividend.

Divide the least number of figures at the left of the dividend that will contain the divisor, by the divisor, and write the quotient under the right-hand figure taken in the dividend; if anything remains, prefix it mentally to the next figure in the dividend, and divide the number thus formed as before, and so proceed till all the figures of the dividend have been employed. Place the decimal point between units and tenths.

NOTE 1. It is best, as soon as the units' figure of the quotient is written, to write the decimal point after it.

NOTE 2. For division with decimals in the divisor, see Art. 162.

26. Divide 24864 by 8.

$$\begin{array}{rl} \text{Divisor, } 8) & \underline{24864} \text{ Dividend.} \\ \text{Quotient, } & 3108 \end{array}$$

(27.)

$$\begin{array}{rl} \text{Divisor, } 8) & \underline{764.128} \text{ Dividend.} \\ \text{Quotient, } & 95.516 \end{array}$$

(28.)

$$\begin{array}{rl} 3) & \underline{243642} \\ & 81214 \end{array}$$

(29.)

$2)\ \underline{924.36}$

(30.)

$6)\ \underline{65814234}$

(31.)

$9)\ \underline{82458.927}$

32. Divide 1326 by 3. Ans. 442.

33. Divide 74.325 by 5.

34. Divide 8323 by 7.

35. Divide 68592 by 8.

36. Divide 965.45 by 5.

37. Divide 93645 by 9.

38. Divide 57.212 by 4.

39. Divide 2781 by 8.

Divisor, 8) 2781 Dividend.
Quotient, 347, and 5 Remainder.

	Quotients,	Rem.
40. Divide 3654 by 4.	913,	2.
41. Divide 72584 by 5.	14516,	4.

42. 86471 ÷ 3 = how many? 43. 40505 ÷ 7 = ?

73. When there is a remainder the division can be continued into decimals by annexing a cipher to the remainder; this reduces the remainder to the next lower denomination in the scale.

44. Divide 7864 by 5.

5) 7864
1572.8

(45.)
4) 87379
21844.75

(46.)
8) 1876.2
234.525

47. 473864 ÷ 5 = ? 48. 986.74 ÷ 4 = ?

49. 234639 ÷ 12 = ?

50. In one week there are 7 days; how many weeks in 255 days? Ans. 36 weeks, Rem. 3 days.

51. How many barrels of sugar, at $8 a barrel, can be bought for $752?

52. If 6 pounds of cheese cost a dollar, how many dollars will 2736 pounds cost?

53. If 4 weeks make a month, how many months are there in 264 weeks?

74. When the divisor is *large*, it is more convenient to write out the work in full. In this case the operation is called *Long Division*.

54. Divide 2875 by 23.

$$
\begin{array}{r@{\,}l}
23) & 2875\ (125 \\
 & \underline{23} \\
 & 57 \\
 & \underline{46} \\
 & 115 \\
 & \underline{115} \\
 & 0
\end{array}
$$

Having written the divisor and dividend as in the margin, we first inquire how many times 23 is contained in 28 (the fewest figures at the left of the dividend that will contain the divisor), and find the quotient to be 1, which we write at the right of the dividend. We then multiply the divisor by the quotient, 1, and write the product, 23, under the 28 of the dividend, and subtract it therefrom. To the remainder, 5, we annex 7, the next figure of the dividend, and then inquire how many times the divisor is contained in 57, the second partial dividend; the result, 2, we write as the second figure of the quotient, and then multiply, subtract, annex, etc., as before, until all the figures of the dividend have been taken.

Since the 28 of the dividend is *hundreds,* the 1 of the quotient is *also hundreds;* since the 57 is *tens,* the 2 is *also tens;* and, *universally, any quotient figure is of the same order as the right-hand figure of the dividend taken to obtain that quotient figure.*

55. Divide 5786.405 by 47.

$$
\begin{array}{r@{\,}l}
47) & 5786.405\ (123.115 \\
 & \underline{47} \\
 & 108 \\
 & \underline{94} \\
 & 146 \\
 & \underline{141} \\
 & 54 \\
 & \underline{47} \\
 & 70 \\
 & \underline{47} \\
 & 235 \\
 & \underline{235} \\
 & 0
\end{array}
$$

The work is done exactly as in Ex. 54. After dividing the 146 units by 47, and obtaining 3 units as the quotient, we make the decimal point in the quotient, and then continue as before.

Hence,

75. To perform Long Division:

Rule.

1. Write the divisor at the left of the dividend, and draw a curved line between divisor and dividend and also at the right of the dividend.

2. By the divisor divide the smallest number of figures in the left of the dividend that will contain the divisor, and write the result as the first figure of the quotient at the right of the dividend.

3. Multiply the divisor by the quotient figure, and write the product under that part of the dividend taken.

4. Subtract the product from the figures over it, and to the remainder annex the next figure of the dividend for a new partial dividend.

5. Divide, and proceed as before, until the whole dividend has been divided.

6. Write the decimal point at the right of the quotient figure obtained from the partial dividend that contains the units' figure of the given dividend.

NOTE 1. It will be seen that the process of dividing consists of four distinct steps, viz. : first, to seek a quotient figure ; second, multiply ; third, subtract ; and, fourth, form a new partial dividend by annexing the next figure of the dividend to the remainder.

NOTE 2. If any partial dividend will not contain the divisor, 0 must be placed in the quotient, and another figure annexed to the partial dividend.

NOTE 3. If the product of the divisor multiplied by the quotient figure is greater than the partial dividend, the quotient figure is too great, and in its place a figure one or more less must be written.

NOTE 4. If the remainder equals or exceeds the divisor, the quotient is too small, and in its place a figure one or more greater must be written.

NOTE 5. For division with decimals in the divisor, see Art. 162.

76. *Division is the reverse of multiplication.* In multiplication the two factors are given, and the product is required; in division the product and one factor are given, and the other factor is required. The dividend is the product, and the divisor and the quotient are the factors; thus,

IN MULTIPLICATION.	IN DIVISION.				
Factors, Product.	Dividend,		Divisor,		Quotient.
$5 \times 4 = 20$	20	÷	5	=	4
	Or, 20	÷	4	=	5

Hence the

77. PROOF. *Multiply the divisor by the quotient, and to the product add the remainder; the result should be the dividend.*

56. Divide 2537 by 53.

```
53) 2537 (47
    212
    ---
     417
     371
     ---
      46
```

PROOF.

```
  53 Divisor.
  47 Quotient.
 ---
 371
212
  46 Remainder.
----
2537 Dividend.
```

(57.)

```
21) 86.4 (4.1
    84
    ---
     2.4
     2.1
     ---
     0.3
```

(58.)

```
87) 3659 (42
    348
    ---
     179
     174
     ---
       5
```

NOTE. When there is a remainder it can be reduced to the next lower denomination by annexing a cipher, and the division continued. When the division has been carried as far as desirable, if there is still a remainder the sign + may be written at the right of the quotient.

59. Divide 7856.3 by 27.

```
27) 7856.3 (290.974+
    54
    ---
    245
    243
    ----
      26.3
      24.3
      ----
       2.00
       1.89
       -----
       0.110
       0.108
       -----
       0.002
```

		Quotients,	Rem.
60. Divide 47086 by 18.	Ans.	2615,	16.
61. Divide 468074 by 46.	Ans.	10175,	24.
62. Divide 340068 by 67.	Ans.	5075,	43.

63. $629483 \div 93 =$ how many?

64. $379048 \div 89 =$ how many?

65. $485376 \div 78 =$ how many?

66. $428796 \div 254 = ?$

67. $9367.85 \div 349 = ?$

68. $658879 \div 247 = ?$

69. $94867.3 \div 978 = ?$

70. $608048 \div 803 = ?$

71. $689.284 \div 458 = ?$

72. $652943 \div 54 = ?$

73. $76984432 \div 4893 = ?$

74. $496428762355 \div 5699 = ?$

75. $7843.2 \div 83 = ?$

76. $2114.37 \div 112 = ?$

77. $8476.3 \div 285 = ?$

78. Divide four hundred eighteen thousand six hundred forty-eight, by twenty-four. Ans. Quo. 17443, Rem. 16.

79. Divide two hundred one thousand five hundred ninety-five acres of land, into twenty-three equal parts.

80. A railroad that cost \$4076500 was divided into 8153 equal shares; what was the cost of each share?

81. A farmer raised 2047 bushels of wheat on 89 acres of land. How many bushels did he raise on an acre?

82. In how many days will a ship sail 3456 miles, if it sails 144 miles a day?

83. A farmer raised 4118 bushels of corn, his crop averaging 58 bushels an acre. How many acres did he plant?

84. A drover paid \$7663 for 79 oxen; how many dollars did he pay for each ox?

85. The product of two numbers is 10707, and one of the numbers is 129; what is the other number?

86. The earth, in its revolution round the sun, moves about 1641600 miles in one day; how far does it move in one second, there being 86400 seconds in a day?

87. Divide \$6024 equally among 24 men.

78. As removing a figure one place to the right divides it by ten, two places divides it by a hundred, etc., to divide by 10, 100, 1000, or 1 with any number of ciphers annexed,

Rule.

Move the decimal point as many places to the left as there are ciphers in the divisor.

NOTE 1. In order to move the decimal point the required number of places to the left, it may be necessary to prefix ciphers. (See Ex. 94.)

NOTE 2. The figures at the right of the decimal point can be left standing as decimals, or written separately as the remainder.

88. Divide 756 by 10. Ans. 75.6, or 75 and 6 Rem.

89. Divide 276403 by 10.
90. Divide 94763 by 100.
91. Divide 98765423 by 100000. Ans. 987 and 65423 Rem.
92. Divide 7830652143 by 100000000.
93. Divide 56.3 by 10.
94. Divide 4.6 by 100. Ans. 0.046.
95. Divide 964.8 by 100.

96. Divide 817468 by 85000.

```
85000) 817.468 (9.617+
       765
        524
        510
         146
          85
          618
          595
           23
```

As $85000 = 85 \times 1000$, we first divide by 1000, obtaining (Art. 79) 817.468; then we divide 817.468 by 85.

If the exact remainder is sought the work is as follows:

```
85000) 817|468 (9
       765
        52468 Rem.
```

Dividing by 1000 as before, we have as a quotient 817 and 468 remainder. Dividing 817 by 85 we have 9 as a quotient and 52 thousands as a remainder. Adding these two remainders together, we have 52468 as the full remainder.

NOTE. If the work is carried into decimals, and there is still a remainder, we can at any point we choose cease dividing and write + at the right of the quotient, as in the first operation above.

97. Divide 77465 by 5700.
98. Divide 74689 by 4200. Ans. 17 and 3289 Rem.
99. Divide 698727 by 3300.
100. $769842 \div 45000 = ?$ Ans. 17 and 4842 Rem.

101. $9999999 \div 33300 = ?$

102. $80407080 \div 40000 = ?$

103. $987654321 \div 90900 = ?$

104. If I pay $1170 for 180 tons of coal, what do I pay a ton?

105. If $7856.50 are divided equally among 50 men, how much will each have?

106. Sahama, one of the highest volcanoes in the world, is 22350 feet high, and there are 5280 feet in a mile. How many miles high is this volcano?

Miscellaneous Examples.

107. A steamer sometimes crosses the Atlantic, 2880 miles, in 8 days of 24 hours each. What is the rate an hour?

108. If a ship sails 10 miles an hour, how many days will it take it to cross the Atlantic?

109. How many barrels of apples, at $2.75 a barrel, must be given for 6 barrels of cranberries, at $8.50 a barrel?

110. If the yearly expenses of a boy at school are $5.50 a week for 40 weeks' board, $50 for tuition, $15 for books, $50 for clothes, $8 for travelling expenses, and $16.50 for other purposes, what will be the expenses of 5 boys for 5 years?

111. If 24 men can build a certain wall in 81 days, how many days will it take 216 men to build the same wall?

112. If it costs a man $5 an acre to secure a title to 640 acres of government land, how much will he make if he sells the whole for $5000? How much on each acre?

113. The product of four factors is 86500, and three of these factors are 25, 4, and 173; what is the fourth?

114. The first of three numbers is 13, the second is 8 times the first, and the third is 10 times the product of the other two divided by 4. What is the difference between the first and third?

115. Sold two cows at $30 apiece, 3 tons of hay at $20 a ton, 100 bushels of corn for $50, and 10 cords of wood at $7 a cord, and received in payment $200 in money, a plough worth $15, 100 pounds of sugar worth $5, and the balance in broadcloth at $4 a yard. How many yards did I receive?

116. If I receive $60 and spend $40 a month, in how many years of 12 months each shall I save $2160?

117. Bought 87 acres of land at $50 an acre, and paid $3150 in cash, and the balance in labor at $240 a year. How many years of labor did it take?

118. Bought 42 yards of cloth at 15 cents a yard, and paid for it with eggs at 30 cents a dozen. How many dozen did it take?

119. If I take 13729 from the sum of 8762 and 14967, divide the remainder by 50, and multiply the quotient by 19, what is the product?

120. How many days will it take a person to walk from Boston to New York and back, if he walks 9 hours a day and 3 miles an hour, and the distance from New York to Boston is 216 miles?

121. A man earns $1065 a year, and each of his two sons $517, and each of the three spends $255 a year. If at the end of 7 years they make an equal division of their savings, what will each receive?

122. If a man sells 19 bushels of potatoes at $0.55 a bushel, 23 bushels of oats at $0.63 a bushel, and with the proceeds buys 8 yards of broadcloth, how much does he pay a yard for the broadcloth?

123. If a farmer exchanges 51 acres of woodland worth $125 an acre for cleared land worth $85 an acre, how many acres will he get?

124. $99 + 75 \div 3 = ?$ Ans. 124.
125. $144 - 27 \div 9 = ?$ Ans. 141.
126. $45 \times 3 - 118 \div 2 = ?$ Ans. 76.
127. $77 \div 11 + 144 \times 3 = ?$
128. $15 \times 3 + 25 \div 5 - 18 \div 6 = ?$
129. $(184 + 27) \times 4 - 6 \times 5 + 183 \div 3 = ?$

GENERAL PRINCIPLES OF DIVISION.

79. The value of a quotient depends upon the *relative* values of the divisor and dividend, and not upon their *absolute* values, as will be seen by the following propositions.

(a.) *If the divisor remains unchanged, multiplying the dividend by any number is, in effect, multiplying the quotient by the same number;* thus,

$$\begin{array}{r@{}l@{}r} 15 & \div 3 = & 5 \\ \underline{4} & & \underline{4} \\ 60 & \div 3 = & 20; \end{array}$$

that is, multiplying the dividend by 4 multiplies the quotient by 4.

(b.) *Dividing the dividend by any number is dividing the quotient by the same number;* thus,

$$\begin{array}{r@{}l} & 24 \div 2 = 12 \\ 3)\, & \underline{24} \\ & \;\;8 \div 2 = \;\; 4 = 12 \div 3; \end{array}$$

that is, dividing the dividend by 3 divides the quotient by 3.

(c.) *Multiplying the divisor divides the quotient;* thus,

$$\begin{array}{l} 30 \div 2 = 15 \\ \quad\;\;\; \underline{3} \\ 30 \div 6 = \;\; 5 = 15 \div 3; \end{array}$$

that is, multiplying the divisor by 3 divides the quotient by 3.

(d.) *Dividing the divisor multiplies the quotient;* thus,

$$40 \div 10 = 4$$
$$5)\,\underline{10}$$
$$40 \div 2 = 20 = 4 \times 5;$$

that is, dividing the divisor by 5 multiplies the quotient by 5.

(e.) It follows, from (a) and (b), *that the greater the dividend, the greater is the quotient; and the less the dividend, the less the quotient.*

(f.) Also, from (c) and (d), *that the greater the divisor, the less is the quotient; and the less the divisor, the greater the quotient.*

80. From the illustrations in Art. 79 we see that any change in the dividend causes a *similar* change in the quotient, and that any change in the divisor causes an *opposite* change in the quotient. Hence,

(a.) *Multiplying both dividend and divisor by the same number does not affect the quotient;* thus,

$$12 \div 3 = 4, \text{ Quotient}$$
$$\underline{2} \quad \underline{2}$$
$$24 \div 6 = 4, \quad \text{" unchanged.}$$

(b.) *Dividing both dividend and divisor by the same number does not affect the quotient;* thus,

$$20 \div 10 = 2, \text{ Quotient}$$
$$5)\,\underline{20} \quad 5)\,\underline{10}$$
$$4 \div 2 = 2, \quad \text{" unchanged.}$$

(c.) It follows from (a) and (b), *that the operations of multiplying and dividing by the same number cancel* (that is, *balance*) *each other;* for example,

If a number is multiplied by any number, and the product divided by the multiplier, the quotient will be the multiplicand; thus,

$8 \times 7 = 56$, and $56 \div 7 = 8$, the multiplicand.

Also, if a number is divided by any number, and the quotient multiplied by the divisor, the product will be the dividend; thus,

$15 \div 3 = 5$, and $5 \times 3 = 15$, the dividend.

81. These general principles may be more briefly stated as follows:

1. *Multiplying the dividend multiplies the quotient; and dividing the dividend divides the quotient* (Art. 79, a and b).

2. *Multiplying the divisor divides the quotient; and dividing the divisor multiplies the quotient* (Art. 79, c and d).

3. *Multiplying both dividend and divisor by the same number, or dividing both by the same number does not affect the quotient* (Art. 80, a and b).

82. Examples.

130. $2764 \div 4 - 2536 \div 4 = (2764 - 2536) \div 4$
$= 228 \div 4 = 57.$

131. $843 \div 3 - 516 \div 3 - 282 \div 3 = ?$

132. $(864 \div 8) \div (144 \div 8) = ?$

133. $2515 - 785 \div 5 = ?$

134. $768 \div 2 - 762 \div 2 = ?$

135. $(423 - 67) \times 4 + 3 \times 4 - 216 \div 6 = ?$

136. $1845 \div 5 - 1720 \div 5 = ?$

137. $2728 \div 4 - 2416 \div 4 = ?$

138. $616 \times 8 \div 4 - 546 \times 8 \div 4 = ?$

139. $714 - 215 \div 5 + 425 \div 5 = ?$

140. $849 \times 4 \div 3 - 714 \times 4 \div 3 - 135 \times 4 \div 3 = ?$

83. Oral Exercises for Review.

1. In a certain orchard there are 43 apple-trees, 10 peach-trees, 7 pear-trees, 8 cherry-trees, and 6 quince-trees; how many trees are there in the orchard?

2. A lady bought a silk dress for 22 dollars, a bonnet for 9 dollars, a shawl for 6 dollars, a pair of shoes for 2 dollars, a pair of gloves for 1 dollar, and had 10 dollars left; how many dollars had she at first?

3. A man paid 75 dollars for a horse, 9 dollars for a saddle, 3 dollars for a bridle, and 1 dollar for a whip; what did they all cost him?

4. A miller bought 54 bushels of wheat, 10 of rye, 8 of corn, and 3 of buckwheat; how many bushels of grain did he buy?

5. David has 37 cents in his purse, 8 in one pocket, 9 in another pocket, and 7 in his hand; how many cents has he?

6. James had 37 cents, and George gave him 7, Charles 6, Samuel 9, John 8, and he bought a top for 10 cents; how many cents had he left?

7. A lady gave 50 dollars for a watch, 9 dollars for a chain, 1 dollar for a key, and 5 dollars for a pin; how many dollars did she give for all?

8. Two men start from the same place and travel in opposite directions, one at the rate of two miles in an hour, and the other six; how far apart are they in 7 hours? How far in 10 hours?

9. If the interest of one dollar is six cents a year, what is the interest of three dollars for two years?

10. How many oranges, at 4 cents apiece, must be given for 12 lemons, at 3 cents apiece?

11. How many pounds of beef, at 9 cents a pound, will pay for 6 pounds of nuts at 12 cents a pound?

12. If 8 men can do a certain piece of work in 6 days, in how many days can 4 men do the same?

13. If 9 men can do a piece of work in 8 days, how many men can do the same in 12 days?

14. If 4 men can do a piece of work in 6 days, in how many days can 3 men do twice as much work?

15. A farmer sold 2 tons of hay at $20 a ton, and for pay received 8 yards of cloth at $4 a yard, and the rest in money; how much money did he receive?

16. If a man earns $24 in 2 months, what will 2 men earn in 6 months?

17. At 12 cents a pound, how many pounds of beef can be bought for 144 cents? For 96 cents?

18. How many wheelbarrows, at 7 dollars apiece, can be bought for 84 dollars? For 63 dollars?

19. A boy sold 6 doves for 72 cents; how many cents did he receive for 1 dove? For 3 doves? For 4 doves?

20. A boy sold chickens at 25 cents apiece, and received 1 dollar, or 100 cents, for them; how many chickens did he sell?

21. If syrup is worth 20 cents a pint, how many pints may be bought for 1 dollar?

22. What is the price of a shawl, if 9 shawls cost $108?

23. A boy divided 80 chestnuts equally among 5 of his companions. How many did he give to each?

24. If 8 quarts make a peck, how many pecks will 96 quarts make? 84 quarts? 48 quarts?

25. A man divided $84 equally among his 3 sons and 4 daughters; how many dollars did he give to each?

26. A boy having 7 melons, gave away 2 of them, and sold the rest for $1; what did he receive for each of those he sold?

27. A man having 75 dollars, bought 7 sheep, and had $5 left; what did he pay for each sheep?

28. A boy bought 5 hens at 20 cents each, and paid for them with apples at 10 cents a dozen; how many dozen did it take?

29. A man had 75 sheep, and bought 5 more; he then distributed them equally into 8 pens; how many sheep did he put in each pen?

84. Written Examples for Review.

30. The area of California is 155980 square miles, and of France 204092 square miles. How many square miles does California lack of being as large as France?

31. The earnings of a man and his three sons are $ 4475 and their expenses are $ 2845. If the balance is equally divided among them, what will each have?

32. How many yards of cloth in 5 bales, each bale containing 25 pieces, and each piece containing 175 yards?

33. From a farm containing 1243 acres there were sold at one time 324 acres and at another time 472 acres. How many acres remain?

34. Paid $ 55 for a harness, $ 147 for a carriage, and for a horse as much as for the carriage and harness. What was the cost of all?

35. Bought 230 acres of land at $ 35 an acre, and built a house that cost $ 1975, and a barn for $ 1187.50. What was the cost of all?

36. From two million seven hundred ninety take one thousand two and forty-nine thousandths?

37. A man paid $ 1775 for a lot, $ 345.50 for digging the cellar and putting in the foundation for a house, $ 4250 for building the house, $ 1844.75 for furniture, $ 215.25 for grading and laying out the grounds. If he had $ 12400 when he began, how much had he left?

38. A man bought three farms; the first contained 648 acres, the second 495 acres, the third as many acres as both the others. How many acres in the three farms?

39. A student had $ 200 with which to fit up his room. He spent for a carpet $ 40, for a set of furniture $ 42.50, for bedding $ 18.25, for curtains $ 6.75, for a mirror $ 4.75, for a bookcase $ 23, for an easy-chair $ 7.50, for a lounge $ 10, for pictures $ 25. How much did he have left?

40. If 3500 is the minuend and 2687 the subtrahend, what is the remainder?

41. The salary of the President of the United States is $ 50000 a year; what sum may he expend each year, and yet save $ 75584 in 4 years, his term of office?

42. If the divisor is 19, the quotient 37, and the remainder 11, what is the dividend?

43. The difference of two numbers is 367854, and the greater number is 897325; what is the smaller? What is the sum of the two numbers?

44. A field of 12 acres yields 16.5 bushels of wheat an acre. The labor and seed cost $ 164.25, and the wheat brought $ 1.12 a bushel. How much profit was made from the field?

45. A cattle dealer deposited in the bank $ 10481 Monday and drew out $ 4550; on Tuesday he deposited $ 2900 and drew out $ 5875; on Wednesday he deposited $ 13470 and drew out $ 8645; how much of this money had he remaining in the bank?

46. If a town containing 23040 acres is divided into farms of 80 acres each, how many farms will there be, and what will they be worth at $ 65 an acre?

47. A drover bought 164 cows at $ 52 each, 95 at $ 59 each. He paid two dollars a head for transportation and sold the lot for $ 15457.50. Did he gain or lose, and how much?

48. The area of Massachusetts is 8315 square miles, of Texas 265780 square miles, and of Spain 191367 square miles. How many States as large as Massachusetts could be made out of what would remain of Texas after taking away an area as large as Spain?

49. If a man's annual income is $ 2250, and he puts in the Savings Bank $ 700, spends $ 350 for rent, $ 427.50 for provisions, $ 211.75 for servant, $ 75 for coal and wood, $ 200 for clothing, $ 76.75 for travelling expenses, $ 153.50 for other expenses, and the balance in books at an average price of $ 1.50 apiece, how many books does he buy?

50. How many pounds of butter at 23 cents a pound must be given for 5 pounds of raisins at 11 cents a pound, 2 pounds of tea at 63 cents a pound, and a barrel of sugar at $ 9?

51. The area of Ireland is 32481 square miles, and 5023984 acres are uncultivated. How many more acres are cultivated than three times the number uncultivated, there being 640 acres in a square mile?

52. If a man earns $ 29 a week and pays $ 3 a week for the board of each of his 2 sons, how much in the course of a year, or 52 weeks, does he have for other expenses?

53. A farmer sold 23 tons of hay at $ 18 a ton, and received in payment a yoke of oxen at $ 125, 5 cows at $ 30 each, 15 sheep at $ 4 each. Then, to balance the account, he agrees to take 2 cows more at the same price, and the rest as far as possible in sheep at $ 4 each, and the balance in money. How many more sheep and how much money did he receive?

54. A flour dealer paid $ 4914 for 819 barrels of flour, and sold it at $ 6.50 a barrel. Did he gain or lose, and how much?

55. Taking the areas as given in Ex. 100, page 26, how many States as large as Massachusetts could be made out of Maine, and how many square miles would be left?

56. How many States as large as Rhode Island could be made out of Maine, and how many square miles would be left?

57. Ohio contains 41060 square miles. How many States like Massachusetts could be made out of it, and how many square miles would be left?

58. A merchant bought 5 barrels of sugar, each barrel containing 235 pounds, at an average price of 9 cents a pound. He sold one barrel at 8 cents a pound, one at 9, two at 10, and one at 11. How much did he gain?

59. In a certain High School building one room will seat 75, 2 will seat 54 each, 4 will seat 48 each, 2 will seat 36 each, and 1 more will seat 40. How many pupils in all can be seated in the building?

PROPERTIES OF NUMBERS.

85. When in division there is *no remainder*, the dividend is said to be *divisible* by the divisor, and the divisor is called an *exact divisor*.

When there *is a remainder*, the dividend is said to be *indivisible* by the divisor.

86. An **Even Number** is a number that is divisible by 2; as, 2, 4, 8, 12.

87. An **Odd Number** is a number that is *not* divisible by 2; as, 1, 3, 5, 11, 19.

88. A **Prime Number** is a number that is not divisible by any whole number except *itself* and *one*; as, 1, 2, 3, 5, 7, 11, 19.

NOTE. *Two* is the only *even* prime number, *for all even numbers are divisible by* 2.

89. Numbers are *mutually* prime (that is, *prime to each other*) when no whole number but *one* will divide all of them without remainder; thus, 8 and 9 are *mutually* prime, although neither 8 nor 9 is *absolutely* prime.

90. A **Composite Number** is a number that is divisible by other numbers besides itself and one; thus, 6 and 33 are composite, as $6 = 3 \times 2$, and $33 = 3 \times 11$.

FACTORING.

91. Oral Exercises.

1. What two numbers multiplied together give 6? 9? 12? 15? 18?

2. What numbers multiplied together give 22? 25? 38? 39? 44?

3. 51 is the product of what numbers? 57? 63? 81? 93?

4. 76 is the product of what numbers? 87? 91? 111? 121?

92. The **Factors** of a number are those numbers whose continued product is the number; thus, 3 and 7 are the factors of 21; 3 and 6, or 3, 3, and 2, are the factors of 18; etc.

NOTE. Every number is a factor of itself, the other factor being 1. In speaking of the factors of a number, we shall exclude the number itself and 1.

93. The *prime* factors of a number are those *prime* numbers whose continued product is the number; thus, the prime factors of 12 are 2, 2, and 3; the prime factors of 36 are 2, 2, 3, and 3; etc.

94. To *factor* a number is to separate it into its factors. In separating a number into its factors,

The following facts will be found convenient:

(a.) Every number whose unit figure is 0, or an *even* number, is divisible by 2.

(b.) Any number is divisible by 3 when the *sum of its figures* is divisible by 3; thus, 4257 is divisible by 3, as the sum of its figures, $4+2+5+7=18$, is divisible by 3.

(c.) Any number is divisible by 4 when 4 will divide the number expressed by the *two right-hand figures;* thus, 4 will divide 32, hence it will divide 7532.

(d.) Any number whose unit figure is 0 or 5 is divisible by 5; as, 90, 1740, 35, 34975, etc.

(e.) Any *even* number which is divisible by 3 is also divisible by 6; thus, 3528 is divisible by 3 and hence by 6.

(f.) For 7 no *general* rule is known.

(g.) Any number is divisible by 8 when 8 will divide the number expressed by the *three right-hand figures*; thus, 8 will divide 816, hence it will divide 175816.

(h.) Any number is divisible by 9 when the *sum of its figures* is divisible by 9; thus, 7146 is divisible by 9 because the sum of its figures, $7 + 1 + 4 + 6 = 18$, is divisible by 9.

(i.) Any number whose unit figure is 0 is divisible by 10.

NOTE. For explanation of these statements, see Appendix, page 336. Also, for table of prime numbers, see Appendix, page 337.

95. Written Exercises.

5. What are the prime factors of 30?

$$\begin{array}{r} 2\underline{)\ 30} \\ 3\underline{)\ 15} \\ 5 \end{array}$$

Ans. 2, 3, 5.

As the unit figure of 30 is 0, it can be divided by 2 (Art. 94, a). The quotient 15 is divisible by 3 (Art. 94, b); the quotient 5 is prime. Hence the prime factors of 30 are 2, 3, and 5. Hence,

To separate a number into its prime factors:

Rule.

Divide the given number by any prime number greater than one, that will divide it; divide the quotient thus found by any prime number greater than one that will divide it, and so on till the quotient is prime. The several divisors and last quotient will be the prime factors sought.

6. What are the prime factors of 40? Ans. 2, 2, 2, and 5.
7. Separate 84 into its prime factors. Ans. 2, 2, 3, and 7.
8. Separate 250 into its prime factors. Ans. 2, 5, 5, and 5.
9. What are the prime factors of 735?
10. What are the prime factors of 9800?

11. What are the prime factors of 441?
12. What are the prime factors of 315?
13. What are the prime factors of 1320?
14. What are the prime factors of 1728?
15. What are the prime factors of 1600?
16. What are the prime factors of 8424?
17. What are the prime factors of 4284?
18. What are the prime factors of 8364?
19. What are the prime factors of 7698?
20. What are the prime factors of 1682?
21. What are the prime factors of 2585?
22. Prove that 907 is prime.

```
31) 907 (29
    62
    ---
    287
    279
    ---
      8
```

From Art. 94, a, b, and d, we see that 2, 3, and 5 are not factors of 907. We then try every other prime number as a divisor to 31, inclusive, and find 907 is not divisible by any of them. There can be no prime factor in 907 greater than 31; for if there were, the quotient would be another factor less than 29; but we have found above that there was no factor in 907 less than 29. Therefore 907 is prime.

23. Prove that 743 is prime.
24. Prove that 997 is prime.
25. Prove that 1997 is prime.

GREATEST COMMON DIVISOR.

96. Oral Exercises.

26. Find a factor common to 8 and 12.
27. Find a factor common to 6, 9, and 15.
28. Find a factor common to 24 and 36.
29. Find a factor common to 18, 27, and 45.

30. What is the greatest factor common to 4, 8, and 12?

31. What is the greatest factor common to 8, 16, and 24?

NOTE. A common factor is generally called a common divisor, and the greatest common factor the greatest common divisor.

97. A **Common Divisor** of two or more numbers is *a number that will divide each of them without remainder;* thus, 3 is a common divisor of 12, 18, and 30.

98. The **Greatest Common Divisor** of two or more numbers is the *greatest* number that will divide each of them without remainder; thus, 6 is the greatest common divisor of 12, 18, and 30.

99. Written Exercises.

32. What is the greatest common divisor of 18, 30, and 48?

$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$
$48 = 2 \times 3 \times 2 \times 2 \times 2$
Ans. $2 \times 3 = 6$.

We see that 2 and 3 are factors *common* to all the numbers, and, furthermore, they are the *only* common factors; hence their product, $2 \times 3 = 6$, is the greatest common divisor of the given numbers.

33. What is the greatest common divisor of 60, 72, 48, and 84?

$60 = 2 \times 2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$84 = 2 \times 2 \times 3 \times 7$
Ans. $2 \times 2 \times 3 = 12$.

Although 2 is a factor *more than twice* in *some* of the given numbers, yet, as it is a factor *only* twice in *others*, we *cannot* take 2 *more than* twice in finding the greatest common divisor. The same remark applies to other factors. Hence, to find the greatest common divisor of two or more numbers,

Rule 1.

Separate each number into its prime factors, and the continued product of all the prime factors that are common to all the given numbers, each factor taken the least number of times that it occurs in any of the numbers, will be the common divisor sought.

34. What is the greatest common divisor of 24, 40, 64, 80, 96, 120, and 192? Ans. $2 \times 2 \times 2 = 8$.

NOTE 1. The greatest common divisor cannot be greater than 24; and as 24 is a divisor of 96, 120, and 192, the answer is not affected by the addition of 96, 120, and 192; also, 40 is a divisor of 80. The question then is at once reduced to this, — What is the greatest common divisor of 24, 40, and 64?

35. Find the greatest common divisor of 15, 45, 75, 105, 135, 150, and 300. Ans. 15.

36. Find the greatest common divisor of 25, 45, and 70.

37. Find the greatest common divisor of 24, 36, and 64.

38. Find the greatest common divisor of 24, 48, 72, and 88.

NOTE 2. As in Ex. 3, this question becomes, — What is the greatest common divisor of 24 and 88?

39. Find the greatest common divisor of 45, 75, 90, 135, 150, and 180.

NOTE 3. This question reduces to, — What is the greatest common divisor of 45 and 75? Apply this principle whenever you can.

40. I have three house lots, the first 136 feet front, the second 184 feet front, and the third 216 feet front; how long is the longest rail which will exactly fence the front of each lot? How many lengths of fence will be required for each lot? Ans. 8 feet.

100. When the given numbers are not readily separated into their prime factors, their greatest common divisor can be found by

Rule 2.

Of two numbers divide the greater by the less, and, if there is a remainder, divide the divisor by the remainder, and continue dividing the last divisor by the last remainder until nothing remains; the last divisor is the greatest common divisor of the two numbers.

If more than two numbers are given, find the greatest divisor of two of them, then of this divisor and a third number,

and so on until all the numbers have been taken; the last divisor will be the divisor sought.

41. What is the greatest common divisor of 18 and 32?

```
18) 32 (1
    18
    14) 18 (1
        14
         4) 14 (3
            12
       Ans. 2) 4 (2
               4
               0
```

(For explanation of this method, see Appendix, page 338.)

42. What is the greatest common divisor of 426 and 784?
Ans. 2.

43. What is the greatest common divisor of 1215 and 1878?

44. What is the greatest common divisor of 1071 and 1870?

45. What is the greatest common divisor of 3696 and 1440?

46. What is the greatest common divisor of 6237 and 2520?

47. What is the greatest common divisor of 16, 24, and 36?

```
       16) 24 (1
           16
            8) 16 (2
               16
                0
Again,  8) 36 (4
           32
      Ans. 4) 8 (2
              8
              0
```

We first find the greatest common divisor of 16 and 24, viz. 8, and then find the greatest common divisor of 8 and 36.

48. What is the greatest common divisor of 42, 48, 216, and 234?

49. What is the greatest common divisor of 154, 210, and 420?

50. What is the greatest common divisor of 18 and 11?

51. What is the greatest common divisor of 36, 60, and 140?

LEAST COMMON MULTIPLE.

101. Oral Exercises.

52. What whole numbers less than 45 will contain 8?

53. What whole numbers less than 50 will contain both 6 and 9?

54. What whole number less than 75 will contain 4, 6, and 10?

55. What is the least number that is divisible by 6 and 8?

56. What is the least number that is divisible by 2, 3, 4, and 6?

57. What is the least number that is divisible by 4, 6, and 8?

102. A **Multiple** of a number is any number which is *divisible* by that number; thus, 15 is a multiple of 5 and also of 3; 21 is a multiple of 7 and of 3.

NOTE. Every number is both a divisor and a multiple of itself.

103. A **Common Multiple** of two or more numbers is any number which is divisible by each of the given numbers; thus, 48 is the common multiple of 4, 6, and 8.

104. The **Least Common Multiple** of two or more numbers is the *least* number that is divisible by each of the given numbers; thus, 24 is the least common multiple of 4, 6, and 8.

105. Written Exercises.

58. What is the least common multiple of 20, 24, and 36?

$$20 = 2 \times 2 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$
Ans. $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360.$

Since 360 contains all the factors of 20, 24, and 36, respectively, it is divisible by each of those numbers. It is also evident that no number less than 360 will contain 20, 24, and 36, for if one of the 2's in the common multiple were omitted, it would not contain 24; if one of the 3's, it would not contain 36; and if the 5 were omitted, it would not contain 20.

Similar reasoning applies to all examples. Hence, to find the least common multiple of two or more numbers,

Rule 1.

Separate each number into its prime factors, and the continued products of all the different prime factors, each taken the greatest number of times it occurs in any of the given numbers, will be the least common multiple.

59. What is the least common multiple of 12, 16, 20, and 30?
Ans. $2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240.$

60. What is the least common multiple of 22, 33, and 55?

61. What is the least common multiple of 16, 36, 40, and 48?

NOTE 1. As 48 contains 16, the least common multiple is not changed by omitting the 16; and the question is reduced to, — What is the least common multiple of 36, 40, and 48?

62. What is the least common multiple of 20, 30, 50, and 80?

63. What is the least common multiple of 15, 25, 45, and 75?

NOTE 2. As 75 contains both 25 and 15, the least common multiple is not changed by omitting 15 and 25. Apply this principle whenever you can.

64. What is the least common multiple of 35, 50, 75, and 90?

65. What is the least common multiple of 24, 36, 48, and 64?

66. What is the least common multiple of 72, 80, 84, and 96?

67. What is the least common multiple of 42, 49, 72, and 88?

68. What is the least common multiple of 12, 20, 24, and 36?

106. The same result is sometimes more easily obtained by

Rule 2.

Having written the given numbers in a line, divide by any prime number that will divide two or more of them, and write the quotients and undivided numbers in a line beneath; proceed with this line as with the first, and so continue until no two of the numbers can be divided by any number greater than one; the continued product of the divisors and numbers in the last line will be the multiple sought.

The second rule may be illustrated by the example already employed in explaining the first rule, viz.:

69. What is the least common multiple of 20, 24, and 36?

$$\begin{array}{r|rrr} 2) & 20, & 24, & 36 \\ \hline 2) & 10, & 12, & 18 \\ \hline 3) & 5, & 6, & 9 \\ \hline & 5, & 2, & 3 \end{array}$$

Ans. $2 \times 2 \times 3 \times 5 \times 2 \times 3 = 360.$

If the process by the 1st rule be examined it will be seen that the factor 2 is found 7 times in the given numbers, and as 2 is taken but 3 times in finding the multiple, it is rejected 4 times. By the 2d rule, also, 2 is rejected 4 times, viz. twice in the 1st division by 2, and twice in the 2d division by 2. The learner may think 2 is rejected *three* times in each of the first two divisions; but he must remember that the *divisor* 2 *is retained* as a factor in the common multiple in each instance.

NOTE 1. This second method is only another way of rejecting the common factors.

70. What is the least common multiple of 5, 16, 24, 32, and 48?

By Rule 1.

$$5 = 5$$
$$32 = 2 \times 2 \times 2 \times 2 \times 2$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Ans. $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 480$.

By Rule 2.

$$\begin{array}{r|rrr}
2) & 5, & 32, & 48 \\ \hline
2) & 5, & 16, & 24 \\ \hline
2) & 5, & 8, & 12 \\ \hline
2) & 5, & 4, & 6 \\ \hline
 & 5, & 2, & 3
\end{array}$$

NOTE 2. As 16 is contained in 32, and 24 in 48, we omit 16 and 24 in the operation.

71. What is the least common multiple of 15, 35, 45, and 210?

72. What is the smallest sum of money with which I can buy horses at \$ 75 each, cows at \$ 50 each, or sheep at \$ 9 each, using the same sum in each case? Ans. \$ 450.

73. I have 4 tin measures; the first holds 4 quarts, the second 5 quarts, the third 6 quarts, and the fourth 8 quarts. What is the size of the smallest cask that can be exactly measured by means of each of these measures? Ans. 120 quarts.

74. What is the least common multiple of 10, 15, 45, 75, and 90?

NOTE 3. If the numbers are mutually prime, their product is their least common multiple.

75. What is the least common multiple of 7 and 11?

Ans. $7 \times 11 = 77$.

76. What is the least common multiple of 4, 9, and 29?

NOTE 4. The least common multiple of *two* numbers is equal to their product divided by their greatest common divisor. This is only another way of rejecting common factors.

77. What is the least common multiple of 12 and 20?

The greatest common divisor of 12 and 20 is 4, and
The least common multiple is $12 \times 20 \div 4 = 60$, Ans.

78. What is the least common multiple of 36 and 45?

79. What is the least common multiple of 44 and 66?

CANCELLATION.

107. Cancellation is the striking out of equal factors from the dividend and divisor.

80. Divide 1155 by 105.

$$\frac{1155}{105} = \frac{\cancel{5} \times \cancel{3} \times \cancel{7} \times 11}{\cancel{5} \times \cancel{3} \times \cancel{7}} = \frac{11}{1} = 11, \text{ Ans.}$$

The principle on which cancellation depends is explained in Art. 80 (b).

The work may also be performed as follows:

$$\begin{array}{c} 11 \\ \cancel{77} \\ \cancel{231} \\ \cancel{1155} \\ \hline \cancel{105} \\ \cancel{21} \\ \cancel{7} \\ 1 \end{array} = 11, \text{ Ans.}$$

In this operation the factor 5 was cancelled from 1155 and 105, leaving 231 as the dividend and 21 as the divisor; then 3 was cancelled from 231 and 21, leaving 77 as the dividend and 7 as the divisor; then 7 was cancelled from 77 and 7, leaving 11 as the dividend and 1 as the divisor. Thus, all the common factors are cancelled from the dividend and the divisor, leaving 11 to be divided by 1. Hence,

To perform examples by cancellation,

Rule.

Cancel all the factors common to both dividend and divisor, then complete the required division.

NOTE 1. In operations involving both multiplication and division very much time and labor are saved by cancellation.

81. Divide $44 \times 8 \times 7$ by $11 \times 16 \times 7$. Ans. 2.

82. Divide $40 \times 18 \times 13 \times 8$ by $10 \times 13 \times 16$.

83. Divide $376 \times 14 \times 21$ by $7 \times 7 \times 16 \times 3$.

84. Divide $5 \times 25 \times 874$ by $2 \times 437 \times 5 \times 5 \times 5$. Ans. 1.

NOTE 2. When a number is entirely cancelled, 1 remains, though it need not be written except when the dividend or divisor is cancelled completely.

85. Divide $108 \times 17 \times 9 \times 4$ by $27 \times 3 \times 16 \times 17$.

86. Divide 87846 by 7986.

87. Divide $15 \times 4 \times 8 \times 9$ by $30 \times 2 \times 6 \times 12$.

108. Miscellaneous Written Examples.

88. What are the prime factors of 180?

89. What are the prime factors of 462?

90. What are the prime factors of 996?

91. What are the prime factors of 2916?

92. Prove that 983 is prime.

93. Prove that 877 is prime.

94. Find the greatest common divisor of 18, 24, 36, 42, 48, and 54.

95. Find the greatest common divisor of 35, 40, 70, and 120.

96. Find the greatest common divisor of 315 and 1215.

97. Find the least common multiple of 16, 20, 32, and 48.

98. Find the least common multiple of 25, 45, 75, and 90.

99. Divide $44 \times 5 \times 9$ by $11 \times 10 \times 3$.

100. Divide $615 \times 24 \times 15$ by $25 \times 8 \times 27$.

101. Divide $3177 \times 18 \times 34$ by $17 \times 54 \times 353$.

102. Divide $4518 \times 343 \times 8$ by $16 \times 63 \times 49$.

103. Divide $1728 \times 757 \times 84$ by $144 \times 1514 \times 7$.

104. Divide $1476 \times 365 \times 14$ by $123 \times 219 \times 56$.

COMMON FRACTIONS.

109. If anything, as a line, is divided into two equal parts, each of these equal parts is called *one half,* and is written $\frac{1}{2}$.

110. If the unit is divided into three equal parts, each of these parts is called *one third,* written $\frac{1}{3}$; two of these parts are called *two thirds,* written $\frac{2}{3}$.

If the unit is divided into four equal parts, each of these parts is called *one fourth,* written $\frac{1}{4}$; two of these parts are called *two fourths,* written $\frac{2}{4}$; three, *three fourths,* $\frac{3}{4}$; and so on.

111. These expressions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, etc., are called **Fractions**; and to distinguish them from *Decimal Fractions,* they are called **Common Fractions.**

112. A **Fraction** is one or more of the equal parts of a unit.

113. A **Common Fraction** is expressed by two numbers, one above and the other below a line; thus, $\frac{1}{2}$ (one half), $\frac{2}{5}$ (two fifths), etc.

114. The number below the line shows *into how many equal parts the unit is divided,* and is called the **Denominator,** because it *denominates* or *gives name* to the parts; thus, if a unit is divided into three equal parts, each part is one *third;* if into eight, each part is one *eighth;* etc.

115. The number above the line is called the **Numerator,** because it *numerates* or *numbers* the parts *taken.*

116. The numerator and the denominator are the **Terms** of the fraction.

117. To distinguish them from fractions whole numbers are called **Integers,** or **Integral Numbers.**

118. A fraction is only an expression of division (Art. 68), the numerator being the dividend, and the denominator the divisor. Hence,

(a.) *The value of a fraction is the quotient of the numerator divided by the denominator; thus,* $\frac{12}{4} = 12 \div 4 = 3$.

(b.) *Any change in the numerator causes a like change in the value of the fraction, and any change in the denominator causes an opposite change in the value of the fraction.* (Art. 79.)

(c.) *Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.* (Art. 80.)

119. A **Proper Fraction** is one whose numerator is less than the denominator; as, $\frac{2}{3}$, $\frac{7}{11}$.

120. An **Improper Fraction** is one whose numerator equals or exceeds its denominator; as, $\frac{4}{4}$, $\frac{8}{5}$. An improper fraction equals or exceeds a unit; hence its name, improper fraction.

121. A **Simple Fraction** is one whose terms are integers; as, $\frac{3}{5}$, $\frac{12}{7}$.

122. A **Mixed Number** is a whole number and a fraction united; as, $3\frac{4}{5}$, $20\frac{3}{8}$.

123. Exercises.

1. Write in figures two fifths; three sevenths; five elevenths; seventeen thirty-thirds. Write a proper fraction; an improper fraction; a mixed number.

2. How many thirds make a unit? how many fifths?

3. Which is greater, a half or a third of the same thing? a third or a fifth?

REDUCTION OF FRACTIONS.

124. Reduction of Fractions is changing the *terms* without changing the *value* of a fraction.

125. To reduce a fraction to its lowest terms.

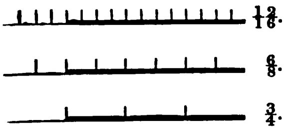
A fraction is in its *lowest terms* when its terms are the least integers that will express its value. In this case its terms are mutually prime.

1. Reduce $\frac{12}{16}$ to its lowest terms.

$\frac{12}{16} = \frac{6}{8} = \frac{3}{4}$, Ans.

or,

$\frac{12}{16} = \frac{3}{4}$, Ans.



Dividing both terms of $\frac{12}{16}$ by 2, we have (Art. 118, c) $\frac{12}{16} = \frac{6}{8}$; dividing both terms of $\frac{6}{8}$ by 2, we have $\frac{6}{8} = \frac{3}{4}$. Or dividing both terms of $\frac{12}{16}$ by 4, their greatest common divisor, we have at once $\frac{12}{16} = \frac{3}{4}$. As 3 and 4 are mutually prime, $\frac{12}{16}$ is reduced to its lowest terms $\frac{3}{4}$. The divided lines also show $\frac{12}{16} = \frac{6}{8} = \frac{3}{4}$.

The principle is the same as in cancellation (Art. 107).

126. Oral Exercises.

2. Reduce to their lowest terms $\frac{2}{4}$; $\frac{3}{6}$; $\frac{4}{8}$; $\frac{3}{9}$; $\frac{6}{9}$; $\frac{4}{10}$; $\frac{6}{8}$; $\frac{6}{10}$; $\frac{8}{10}$; $\frac{6}{12}$.

3. Reduce to their lowest terms $\frac{5}{10}$; $\frac{3}{12}$; $\frac{9}{15}$; $\frac{7}{14}$; $\frac{8}{14}$; $\frac{10}{14}$; $\frac{12}{14}$; $\frac{5}{15}$; $\frac{6}{15}$; $\frac{9}{15}$.

4. Reduce to their lowest terms $\frac{10}{15}$; $\frac{12}{15}$; $\frac{2}{16}$; $\frac{4}{16}$; $\frac{6}{16}$; $\frac{8}{16}$; $\frac{10}{16}$; $\frac{12}{16}$; $\frac{14}{16}$; $\frac{2}{18}$.

5. Reduce to their lowest terms $\frac{2}{12}$; $\frac{3}{15}$; $\frac{7}{21}$; $\frac{8}{28}$; $\frac{9}{33}$; $\frac{4}{18}$; $\frac{9}{36}$; $\frac{5}{20}$; $\frac{5}{25}$.

6. Reduce to their lowest terms $\frac{8}{22}$; $\frac{7}{28}$; $\frac{8}{56}$; $\frac{9}{63}$; $\frac{6}{22}$; $\frac{7}{8}$; $\frac{8}{20}$; $\frac{8}{24}$; $\frac{9}{21}$; $\frac{9}{30}$.

127. Hence, to reduce fractions to their lowest terms,

Rule.

Cancel all the factors common to both numerator and denominator. Or,

Divide both terms by their greatest common divisor.

128. Written Exercises.

Reduce to their lowest terms:

7. $\frac{25}{150}$.	11. $\frac{75}{325}$.	15. $\frac{495}{825}$.	19. $\frac{1728}{8640}$.
8. $\frac{480}{560}$.	12. $\frac{42}{826}$.	16. $\frac{162}{594}$.	20. $\frac{814}{1628}$.
9. $\frac{82}{178}$.	13. $\frac{84}{126}$.	17. $\frac{441}{819}$.	21. $\frac{209}{247}$.
10. $\frac{94}{141}$.	14. $\frac{315}{420}$.	18. $\frac{924}{1617}$.	22. $\frac{2346}{4669}$.

129. To reduce an integer or a mixed number to an improper fraction.

23. In 3 oranges how many fourths of an orange are there?

Solution. In one orange there are 4 fourths; therefore, in 3 oranges there are 3 times 4 fourths, or 12 fourths; $\frac{12}{4}$, Ans.

24. How many sixths are there in $2\frac{5}{6}$?

Solution. As 6 sixths make a unit, in 2 units there are 2 times 6 sixths, or 12 sixths; 12 sixths and 5 sixths are 17 sixths; $\frac{17}{6}$, Ans.

130. Oral Exercises.

25. How many fifths in 6 apples?

26. Reduce 7 to fourths; 9 to sevenths; 6 to elevenths; 5 to eighths; 8 to fifths; 4 to ninths; 9 to fourths.

27. How many quarters in $ 8?

28. Reduce 12 to a fraction whose denominator is 7.

29. Reduce 9 to a fraction whose denominator is 8.

30. Reduce 7 to a fraction whose denominator is 1. **Ans.** $\frac{7}{1}$.

Note. Any integer may be expressed as a fraction by writing 1 under it as its denominator.

31. Reduce 16 to a fraction whose denominator is 1.

32. Reduce to improper fractions in their lowest terms $2\frac{1}{5}$; $8\frac{3}{7}$; $9\frac{4}{6}$; $3\frac{1}{7}$; $8\frac{4}{12}$; $6\frac{1}{3}$; $7\frac{4}{7}$; $11\frac{5}{10}$; $5\frac{8}{9}$; $9\frac{5}{11}$; $5\frac{4}{7}$; $8\frac{3}{5}$; $6\frac{5}{7}$; $9\frac{5}{7}$; $5\frac{7}{8}$; $9\frac{3}{5}$; $11\frac{5}{7}$; $12\frac{8}{9}$; $8\frac{4}{9}$; $9\frac{7}{9}$.

131. Written Exercises.

33. Reduce $714\frac{3}{5}$ to an improper fraction.

$$\begin{array}{r} 714\frac{3}{5} \\ 5 \\ \hline \frac{3573}{5}, \text{ Ans.} \end{array}$$

As there are 5 fifths in a unit, in 714 there are 714 times 5 fifths, or 3570 fifths; 3570 fifths and 3 fifths are 3573 fifths; or $\frac{3573}{5}$, Ans. As the product is numerically the same whether we multiply 5 by 714, or 714 by 5, and the denominator of the fractional part is usually the smaller number, in practice we use the denominator as the multiplier. Hence,

To reduce an integer, or a mixed number, to an improper fraction,

Rule.

Multiply the whole number by the denominator of the fraction, to the product add the numerator, and under the result write the denominator.

34. Reduce 87 to a fraction whose denominator is 87.

Reduce to improper fractions in their lowest terms:

35. $13\frac{5}{8}$.	39. $16\frac{4}{20}$.	43. $314\frac{7}{21}$.
36. $14\frac{8}{20}$.	40. $18\frac{5}{19}$.	44. $278\frac{4}{9}$.
37. $15\frac{5}{60}$.	41. $19\frac{3}{100}$.	45. $673\frac{8}{12}$.
38. $17\frac{3}{8}$.	42. $272\frac{5}{11}$.	46. $946\frac{3}{13}$.

132. To reduce an improper fraction to an integer or mixed number.

47. In 8 fourths of a dollar how many dollars are there?

Solution. As 4 fourths make a unit, there are as many dollars in 8 fourths of a dollar as the number of times 4 fourths is contained in 8 fourths, that is, 2, Ans. See also Art. 118 a.

Oral Exercises.

48. In 12 sixths how many units? Ans. 2.

49. In 27 fifths how many units? Ans. 5 and 2 fifths.

50. Reduce to integers or mixed numbers $\frac{14}{3}$; $\frac{17}{8}$; $\frac{18}{6}$; $\frac{27}{4}$; $\frac{57}{12}$; $\frac{121}{10}$; $\frac{84}{7}$; $\frac{116}{11}$; $\frac{47}{8}$; $\frac{59}{12}$.

133. Hence, to reduce an improper fraction to an integer or mixed number,

Rule.

Divide the numerator by the denominator; if there is any remainder, place it over the divisor, and annex the fraction so formed to the quotient.

134. Written Exercises.

Reduce to integers or mixed numbers:

51. $\frac{67}{37}$.	55. $\frac{178}{61}$.	59. $\frac{872}{25}$.
52. $\frac{84}{19}$.	56. $\frac{828}{173}$.	60. $\frac{1754}{82}$.
53. $\frac{75}{29}$.	57. $\frac{174}{17}$.	61. $\frac{651}{93}$.
54. $\frac{83}{15}$.	58. $\frac{785}{37}$.	62. $\frac{9143}{468}$.

135. To reduce fractions to fractions having a common denominator.

Fractions have a *Common Denominator* when their denominators are equal.

Fractions have their *Least Common Denominator* when the common denominator is the least possible.

136. Oral Exercises.

63. Change $\frac{1}{2}$ and $\frac{2}{3}$ to sixths. $\frac{1}{2}=\frac{3}{6}$; $\frac{2}{3}=\frac{4}{6}$.

64. Change $\frac{1}{3}$ and $\frac{1}{4}$ to twelfths.

65. Reduce $\frac{1}{2}$ and $\frac{3}{4}$ to fractions having a common denominator.

66. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to fractions having a common denominator.

67. Reduce the following pairs of fractions to fractions having a common denominator: $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{1}{5}$ and $\frac{1}{6}$; $\frac{1}{6}$ and $\frac{1}{7}$; $\frac{1}{7}$ and $\frac{1}{8}$; $\frac{1}{8}$ and $\frac{1}{9}$; $\frac{2}{3}$ and $\frac{3}{5}$; $\frac{5}{7}$ and $\frac{4}{9}$; $\frac{3}{11}$ and $\frac{4}{5}$; $\frac{2}{5}$ and $\frac{4}{15}$.

68. Reduce the following pairs of fractions to fractions having their least common denominator: $\frac{1}{2}$ and $\frac{5}{6}$; $\frac{2}{3}$ and $\frac{4}{9}$; $\frac{3}{8}$ and $\frac{5}{12}$; $\frac{2}{5}$ and $\frac{4}{10}$; $\frac{3}{7}$ and $\frac{2}{8}$; $\frac{5}{9}$ and $\frac{5}{6}$; $\frac{7}{8}$ and $\frac{9}{12}$; $\frac{3}{8}$ and $\frac{1}{16}$; $\frac{7}{5}$ and $\frac{7}{7}$; $\frac{3}{11}$ and $\frac{3}{12}$.

69. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{8}$ to fractions having their least common denominator, that is, to eighths.

70. Reduce $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$ to fractions having their least common denominator.

71. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ to fractions having their least common denominator.

NOTE. When the denominators are mutually prime, the least common denominator is the product of all the denominators, and the new numerators are found by multiplying each numerator by all the denominators except its own.

72. Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ to fractions having their least common denominator.

73. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{8}$ to fractions having their least common denominator.

137. From these examples, to reduce fractions to fractions having their least common denominator, we derive the following

Rule.

Reduce each fraction to its lowest terms.

Find the least common multiple of the denominators for the common denominator. For the new numerators multiply each numerator by the quotient arising from dividing this multiple by its denominator.

138. Written Exercises.

Reduce the following sets of fractions to fractions having their least common denominator:

74. $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{3}{7}$.

75. $\frac{7}{8}$, $\frac{1}{9}$, and $\frac{5}{7}$.

76. $\frac{6}{10}$, $\frac{4}{8}$, $\frac{9}{12}$, and $\frac{4}{5}$.

77. $\frac{8}{9}$, $\frac{12}{15}$, $\frac{4}{12}$, and $\frac{9}{18}$.

NOTE. As we know the *factors* of the least common multiple and of each denominator, we can tell what "the quotient arising from dividing this multiple" is, without actually performing the division.

78. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{2}{3}$, and $\frac{5}{9}$.
79. $\frac{7}{8}$, $\frac{8}{9}$, $\frac{10}{21}$, and $\frac{6}{7}$.
80. $\frac{18}{30}$, $\frac{15}{45}$, $\frac{9}{15}$, and $\frac{18}{72}$.
81. $\frac{3}{11}$, $\frac{4}{7}$, and $\frac{10}{28}$.
82. $\frac{7}{65}$, $\frac{3}{91}$, and $\frac{5}{637}$.
83. $\frac{8}{13}$, $\frac{7}{51}$, $\frac{11}{17}$, and $\frac{2}{39}$.
84. $\frac{17}{31}$, $\frac{15}{46}$, $\frac{8}{12}$, and $\frac{3}{62}$.
85. $\frac{7}{8}$, $\frac{13}{26}$, $\frac{7}{49}$, and $\frac{5}{312}$.
86. $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{24}$, and $\frac{3}{48}$.
87. $\frac{1}{3}$, $\frac{22}{18}$, $\frac{9}{24}$, and $\frac{7}{72}$.

ADDITION OF FRACTIONS.

139. Oral Exercises.

88. James had 2 quarters of a dollar, and his father gave him another quarter; how many quarters did he have? $\frac{2}{4}+\frac{1}{4}=?$

89. William had half a dollar, and his father gave him a quarter; how much money did William have then?

$$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=?$$

90. Add $\frac{1}{3}$ and $\frac{2}{3}$ together.
91. Add $\frac{1}{3}$ and $\frac{1}{4}$ together. $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=?$
92. Add $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ together.
93. Add $\frac{1}{6}$ and $\frac{5}{6}$ together.
94. Add $\frac{1}{2}$ and $\frac{1}{8}$ together.
95. Add $\frac{3}{7}$ and $\frac{2}{7}$ together.
96. Add $\frac{1}{4}$ and $\frac{3}{8}$ together.
97. Add $\frac{3}{4}$ and $\frac{5}{8}$ together.
98. Add $\frac{5}{6}$ and $\frac{2}{3}$ together.
99. Add $\frac{1}{5}$, $\frac{4}{5}$, and $\frac{1}{10}$ together.

140. From these examples in addition of fractions we derive the following

Rule.

Reduce the fractions, if necessary, to fractions having a common denominator; then write the sum of the new numerators over the common denominator.

141. Written Exercises.

100. Add $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{24}$, and $\frac{3}{48}$.

101. Add $\frac{5}{6}$, $\frac{3}{13}$, and $\frac{7}{17}$.

102. Add $\frac{2}{9}$, $\frac{16}{21}$, and $\frac{4}{13}$.

103. Add $\frac{1}{2}$, $\frac{5}{6}$, $\frac{7}{15}$, and $\frac{11}{18}$.

104. Add $\frac{7}{10}$, $\frac{11}{12}$, $\frac{4}{15}$, $\frac{6}{25}$, and $\frac{1}{60}$.

105. $\frac{3}{5} + \frac{5}{6} + \frac{2}{18} = ?$

106. $\frac{2}{5} + \frac{13}{15} + \frac{5}{3} = ?$

107. $\frac{7}{9} + \frac{1}{6} + \frac{7}{36} + \frac{5}{8} = ?$

108. $\frac{3}{25} + \frac{1}{8} + \frac{3}{5} + \frac{3}{4} = ?$

109. $\frac{1}{11} + \frac{1}{7} + \frac{2}{3} + \frac{1}{21} = ?$

110. $\frac{2}{7} + \frac{9}{11} + \frac{8}{24} = ?$

111. $\frac{3}{90} + \frac{1}{15} + \frac{3}{7} = ?$

112. $\frac{7}{18} + \frac{7}{13} + \frac{3}{24} = ?$

113. What is the sum of $16\frac{4}{5}$ and $35\frac{3}{4}$?

$$\begin{aligned} 16\tfrac{4}{5} &= 16\tfrac{16}{20} \\ 35\tfrac{3}{4} &= \underline{35\tfrac{15}{20}} \\ \text{Ans. } & \ 52\tfrac{11}{20} \end{aligned}$$

Reduce the fractions to fractions having a common denominator; $\frac{16}{20} + \frac{15}{20} = \frac{31}{20} = 1\frac{11}{20}$; write the $\frac{11}{20}$ in its place, and add the 1 unit to the sum of the units; and proceed as in simple addition.

114. $5\frac{4}{5} + 18\frac{7}{15} + 25\frac{9}{30} = ?$

115. $187\frac{1}{3} + 1976\frac{2}{5} + 746\frac{1}{9} = ?$

116. $1764\frac{3}{4} + 7867\frac{2}{3} + \frac{3}{27} = ?$

117. A man had seven horses sold at auction. The first brought \$ $141\frac{1}{2}$, the second \$ $173\frac{2}{3}$, the third \$ $131\frac{1}{4}$, the fourth \$ $217\frac{3}{4}$, the fifth \$ $183\frac{7}{8}$, the sixth \$ $224\frac{1}{3}$, the seventh \$ $257\frac{1}{8}$. What did he receive for all?

118. A merchant bought five hogsheads of sugar weighing respectively, $417\frac{1}{2}$, $487\frac{1}{4}$, $516\frac{3}{8}$, $419\frac{3}{4}$, and $502\frac{1}{8}$ pounds. How many pounds of sugar did he buy?

SUBTRACTION OF FRACTIONS.

142. Oral Exercises.

119. John had 3 quarters of a dollar and lost 2 of them; how many quarters had he left? $\frac{3}{4} - \frac{2}{4} = ?$

120. Henry had half a dollar and spent a quarter; how much did he have left? $\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = ?$

121. From $\frac{5}{9}$ take $\frac{2}{9}$.

122. From $\frac{3}{5}$ take $\frac{1}{5}$.

123. Take $\frac{3}{7}$ from $\frac{5}{7}$.

124. Take $\frac{1}{3}$ from $\frac{3}{5}$. $\frac{3}{5} - \frac{1}{3} = \frac{9}{15} - \frac{5}{15} = ?$

125. $\frac{5}{6} - \frac{4}{5} = ?$ $\frac{1}{6} - \frac{1}{7} = ?$ $\frac{5}{8} - \frac{2}{9} = ?$ $\frac{1}{3} - \frac{1}{6} = ?$ $\frac{1}{2} - \frac{1}{3} = ?$

126. $\frac{2}{3} - \frac{1}{2} = ?$ $\frac{3}{5} - \frac{1}{2} = ?$ $\frac{4}{9} - \frac{4}{10} = ?$ $\frac{7}{10} - \frac{2}{5} = ?$ $\frac{7}{8} - \frac{5}{6} = ?$

127. If a boy who has $\frac{4}{5}$ of a dollar gives away $\frac{3}{10}$ of a dollar, how much has he left?

128. If $\frac{1}{3}$ of a pole is above the surface of the water, and $\frac{1}{5}$ of it is in the mud, what part of it is in the water?

143. From these examples in subtraction of fractions we derive the following

Rule.

Reduce the fractions, if necessary, to fractions having a common denominator; then subtract the numerator of the subtrahend from that of the minuend, and write the result over the common denominator.

144. Written Exercises.

129. $\frac{53}{85} - \frac{29}{85} = ?$

130. $\frac{214}{222} - \frac{111}{222} = ?$

131. $\frac{127}{900} - \frac{3}{30} = ?$

132. $\frac{27}{84} - \frac{6}{36} = ?$

133. $\frac{13}{60} - \frac{1}{14} = ?$

134. $\frac{77}{336} - \frac{5}{300} = ?$

135. $\frac{87}{252} - \frac{25}{330} = ?$

136. $\frac{5}{80} - \frac{15}{720} = ?$

137. $\frac{4}{176} - \frac{1}{88} = ?$

138. $\frac{14}{72} - \frac{13}{78} = ?$

139. From $187\frac{3}{4}$ take $75\frac{5}{6}$.

$$\begin{aligned} 187\tfrac{3}{4} &= 187\tfrac{9}{12} \\ 75\tfrac{5}{6} &= 75\tfrac{10}{12} \\ \text{Ans. } & \ 111\tfrac{11}{12} \end{aligned}$$

Reduce the fractions to fractions having a common denominator; as we cannot take $\frac{10}{12}$ from $\frac{9}{12}$, we take a unit from the 7 units of 187, and reduce it to twelfths; adding this to the $\frac{9}{12}$, we have $\frac{21}{12}$, from which subtracting $\frac{10}{12}$ we have $\frac{11}{12}$; then we proceed as in simple subtraction.

140. $1198\frac{3}{5} - 149\frac{2}{3} = ?$

141. $1786\frac{2}{11} - 417\frac{3}{10} = ?$

142. $6478\frac{1}{3} - 819\frac{2}{9} = ?$

143. $77\frac{1}{6} - 47\frac{5}{9} = ?$

MULTIPLICATION OF FRACTIONS.

145. Oral Exercises.

144. At $\$\frac{2}{5}$ a meter what will 4 meters of cloth cost?

Solution. If one meter costs two fifths of a dollar, 4 meters will cost 4 times 2 fifths, or 8 fifths of a dollar.

$$\$\tfrac{2}{5} \times 4 = \$\tfrac{8}{5}, \text{ Ans.} \quad \text{(Art. 118 b.)}$$

145. Alfred has $\frac{1}{8}$ of a dollar and James 4 times as much; what part of a dollar has James?

$$\$\tfrac{1}{8} \times 4 = \$\tfrac{1}{2}, \text{ Ans.} \quad \text{(Art. 118 b.)}$$

How can a fraction be multiplied by an integer?

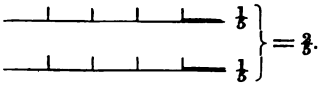
146. Multiply $\frac{2}{7}$ by 3; $\frac{2}{5}$ by 2; $\frac{3}{11}$ by 5; $\frac{3}{13}$ by 4; $\frac{2}{9}$ by 5.

147. Multiply $\frac{1}{6}$ by 3; $\frac{3}{10}$ by 2; $\frac{4}{15}$ by 5; $\frac{7}{18}$ by 9; $\frac{3}{20}$ by 4.

148. Multiply $\frac{5}{6}$ by 4; $\frac{7}{12}$ by 10; $\frac{3}{8}$ by 6; $\frac{7}{10}$ by 8; $\frac{5}{12}$ by 9.

Does $\frac{1}{3}$ multiplied by 2 equal 2 multiplied by $\frac{1}{3}$?

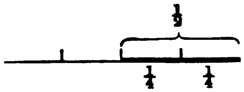
149. Henry and James each had 1 dollar, and each gave $\frac{1}{5}$ of what he had to William; what part of a dollar did William receive? $\frac{1}{5}$ of $\$2 = ?$



This illustration shows that $\frac{1}{5}$ of 2 equal things $= \frac{2}{5}$ of either one of them, or that $\frac{1}{5}$ of $2 = \frac{2}{5}$ of 1.

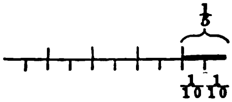
150. What is $\frac{1}{3}$ of 5? $\frac{1}{2}$ of 7? $\frac{1}{4}$ of 3? $\frac{1}{5}$ of 4? $\frac{1}{7}$ of 5?

151. If John has $\frac{1}{2}$ of a dollar and gives away $\frac{1}{2}$ of it, what part of a dollar does he give away? $\frac{1}{2}$ of $\frac{1}{2} = ?$



This illustration shows that $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$.

152. If Oliver has $\frac{1}{5}$ of a dollar and gives away $\frac{1}{2}$ of it, what part of a dollar does he give away? $\frac{1}{2}$ of $\frac{1}{5} = ?$



This illustration shows that $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$.

153. Mary had 5 quarts of berries, and sold $\frac{1}{3}$ of them; how many quarts did she sell?

154. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{1}{7}$?

155. What is $\frac{1}{3}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{2}{5}$? $\frac{1}{3}$ of $\frac{4}{5}$?

156. What is $\frac{2}{5}$ of 6? $\frac{3}{7}$ of $\frac{1}{2}$? $\frac{1}{3}$ of 7? $\frac{1}{4}$ of 11? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{1}{7}$ of $\frac{1}{3}$? $\frac{3}{5}$ of $\frac{3}{8}$? $\frac{5}{9}$ of $\frac{3}{7}$? $\frac{7}{11}$ of $\frac{5}{8}$? $\frac{4}{9}$ of $\frac{3}{10}$?

146. Written Exercises.

157. Multiply $\frac{5}{7}$ by $\frac{3}{4}$.

$$\frac{5}{7} \times 3 = \frac{15}{7}$$

$$\frac{15}{7} \div 4 = \frac{15}{28}, \text{ Ans.}$$

Hence,

$$\frac{5}{7} \times \frac{3}{4} = \frac{15}{28}, \text{ Ans.}$$

By Art. 118 b, $\frac{5}{7} \times 3 = \frac{15}{7}$; but the multiplier is not 3, but $\frac{3}{4}$, $3 \div 4$, that is, 3 is 4 times the given multiplier; therefore the product $\frac{15}{7}$ is 4 times the product sought, and must be divided by 4 to obtain the correct result. By Art. 118 b, $\frac{15}{7} \div 4 = \frac{15}{28}$, Ans. Hence,

To multiply a fraction by a fraction,

Rule.

Multiply the numerators together for a new numerator, and the denominators for a new denominator.

Note 1. As a mixed number can be reduced to an improper fraction (Art. 131), and an integer expressed as a fraction by writing under it 1 as its denominator, this rule covers all possible cases in Multiplication of Fractions.

158. Multiply $\frac{13}{15}$ by $\frac{7}{11}$.

159. Multiply $\frac{23}{17}$ by $\frac{18}{25}$.

160. Multiply $\frac{12}{35}$ by $\frac{5}{6}$.

$$\frac{\overset{2}{\cancel{12}}}{\underset{7}{\cancel{35}}} \times \frac{\cancel{5}}{\cancel{6}} = \frac{2}{7}, \text{ Ans.}$$

Before multiplying, we cancel according to Art. 107.

Note 2. *Always cancel, if possible.*

161. Multiply $\frac{46}{85}$ by $\frac{25}{23}$.

162. Multiply $\frac{36}{125}$ by $\frac{75}{84}$.

163. Multiply $\frac{25}{44}$ by $\frac{8}{75}$.

164. Multiply $\frac{33}{54}$ by $\frac{12}{44}$.

165. Multiply $\frac{24}{35}$ by $\frac{28}{84}$.

166. Multiply $\frac{144}{365}$ by $\frac{73}{288}$.

167. Multiply $8\frac{1}{3}$ by $2\frac{2}{5}$.

168. Multiply $11\frac{9}{11}$ by $7\frac{1}{3}$.

169. Multiply $177\frac{2}{3}$ by 3.

$$\begin{array}{r} 177\frac{2}{3} \\ 3 \\ \hline \text{Ans. } 533 \end{array}$$

Three times $\frac{2}{3} = 2$; 3 times 7 are 21, and 2 to be added give 23; and so on, as in multiplication of simple numbers.

Note 3. If one of the factors is integral and the other a mixed number, it is shorter to multiply the fraction and the integral part of the mixed number separately, as in Ex. 169.

170. Multiply 1875 by $7\frac{1}{5}$.

$$
\begin{array}{rr}
 & 1875 \\
 & 7\frac{1}{5} \\
\hline
 & 13125 \\
 & 375 \\
\hline
\text{Ans.} & 13500
\end{array}
$$

Multiply 1875 first by 7; then take $\frac{1}{5}$ of 1875 and add these partial products together.

171. Multiply $176\frac{3}{7}$ by 8.

172. Multiply 789 by $16\frac{1}{7}$.

173. Multiply $6344\frac{1}{11}$ by 18.

174. Multiply 683 by $17\frac{1}{5}$.

175. What is $\frac{3}{11}$ of $\frac{5}{8}$ of $\frac{11}{12}$ of $8\frac{1}{5}$?

176. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$?

177. What is $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{5}{11}$ of $\frac{7}{5}$?

178. What is $\frac{5}{7}$ of $\frac{6}{5}$ of $\frac{13}{17}$ of $\frac{17}{12}$ of $\frac{8}{13}$?

179. Multiply $\frac{1728}{177}$ by 177. Ans. 1728.

NOTE 4. As in Ex. 179, if we multiply a fraction by its denominator, the product is its numerator.

180. Multiply 9786 by $\frac{111}{9786}$.

181. At \$ $2\frac{3}{4}$ a yard what will $5\frac{1}{4}$ yards of broadcloth cost?

182. If an acre of land costs \$ 75, what will $17\frac{3}{4}$ acres cost?

183. If a horse is driven $27\frac{1}{3}$ miles each day, how far will he go in $17\frac{3}{5}$ days?

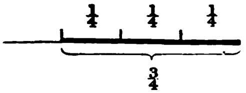
184. At \$ $7\frac{7}{8}$ a cord what will $18\frac{8}{9}$ cords of wood cost?

DIVISION OF FRACTIONS.

147. Oral Exercises.

185. How can $\$\frac{3}{4}$ be divided equally among 3 boys?

$$\frac{3}{4} \div 3 = ?$$

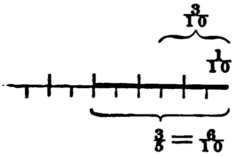


$$\frac{3}{4} \div 3 = \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}} = \frac{1}{4}.$$

If $\$\frac{3}{4}$ is divided equally among 3 boys, each boy must have 1 third of $\$\frac{3}{4}$; $\frac{1}{3}$ of $\$\frac{3}{4} = \$\frac{1}{4}$, Ans.

Dividing by 3 is equivalent to multiplying by what?

186. How can $\$\frac{3}{5}$ be divided equally between 2 boys?



$$\frac{3}{5} \div 2 = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}.$$

If $\$\frac{3}{5}$ is divided equally between 2 boys, each boy must have 1 half of $\$\frac{3}{5}$; $\frac{3}{5} = \frac{6}{10}$; therefore $\frac{1}{2}$ of $\$\frac{3}{5} = \frac{1}{2}$ of $\$\frac{6}{10} = \$\frac{3}{10}$, Ans.

Dividing by 2 is equivalent to multiplying by what?

How can a fraction be divided by an integer?

187. Divide $\frac{4}{7}$ by 4; $\frac{10}{11}$ by 5; $\frac{9}{10}$ by 3; $\frac{8}{9}$ by 2; $\frac{6}{7}$ by 3.

188. Divide $\frac{2}{3}$ by 3; $\frac{3}{5}$ by 2; $\frac{2}{7}$ by 2; $\frac{4}{5}$ by 5; $\frac{5}{6}$ by 3.

189. Divide $\frac{2}{3}$ by 4; $\frac{3}{5}$ by 6; $\frac{4}{7}$ by 6; $\frac{3}{8}$ by 6; $\frac{2}{9}$ by 4.

190. If James has $\$2$ and spends $\frac{1}{5}$ of a dollar a day, how many days will it last him?

How many fifths of a dollar are there in $\$2$? How many times is $\frac{1}{5}$ contained in 2? $2 \div \frac{1}{5} = ?$

How can an integer be divided by a fraction?

191. Six are how many times $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$?

192. Eight are how many times $\frac{1}{2}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

193. Divide 7 by $\frac{1}{2}$; 8 by $\frac{1}{3}$; 9 by $\frac{2}{3}$; 10 by $\frac{2}{5}$; 12 by $\frac{3}{4}$.

194. Divide 5 by $\frac{1}{3}$; $\frac{1}{3}$ by 5.

195. If John spends $\frac{2}{5}$ of a dollar a day, how many days will it take him to spend $\frac{8}{5}$ of a dollar?

How many times is 2 fifths contained in 8 fifths? $\frac{8}{5} \div \frac{2}{5} = ?$

NOTE. If the denominator of the dividend and of the divisor are alike, they may both be disregarded; for, evidently, 2 fifths is contained in 8 fifths just as many times as 2 dollars is contained in 8 dollars, or 2 in 8.

196. Divide $\frac{9}{11}$ by $\frac{3}{11}$; $\frac{6}{7}$ by $\frac{3}{7}$; $\frac{10}{13}$ by $\frac{2}{13}$; $\frac{8}{9}$ by $\frac{4}{9}$.

197. How many times is $\frac{1}{8}$ contained in $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{1}{8} = \frac{4}{8} \div \frac{1}{8} = ?$$

198. Divide $\frac{1}{5}$ by $\frac{1}{10}$; $\frac{3}{7}$ by $\frac{3}{14}$; $\frac{4}{9}$ by $\frac{2}{27}$; $\frac{6}{7}$ by $\frac{2}{21}$.

199. How many times is $\frac{1}{5}$ contained in $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{1}{5} = \frac{5}{10} \div \frac{2}{10} = 5 \div 2 = \frac{5}{2} = 2\frac{1}{2}, \text{ Ans.}$$

200. How many times is $\frac{1}{3}$ contained in $\frac{1}{5}$?

$$\frac{1}{5} \div \frac{1}{3} = \frac{3}{15} \div \frac{5}{15} = 3 \div 5 = \frac{3}{5}, \text{ Ans.}$$

How then can one fraction be divided by another?

201. Divide $\frac{1}{2}$ by $\frac{1}{6}$; $\frac{1}{5}$ by $\frac{1}{2}$; $\frac{2}{3}$ by $\frac{1}{6}$; $\frac{3}{7}$ by $\frac{1}{2}$.

202. Divide $\frac{1}{5}$ by $\frac{2}{7}$; $1\frac{1}{2}$ by $\frac{1}{3}$; $3\frac{1}{3}$ by $\frac{1}{4}$; $4\frac{1}{5}$ by $\frac{1}{6}$; $\frac{1}{6}$ by $4\frac{1}{5}$.

148. Written Exercises.

203. Divide $\frac{18}{35}$ by $\frac{6}{7}$.

$$\frac{18}{35} \div \frac{6}{7} = \frac{3}{5}, \text{ Ans.}$$

As in multiplication of fractions we multiply the numerators together for the numerator of the product, and the denominators for the denominator, and division is the reverse of multiplication, therefore, if we divide the numerator of the dividend by the numerator of the divisor, and the denominator of the dividend by the denominator of the divisor, we shall have the numerator and denominator respectively of the quotient.

The practical difficulty with this method is that the terms of the dividend are rarely both divisible by the respective terms of the divisor.

204. Divide $\frac{3}{7}$ by $\frac{2}{5}$.

$$\frac{3}{7} \div 2 = \frac{3}{14}$$

$$\frac{3}{14} \times 5 = \frac{15}{14}, \text{ Ans.}$$

Hence,

$$\frac{3}{7} \div \frac{2}{5} = \frac{3}{7} \times \frac{5}{2} = \frac{15}{14}, \text{ Ans.}$$

By Art. 118 b, $\frac{3}{7} \div 2 = \frac{3}{14}$; but the divisor is not 2 but $\frac{2}{5}$, or $2 \div 5$, that is, 2 is 5 times the given divisor, therefore the quotient $\frac{3}{14}$ is 1 fifth the quotient sought, and must be multiplied by 5 to obtain the correct result. By Art. 118 b, $\frac{3}{14} \times 5 = \frac{15}{14}$, Ans.

It will be seen that we multiply the denominator of the dividend by the numerator of the divisor for the denominator of the quotient, and the numerator of the dividend by the denominator of the divisor for the numerator of the quotient. Hence,

To divide a fraction by a fraction,

Rule.

Invert the divisor and proceed as in multiplication of fractions.

NOTE 1. As an integer can be expressed as a fraction by writing 1 under it as its denominator, and a mixed number can be reduced to an improper fraction, this rule covers all possible cases in Division of Fractions.

205. Divide $\frac{13}{47}$ by $\frac{8}{11}$.

206. Divide $\frac{14}{15}$ by $\frac{11}{13}$.

207. Divide $\frac{117}{873}$ by $\frac{13}{1746}$.

$$\frac{117}{873} \div \frac{13}{1746} = \frac{\overset{9}{\cancel{117}}}{\cancel{873}} \times \frac{\overset{2}{\cancel{1746}}}{\cancel{13}} = 18, \text{ Ans.}$$

NOTE 2. *Always cancel, if possible.*

208. Divide $\frac{52}{189}$ by $\frac{13}{126}$.

209. Divide $\frac{177}{814}$ by $\frac{3}{11}$.

210. Divide $\frac{1807}{4359}$ by $\frac{13}{4359}$.

211. Divide $\frac{144}{896}$ by $\frac{1728}{896}$.

212. Divide $17\frac{6}{7}$ by 14.

213. Divide $178\frac{1}{3}$ by $16\frac{1}{5}$.

214. Divide $148\frac{1}{7}$ by $\frac{6}{7}$.

215. Divide $\frac{6}{7}$ by $148\frac{1}{7}$.

216. Divide $1876\frac{2}{3}$ by 3.

$$\begin{array}{r} 3\underline{)\ 1876\frac{2}{3}} \\ \text{Ans. } 625\frac{5}{9} \end{array}$$

Dividing as in short division, we have 1 unit remainder; this 1 unit is equal to $\frac{3}{3}$, which added to the $\frac{2}{3}$ gives $\frac{5}{3}$; $\frac{5}{3} \div 3 = \frac{5}{9}$, which added to the quotient 625 gives $625\frac{5}{9}$, Ans.

217. Divide $14789\frac{1}{5}$ by 3.

218. Divide $8763\frac{7}{11}$ by 11.

219. Divide $\frac{161}{6}$ by 42.

220. Divide $\frac{7}{8}$ of $\frac{16}{15}$ by $\frac{4}{11}$ of $\frac{33}{45}$.

221. Reduce $\dfrac{18\frac{3}{7}}{\frac{3}{5}}$ to its simplest form.

$$\frac{18\frac{3}{7}}{\frac{3}{5}} = 18\tfrac{3}{7} \div \frac{3}{5} = \frac{\overset{43}{\cancel{129}}}{7} \times \frac{5}{\cancel{3}} = \frac{215}{7} = 30\tfrac{5}{7}, \text{ Ans.}$$

NOTE 3. Such expressions as this in Ex. 221 are called complex fractions. Their reduction consists in performing the division expressed.

222. Reduce $\dfrac{175\frac{1}{3}}{184\frac{1}{2}}$ to its simplest form.

$$\frac{175\frac{1}{3}}{184\frac{1}{2}} = \frac{1052}{1107}, \text{ Ans.}$$

We multiply both terms of the complex fraction by 6, the least common multiple of the denominators of the fractional parts $\frac{1}{3}$ and $\frac{1}{2}$.

223. Reduce $\dfrac{\frac{3}{4} \times \frac{2}{3} \times 8}{\frac{2}{7} \times 9\frac{1}{3} \times \frac{7}{8}}$ to its simplest form.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{2}{\cancel{3}} \times \frac{\cancel{8}}{1} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{3}{\underset{7}{\cancel{28}}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{7}} = \frac{12}{7} = 1\tfrac{5}{7}, \text{ Ans.}$$

224. Reduce $\dfrac{18\frac{3}{5}}{85\frac{1}{2}}$ to its simplest form.

225. Reduce $\dfrac{4\frac{3}{5}}{6\frac{9}{10}}$.

226. Reduce $\dfrac{8\frac{4}{9}}{12\frac{2}{3}}$.

227. Reduce $\dfrac{4\frac{3}{8}}{1\frac{3}{8}}$.

228. Reduce $\dfrac{3\frac{5}{8}}{6\frac{3}{4}}$.

229. Reduce $\dfrac{5\frac{3}{8}}{7\frac{5}{6}}$.

230. Reduce $\dfrac{5\frac{3}{5}}{\frac{3}{5}}$.

231. Reduce $\dfrac{\frac{2}{5} \text{ of } \frac{3}{7} \text{ of } 2\frac{1}{3}}{3\frac{4}{5}}$.

232. Reduce $\dfrac{7\frac{2}{3}}{4\frac{4}{9}}$.

233. Reduce $\dfrac{18}{2\frac{1}{3}}$.

234. Reduce $\dfrac{\frac{3}{5} \text{ of } \frac{2}{6} \text{ of } \frac{7}{8} \text{ of } \frac{5}{4}}{\frac{14}{6} \text{ of } \frac{3}{16} \text{ of } \frac{1}{4} \text{ of } 2}$.

235. If $3\frac{1}{2}$ pounds of beef cost $43\frac{3}{4}$ cents, what is the price of a pound?

236. If a man travels by rail $376\frac{1}{3}$ miles in $15\frac{1}{2}$ hours, how far does he travel in an hour?

237. If a man travels at the rate of $27\frac{3}{8}$ miles an hour, how long will it take him to travel $784\frac{3}{4}$ miles?

149. Miscellaneous Oral Exercises.

238. Reduce to lowest terms, $\frac{16}{24}$; $\frac{7}{56}$; $\frac{8}{48}$; $\frac{17}{51}$; $\frac{121}{132}$; $\frac{117}{126}$; $\frac{13}{39}$; $\frac{8}{50}$; $\frac{21}{56}$; $\frac{18}{81}$.

239. Reduce to improper fractions, $4\frac{1}{5}$; $7\frac{3}{7}$; $6\frac{2}{3}$; $8\frac{1}{9}$; $5\frac{2}{5}$; $9\frac{2}{7}$; $11\frac{2}{9}$; $3\frac{11}{12}$; $10\frac{6}{7}$; $12\frac{2}{11}$.

240. Reduce to integral, or mixed numbers, $\frac{18}{4}$; $\frac{51}{8}$; $\frac{17}{5}$; $\frac{19}{7}$; $\frac{56}{11}$; $\frac{47}{9}$; $\frac{82}{3}$; $\frac{97}{10}$; $\frac{63}{7}$; $\frac{75}{6}$.

241. Add $\frac{1}{2}$ to $\frac{1}{3}$; $\frac{1}{3}$ to $\frac{1}{4}$; $\frac{2}{3}$ to $\frac{2}{5}$; $\frac{6}{7}$ to $\frac{5}{6}$; $\frac{3}{4}$ to $\frac{4}{5}$; $\frac{7}{8}$ to $\frac{3}{7}$; $\frac{3}{5}$ to $\frac{3}{4}$; $\frac{2}{9}$ to $\frac{2}{5}$; $\frac{3}{11}$ to $\frac{2}{3}$; $\frac{5}{8}$ to $\frac{1}{6}$.

242. Add $5\frac{1}{5}$ to $4\frac{1}{3}$; $6\frac{2}{3}$ to $7\frac{2}{5}$; $3\frac{2}{7}$ to $8\frac{1}{3}$; $7\frac{1}{8}$ to $6\frac{1}{4}$; $9\frac{2}{3}$ to $3\frac{5}{6}$.

243. Subtract $\frac{1}{3}$ from $\frac{1}{2}$; $\frac{2}{3}$ from $\frac{5}{6}$; $\frac{1}{7}$ from $\frac{1}{6}$; $\frac{2}{9}$ from $\frac{2}{3}$; $\frac{3}{5}$ from $\frac{7}{8}$.

244. $8\frac{2}{3} - 2\frac{1}{2} = ?$ $7\frac{4}{5} - 4\frac{3}{4} = ?$ $12\frac{1}{4} - 7\frac{1}{5} = ?$ $9\frac{1}{2} - 8\frac{7}{8} = ?$ $6\frac{3}{5} - 4\frac{4}{5} = ?$

245. Multiply $\frac{5}{8}$ by 3; $\frac{5}{9}$ by 3; 7 by $\frac{2}{3}$; 8 by $\frac{3}{5}$; 18 by $\frac{2}{9}$; $\frac{3}{4}$ by $\frac{1}{3}$; $\frac{7}{8}$ by $\frac{2}{5}$; $\frac{5}{7}$ by $\frac{3}{5}$; $\frac{4}{9}$ by $\frac{3}{8}$; $\frac{5}{6}$ by $\frac{4}{5}$.

246. Divide $\frac{8}{9}$ by 4; $\frac{8}{9}$ by 3; 4 by $\frac{8}{9}$; 3 by $\frac{8}{9}$; 7 by $\frac{1}{2}$; $33\frac{1}{3}$ by 3; $\frac{1}{2}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{1}{2}$; $\frac{2}{7}$ by $\frac{3}{5}$; $\frac{3}{5}$ by $\frac{3}{4}$.

247. What part of 6 is 1? 3 is what part of 6? 5 is what part of 6?

248. What part of $\frac{5}{6}$ is $\frac{2}{3}$? Ans. $\frac{4}{5}$.

NOTE. $\frac{2}{3}$ is the product and $\frac{5}{6}$ is one of two factors that produce it. The question is, — what is the other factor?

249. What part of $\frac{4}{7}$ is $\frac{3}{8}$?

250. Reduce $\frac{1\frac{1}{2}}{1\frac{2}{3}}$ to its simplest form.

251. Multiply $\frac{25}{33}$ by 33.

252. If $\frac{1}{4}$ of a ton of hay cost $ 5, what will 8 tons cost?

253. If $\frac{3}{5}$ of an acre of land cost $ 60, what will $2\frac{1}{2}$ acres cost?

254. If 4 barrels of beans cost $ 36, what will $\frac{2}{5}$ of a barrel cost?

255. If 2 men can do a piece of work in 4 days, how long will it take 1 man to do the same work?

256. If 2 tons of hay cost $ 40, what will $\frac{3}{5}$ of a ton cost?

257. At $ 72 an acre, what is the value of $\frac{7}{8}$ of an acre of land?

258. If I own $\frac{5}{6}$ of a farm and sell $\frac{1}{3}$ of my share, how much have I left?

259. Mr. James sold a horse for $240, which was $\frac{5}{6}$ of what he paid; what did he pay? what did he lose?

260. If the difference between $\frac{2}{3}$ and $\frac{3}{5}$ of my age is 3 years, how old am I?

261. If $\frac{2}{5}$ of my age added to my age is 35 years, how old am I?

262. If a boy spends $\frac{2}{3}$ of $\frac{3}{5}$ of his money and then has 60 cents left, how much did he have at first?

263. $\frac{2}{3}$ of 60 is $\frac{4}{5}$ of what number?

264. A boy bought a ball and a bat for 91 cents. The bat cost $\frac{3}{4}$ as much as the ball; what did each cost?

265. If $\frac{3}{4}$ of an acre of land is to be divided into 4 equal gardens, what part of an acre will there be in each garden?

266. If 6 yards of cloth cost $2\frac{1}{4}$, what is the price of 1 yard?

267. If 1 man can build a wall in $8\frac{4}{5}$ days, how long will it take 11 men to build it?

268. At $7 a barrel what will $6\frac{3}{7}$ barrels of flour cost?

269. At $8 a yard what will $4\frac{1}{2}$ yards of cloth cost?

270. A man sold a cow for $35, which was $\frac{5}{8}$ of what she cost. What did she cost?

271. A man bought a sheep for $9, and a cow for $2\frac{2}{3}$ times as many dollars. What did he give for both?

272. If you are 12 years old, and $\frac{1}{3}$ of your age is $\frac{1}{5}$ of your brother's age, how old is your brother?

273. A man sold a horse for $\frac{5}{6}$ of what it cost, losing $10. What did the horse cost?

274. John has 15 cents, and $\frac{2}{3}$ of the number of cents John has is 2 less than $\frac{3}{5}$ of the number Charles has. How many cents has Charles?

275. If a man buys $\frac{2}{3}$ of a ton of coal in November, $\frac{3}{4}$ of a ton in December, and $\frac{5}{6}$ of a ton in January, how many tons does he buy in the three months?

276. At $ 6 a ton what does the coal mentioned in Ex. 275 cost?

277. Among how many boys can 5 apples be divided by giving $\frac{5}{8}$ of an apple to each boy?

278. What will a dozen eggs cost if 3 cost 5 cents?

279. If 7 men can do a piece of work in $4\frac{2}{7}$ days, how long will it take 5 men?

280. If $\frac{3}{14}$ of a pole is standing in mud, $\frac{3}{7}$ in water, and 15 feet above the water, how long is the pole?

281. If $\frac{3}{4}$ of a bushel of corn can be bought for $ $\frac{3}{8}$, how many bushels can be bought for $ 4?

282. A boy being asked how many chickens he had, replied, that if he had as many more and $\frac{1}{3}$ as many more and 4 more he should have 25. How many had he?

283. I spent $ 0.72 for a bat and a ball, giving $\frac{3}{5}$ as much for the bat as for the ball. How much did I pay for the bat, and how much for the ball?

284. William spent $\frac{3}{5}$ of his money and lost $\frac{1}{10}$, and then found he had 18 cents left. How much did he have at first?

285. James lost $\frac{1}{4}$ of his money and spent $\frac{2}{3}$ of the remainder, and then had 8 cents left. How much did he have at first?

286. A farmer sold $\frac{2}{5}$ of his sheep and bought $\frac{1}{2}$ as many as he had left, when he had 27 sheep. How many had he at first?

287. A boy 14 years old, being asked how old his father was, said that $\frac{4}{7}$ of his own age was exactly $\frac{2}{9}$ of his father's age. How old was his father?

288. How many dozen eggs at $ $\frac{1}{6}$ a dozen must be given for 7 pounds of tea at $ $\frac{4}{7}$ a pound?

289. Richard sold his knife for 54 cents, which was $\frac{1}{8}$ more than he gave for it. What did he give for it?

290. A sold a horse to B for $ 84, which was $\frac{1}{6}$ more than he paid. What did he pay for the horse?

291. If 5 horses eat $8\frac{1}{3}$ tons of hay in 6 months, how many tons will 8 horses eat in the same time?

292. A can do a piece of work in 4 days and B in 5 days. What part can each do in a day? What part can both do in one day? How many days will it take both together to do the work?

293. If A can do a piece of work in 6 days and B in 8 days, how long will it take both together to do it?

294. If A can build a wall in 3 days, B in 4 days, and C in 6 days, how long will it take A, B, and C together to build it?

150. Miscellaneous Written Examples.

295. Find the sum of $187\frac{1}{2}$, $734\frac{2}{3}$, and $4145\frac{5}{8}$.

296. Take $187\frac{3}{5}$ from $754\frac{1}{7}$.

297. Multiply $17\frac{1}{6}$ by $8\frac{1}{3}$.

298. Divide $\frac{3}{5}$ of $\frac{5}{7}$ by $7\frac{6}{11}$.

299. Reduce $\dfrac{14\frac{3}{8}}{17\frac{4}{5}}$ to its simplest form.

300. Reduce $\dfrac{\frac{4}{5} \text{ of } \frac{2}{3} \text{ of } \frac{7}{8} \text{ of } 1\frac{4}{11}}{\frac{8}{9} \text{ of } \frac{3}{4} \text{ of } \frac{5}{11} \text{ of } \frac{7}{12}}$. Ans. $3\frac{3}{5}$.

301. Divide $\dfrac{\frac{7}{5}}{\frac{3}{11}}$ by $\dfrac{\frac{2}{3}}{15}$.

302. Divide $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{7}$ by $\frac{7}{9} \times \frac{2}{5} \times 7$.

303. Divide $\frac{7}{8}$ of $\frac{16}{19}$ by $\frac{4}{11}$ of $\frac{33}{29}$.

304. $\dfrac{18\frac{2}{7}}{\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } \frac{5}{2}} = ?$

305. $\frac{1}{5}$ of $\frac{6}{7}$ of $2\frac{1}{2} \div \dfrac{1\frac{6}{7}}{3\frac{1}{3}} = ?$

306. $\dfrac{\frac{1}{2} \text{ of } \frac{5}{7} \text{ of } 7\frac{3}{8}}{21\frac{1}{14}} = ?$

307. $\frac{2}{15}$ of $\frac{28}{3}$ of $3\frac{1}{8} \div \dfrac{24\frac{1}{2}}{1\frac{8}{55} \times 1\frac{1}{2}} = ?$ Ans. $\frac{3}{11}$.

308. Four loads of hay weigh respectively $1723\frac{5}{8}$, $2317\frac{3}{4}$, $1547\frac{3}{8}$, and $1357\frac{1}{2}$ pounds. What is the total weight of the hay?

309. What number is that from which if $288\frac{5}{12}$ is taken the remainder is $759\frac{7}{8}$?

310. Find the cost of $\frac{4}{5}$ of $156\frac{2}{3}$ acres of land at $\frac{5}{8}$ of \$54 an acre.

311. What number multiplied by $33\frac{5}{6}$ will produce $297\frac{1}{2}$?

312. If $3\frac{1}{3}$ acres of land are worth \$450, what is $\frac{2}{15}$ of an acre worth?

$$\$\frac{\overset{3}{\cancel{450}}}{1} \times \frac{3}{\cancel{10}} \times \frac{2}{\cancel{15}} = \$18, \text{ Ans.}$$

If $3\frac{1}{3}$ acres are worth \$450, 1 acre would be worth \$450 divided by $3\frac{1}{3}$; $\frac{2}{15}$ of an acre would be worth $\frac{2}{15}$ of the quotient.

NOTE. If there are to be a series of multiplications and divisions, *express* them all before performing any; then cancel.

313. If $5\frac{1}{3}$ tons of hay cost \$80, what will $35\frac{1}{5}$ tons cost?

314. What will $236\frac{1}{7}$ yards of cloth cost at $\frac{3}{16}$ of a dollar a yard?

315. If I give $18\frac{3}{4}$ bushels of potatoes at 60 cents a bushel for butter at $29\frac{1}{2}$ cents a pound, how many pounds of butter do I receive?

316. What will $33\frac{1}{2}$ yards of cloth cost at 75 cents a yard?

317. What will $49\frac{3}{4}$ tons of hay cost at $\$18\frac{7}{8}$ a ton?

318. If for 24800 square feet of land I pay \$4340, how much do 12000 feet of this lot cost me?

319. How much shall I make on the whole lot, named above, if I sell it at once for $\frac{3}{13}$ of a dollar a square foot?

320. What number divided by $\frac{7}{11}$ gives as the quotient $347\frac{7}{8}$?

321. If the difference between $\frac{3}{8}$ and $\frac{2}{3}$ of a certain number is $177\frac{1}{3}$, what is the number?

322. A man lost $\frac{3}{16}$ of his money, and afterwards found $\frac{1}{3}$ of what he had lost; he then had \$735; how much had he at first?

323. If $\frac{3}{4}$ of a farm cost \$4200, what is $\frac{5}{7}$ of it worth?

324. If $\frac{5}{8}$ of a cord of wood is worth \$$3\frac{3}{4}$, what will $\frac{3}{4}$ of a cord cost?

325. If $\frac{3}{16}$ of a ship is worth \$8769, what is $\frac{5}{8}$ of the ship worth?

326. If $3\frac{3}{4}$ bushels of oats will sow an acre, how many bushels will it take to sow $7\frac{1}{5}$ acres?

327. In a certain school $\frac{2}{5}$ of the scholars belong to the fourth class, $\frac{1}{6}$ to the third class, $\frac{1}{7}$ to the second class, and the remainder, 61, to the first class. How many scholars are there in the school?

328. A merchant bought $48\frac{3}{4}$ pounds of butter of one man, $28\frac{3}{4}$ of another, $25\frac{3}{16}$ of another, and $56\frac{5}{16}$ of another; how many pounds did he buy, and what must he pay at \$$\frac{1}{4}$ a pound?

329. Six men are to be clothed with cloth that is $1\frac{1}{2}$ yards wide. Now if it takes $2\frac{2}{3}$ yards of this cloth for each man, how many yards of cloth $\frac{3}{4}$ yards wide will be sufficient to line all the garments?

330. A gentleman gave $\frac{1}{2}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and $\frac{1}{2}$ of what then remained to his daughter, who received \$$376\frac{1}{8}$; what was the value of the estate?

331. If a man can do a piece of work in 9 days by working $14\frac{2}{3}$ hours per day, in how many days, if he works $8\frac{1}{4}$ hours each day, can he do the same work?

332. If $1\frac{7}{8}$ yards of cloth are required for 1 coat, how many coats may be made from $16\frac{7}{8}$ yards?

333. A man bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{3}$ of the remainder to his daughter, and the rest to his son. The difference between the wife's and the son's share was \$$2787\frac{1}{3}$. What did each receive?

334. If $\frac{7}{12}$ of a lot of land is worth \$ $785\frac{3}{8}$, what is $\frac{8}{9}$ of it worth?

335. If $\frac{2}{3}$ of a farm is worth \$ 9000, what is $\frac{5}{12}$ of it worth?

336. A staff 3 feet long cast a shadow $\frac{3}{4}$ of a foot at 12 o'clock; what is the length of a shadow cast by a steeple $125\frac{1}{2}$ feet high, at the same time?

337. If a staff 3 feet long casts a shadow of $\frac{3}{4}$ of a foot at 12 o'clock, what is the height of a steeple that casts a shadow $31\frac{3}{8}$ feet, at the same time?

338. Sold a watch for \$ $43\frac{3}{4}$, which was $\frac{7}{8}$ of its cost; what was its cost?

339. If a man earns \$ $1\frac{3}{5}$ a day, in how many days will he earn \$ 100?

340. Divide 98 by $11\frac{1}{13}$, and multiply the quotient by $\frac{3}{7}$ of $8\frac{3}{4}$.

341. What is the smallest sum of money with which I can buy horses at \$ 50 each, cows at \$ 30 each, or sheep at \$ 8 each, using the same sum in each case?

342. If $\frac{7}{9}$ of a yard of cloth cost $\frac{7}{25}$ of a dollar, what will $\frac{5}{6}$ of a yard cost?

343. A man sold a wood-lot for \$ 2250, which was $\frac{3}{5}$ of what it cost him; how much did he lose by the bargain?

344. A man lost $\frac{3}{5}$ of his money, and then earned \$ 117, when he found he had $\frac{5}{6}$ of his original sum; how much had he at first?

345. If a man can perform a journey in $8\frac{9}{14}$ days, what part of it can he perform in $3\frac{1}{7}$ days?

346. A merchant owned $\frac{3}{5}$ of a cargo of tea. The value of the whole cargo was \$ 6500. If he sells $\frac{2}{3}$ of his share for \$ 2750, how much does he make, or lose?

347. A man gave, by will, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{2}{5}$ of his estate to various objects, and the residue, which was \$ 200, he ordered to be expended in the purchase and care of his burial-place. What was the value of the whole estate?

348. A painter worked $27\frac{1}{2}$ days, and after paying $\frac{2}{5}$ of his earnings for board and other expenses, had $ $41\frac{1}{4}$ left. What did he receive a day?

349. How many bushels of oats at $ $\frac{5}{8}$ a bushel must be given for $\frac{3}{8}$ of a barrel of sugar at $ $10\frac{1}{2}$ a barrel?

350. If $ $16\frac{1}{4}$ will buy $2\frac{1}{2}$ cords of wood, how many cords will $ $74\frac{3}{4}$ buy?

351. If you can buy $30\frac{1}{4}$ pounds of raisins at 10 cents a pound for $9\frac{1}{6}$ dozen of eggs, what are eggs a dozen?

352. If a man's expenses are $ $1784\frac{4}{5}$ a year, and he saves $\frac{3}{7}$ of his income, what is his annual income?

353. A man engaging in trade lost $\frac{2}{5}$ of his investment and then gained $ $834\frac{3}{5}$, when he had as much as he put in and $\frac{1}{4}$ as much more. How much did he put in?

354. If a man loses $\frac{1}{3}$ of an investment, then gains $ $542\frac{2}{3}$, and has $ $1287\frac{1}{2}$, how much was his investment, and how much was his loss?

355. A farmer sold 25 barrels of apples for $ $37\frac{1}{2}$, which was $\frac{3}{8}$ as much as he afterward received for all the rest at $ $1\frac{3}{4}$ a barrel. How many barrels did he sell in all?

356. If $25\frac{1}{2}$ tons of coal cost $ $216\frac{3}{4}$, how much will $9\frac{1}{2}$ tons cost?

357. If $\frac{3}{4}$ of a pound of tea cost $ $\frac{1}{2}$, what will $75\frac{1}{2}$ pounds cost?

358. If $ $67\frac{5}{6}$ will buy $9\frac{1}{4}$ barrels of flour, how many barrels can be bought for $ 55?

359. If $6\frac{6}{7}$ yards of cloth cost $ $16\frac{4}{5}$, what will $9\frac{3}{5}$ yards cost?

360. If $ $2\frac{3}{5}$ will buy $\frac{3}{4}$ of a yard of cloth, how many yards can be bought for $ $147\frac{1}{5}$?

361. If $10\frac{3}{4}$ bushels of corn cost $ $8\frac{3}{5}$, how many bushels can be bought for $ $2\frac{1}{2}$?

362. How many pounds of coffee can be bought for $ $27\frac{1}{2}$, if $6\frac{1}{2}$ pounds cost $ $2\frac{11}{40}$?

363. If I pay \$ $5\frac{5}{8}$ for $\frac{7}{8}$ of a ton of coal, how much can I buy for \$ $2\frac{13}{16}$?

364. If 7 horses eat $22\frac{4}{5}$ tons of hay in a certain time, how many horses will eat $81\frac{3}{7}$ tons in the same time?

365. If $4\frac{2}{7}$ yards of cloth cost \$ $10\frac{5}{7}$, how many yards can be bought for \$ $33\frac{1}{3}$?

366. If $\frac{7}{18}$ of a cheese that weighs 40 pounds is sold for \$ $2\frac{3}{8}$, how much ought $\frac{8}{19}$ of the same cheese to be sold for?

367. If $\frac{3}{8}$ of a cord of wood cost \$ $2\frac{1}{4}$, how many cords can be bought for \$ $25\frac{1}{3}$?

368. If I paid \$ $487\frac{1}{2}$ for a lot of apples, at the rate of \$ $\frac{3}{5}$ for $\frac{2}{5}$ of a barrel, how many barrels did I buy?

369. A gentleman left his son an estate, $\frac{1}{4}$ of which he spent in 7 months, and $\frac{1}{6}$ of the remainder in 3 months more, when he had only \$ 5000 remaining; what was the value of the estate?

370. A can build a certain wall in 12 days, and B in 15 days. How long will it take A and B together to build the wall?

Solution. If A can build it in 12 days, he can build $\frac{1}{12}$ of it in 1 day; and if B can build it in 15 days, he can build $\frac{1}{15}$ of it in 1 day; therefore, A and B together can build $\frac{1}{12} + \frac{1}{15}$, or $\frac{9}{60}$ of it in 1 day. If they can build $\frac{9}{60}$ of it in 1 day, it will take as many days to do the whole as $\frac{9}{60}$ is contained times in $\frac{60}{60}$, or 9 in 60, that is, $6\frac{2}{3}$ days.

371. Two men and a boy engage to reap a field of rye; one of the men can reap it in 10 days, the other in 12, and the boy in 15 days. In how many days can the three together reap it?

372. How long will it take the two men together?

373. How long will it take the first man and the boy?

374. How long will it take the second man and the boy?

375. Three men, A, B, and C, can do a piece of work in 10 days. If A and B together can do it in 14 days, how many days will it take C to do it alone? If A and C together can do

it in 16 days, how long will it take B to do it alone? How long will it take A alone?

376. A merchant bought a number of bales of hops, each bale containing $247\frac{1}{2}$ lb., at the rate of \$3 for 10 lb., and sold them at the rate of \$5 for 12 lb., and gained \$693. How many bales did he buy?

377. If a railway train goes $75\frac{1}{4}$ miles in 2 hours and 15 minutes, how many miles does it go in an hour?

378. Arrange $\frac{3}{4}$, $\frac{7}{10}$, $\frac{31}{40}$, $\frac{41}{60}$, in the order of their magnitude.

379. If $40\frac{1}{3}$ yards of silk cost $\$30\frac{1}{4}$, how many yards can be bought for $\$41\frac{3}{4}$?

380. Freight is carried by canal at the rate of a mile in $22\frac{2}{3}$ minutes, by common road $1\frac{1}{3}$ times as fast, and by rail 8 times as fast. If it is carried by common road $8\frac{1}{2}$ hours, by canal $25\frac{1}{2}$ hours, and by rail $45\frac{1}{3}$ hours, how many miles is it carried?

381. If 8 men can reap a field in $14\frac{1}{2}$ hours, how many hours will it take for 5 men to reap it?

382. If telegraph posts are set 66 yards apart, how often do telegraph posts and mile stones come at the same point? (There are 1760 yards in a mile.)

383. If it costs $\$7\frac{1}{4}$ to carry $9\frac{1}{2}$ tons $92\frac{2}{3}$ miles, how far can $28\frac{1}{2}$ tons be carried for $\$130\frac{1}{2}$?

384. A clerk earns $\$1\frac{3}{5}$ a day and spends $\$\frac{2}{3}$ a day. How much does he save in a year?

385. A policeman starts after a thief who is 418 yards ahead of him. If the policeman runs at the rate of a mile in 8 minutes and the thief at the rate of a mile in 10 minutes, how long will it take the policeman to catch the thief?

386. A man walks from home to a neighboring town at the rate of $2\frac{3}{4}$ miles an hour and rides back at the rate of $5\frac{1}{2}$ miles an hour. He started at 7 A.M., remained in town 5 hours, and arrived at home at 5 h. 30 m. P.M. How far does he live from town?

DECIMALS.

152. The following table is a continuation to the right of the table in Art. 21.

NUMERATION TABLE.

Thousands.	Hundreds,	Tens,	Units.		Tenths,	Hundredths,	Thousandths,	Ten-Thousandths,	Hundred-Thousandths,	Millionths,	Ten-Millionths,	Hundred-Millionths,	Billionths.
1	**8**	**4**	**3**	**.**	**5**	**4**	**8**	**7**	**6**	**3**	**2**	**4**	**5**

NOTE. This table can be extended to any number of periods to the right, as shown in the Appendix.

153. A **Decimal Fraction,** or a **Decimal,** is a fraction whose denominator is 10, 100, 1000, or 1 with one or more ciphers annexed. As its denominator is known, the numerator alone may be written with the decimal point, and read, according to the Rules in Arts. 24 and 26.

154. The value of the decimals in the table, expressed in words, is five hundred forty-eight million seven hundred sixty-three thousand two hundred forty-five billionths.

Exercises in the Numeration of Decimals.

155. Read the following numbers :

1. 7.6534	7. 3004.0087
2. 14.8765	8. 1406.70909
3. 0.31459	9. 703.1376589
4. 7.64328	10. 444.4444444
5. 10.00064	11. 37.0800764
6. 8.76007	12. 700.0000984

Exercises in Notation and Numeration of Decimals.

156. Write the following numbers in figures :

13. Seven hundred eighteen ten-thousandths.
14. Nine millionths.
15. Eight, and eight hundred four ten-millionths.
16. Fifteen thousand eighty, and four thousand six millionths.
17. One thousand, and one hundred-millionth.
18. Seventy million, and seven millionths.
19. Sixteen thousand, and fourteen hundred-thousandths.
20. Eight million, and eighteen ten-millionths.

157. Though decimals can be added, subtracted, multiplied, and divided as integral quantities, yet as parts of a unit, they are fractions. Thus, 0.3 is $\frac{3}{10}$; 0.37 is $\frac{37}{100}$; 0.375 is $\frac{375}{1000}$; etc.

158. When the denominators of Decimals are written, the Decimals appear as Common Fractions.

Every principle and operation in Common Fractions is equally applicable to Decimals.

159. The denominator of a decimal is 1 with as many ciphers annexed as there are figures in the decimal.

160. For Addition and Subtraction of Decimals, see pages 12 – 27.

MULTIPLICATION OF DECIMALS.

161. Written Exercises.

21. Multiply 0.485 by 0.7.

$$\begin{array}{rr} & 0.485 \\ & 0.7 \\ \hline \text{Ans.} & 0.3395 \end{array}$$

As 0.485 $= \frac{485}{1000}$, and 0.7 $= \frac{7}{10}$, 0.485 $\times$ 0.7 $= \frac{485}{1000} \times \frac{7}{10} = \frac{3395}{10000}$, or 0.3395, Ans.

22. Multiply 0.0181 by 0.006.

	0.0181
	0.006
Ans.	0.0001086

As $0.0181 = \frac{181}{10000}$, and $0.006 = \frac{6}{1000}$, $0.0181 \times 0.006 = \frac{181}{10000} \times \frac{6}{1000} = \frac{1086}{10000000} = 0.0001086$, Ans.

From these examples we derive the following

Rule.

Multiply as in whole numbers, and point off as many figures for decimals in the product as there are decimal places in both factors, counted together.

NOTE 1. If the number of figures in the product is less than the number of decimal places in the two factors, the deficiency must be supplied by *prefixing ciphers to the product*, as in Exs. 22 and 24.

NOTE 2. The pointing off is in reality the multiplying of the denominators of the factors, or it shows what the product of the denominators is.

	(23.)	(24.)
Multiplicand,	26.2983	0.32
Multiplier,	8.4	0.23
	1051932	96
	2103864	64
Product,	220.90572	0.0736

	(25.)	(26.)
Multiplicand,	423.6	0.3259
Multiplier,	0.54	.000025
	16944	16295
	21180	6518
Product,	228.744	0.0000081475

27. Multiply 34.87 by 4.5.

28. Multiply 2769 by 0.84.

29. Multiply 0.2436 by 0.034.

30. Multiply 0.0068 by 0.003.

31. Multiply 36.874 by 0.5421.

32. Multiply 0.14687 by 0.00054.

33. Multiply 0.17288 by 0.14403.

34. Multiply 0.00369 by 0.24683.

35. Multiply three hundred fifty-six thousandths by one hundred forty-five ten-thousandths.

36. Multiply thirty-four millionths by twenty-six ten-millionths.

37. Multiply eight hundred forty-two thousandths by five hundred thousand.

DIVISION OF DECIMALS.

162. Written Exercises.

38. Divide 0.625 by 0.25.

$$
\begin{array}{r}
0.25)\ 0.625\ (2.5,\ \text{Ans.} \\
\underline{50}\phantom{\ (2.5,\ \text{Ans.}} \\
125\phantom{\ (2.5,\ \text{Ans.}} \\
\underline{125}\phantom{\ (2.5,\ \text{Ans.}} \\
0\phantom{\ (2.5,\ \text{Ans.}}
\end{array}
$$

As $0.625 = \frac{625}{1000}$, and $0.25 = \frac{25}{100}$, $0.625 \div 0.25 = \frac{625}{1000} \div \frac{25}{100} =$ (Art. 148) $\frac{25}{10}$, or 2.5, Ans.

39. Divide 0.7 by 0.175.

$$
\begin{array}{r}
0.175)\ 0.700\ (4,\ \text{Ans.} \\
\underline{700}\phantom{\ (4,\ \text{Ans.}} \\
0\phantom{\ (4,\ \text{Ans.}}
\end{array}
$$

As $0.7 = \frac{7}{10} = \frac{700}{1000}$, and $0.175 = \frac{175}{1000}$, $0.7 \div 0.175 = \frac{700}{1000} \div \frac{175}{1000} = \frac{4}{1}$, or 4, Ans.

40. Divide 0.0074 by 3.7.

$$
\begin{array}{r}
3.7)\ \underline{0.0074} \\
0.002,\ \text{Ans.}
\end{array}
$$

As $0.0074 = \frac{74}{10000}$, and $3.7 = \frac{37}{10}$, $0.0074 \div 3.7 = \frac{74}{10000} \div \frac{37}{10} = \frac{2}{1000} = 0.002$, Ans.

From these examples we derive the following

Rule.

Divide as in whole numbers, and point off as many figures for decimals in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.

NOTE 1. If there are not as many decimal places in the dividend as in the divisor, make as many by annexing ciphers.

NOTE 2. If the number of figures in the quotient is less than the excess of decimal places in the dividend over those of the divisor, supply the deficiency by *prefixing ciphers to the quotient.*

NOTE 3. The pointing off is in reality the dividing of the denominator of the dividend by the denominator of the divisor, or it shows what that quotient is.

NOTE 4. The rule for pointing in the quotient is also evident from the rule in multiplication if we notice that the dividend is a product whose factors are the divisor and quotient.

41. Divide 1.2575125 by 2.5.
42. Divide 8.43648108 by 0.06.
43. Divide 38.7425 by 0.25.
44. Divide 0.09936 by 0.276.
45. Divide 0.000975 by 0.15.
46. Divide 17.472 by 0.48.
47. Divide 234.7744 by 62.44.
48. Divide 58.794 by 12.3.
49. Divide 3647 by 0.125.
50. Divide 90321.6 by 3.642.
51. Divide 72 by 0.064.
52. Divide 0.13 by 8.
53. Divide 7.2 by 0.16.
54. Divide 8.7 by 0.25.
55. Divide 3.6 by 7.5.

56. Divide 0.34 by 0.24.

57. Divide 0.73 by 1.5.

58. Divide 4.63 by 2.9.

59. Divide three thousand eight hundred fifty-three hundred-thousandths by thirty-two millionths. Ans. 1204.0625.

60. Divide eighty-four, and eighty-four hundredths by forty-eight thousandths.

61. Divide four hundred by four hundredths.

62. Divide sixteen thousandths by forty-five hundred.

63. Divide seventy-five thousand eight hundred one by two thousand two hundred ninety-seven ten-thousandths.

163. To reduce a common fraction to a decimal.

64. Reduce $\frac{7}{8}$ to a decimal.

$$\begin{array}{r|l} 8) & 7.000 \\ \hline & 0.875, \text{ Ans.} \end{array}$$

The value of a fraction is the quotient arising from dividing the numerator by the denominator (Art. 118 a). $\frac{7}{8} = 7 \div 8$; but as 8 is not contained in 7 we reduce the 7 to tenths, viz. to 70 tenths; 70 tenths divided by 8 gives 8 tenths, the first quotient figure, and 6 tenths remainder; 6 tenths = 60 hundredths; 60 hundredths divided by 8 gives 7 hundredths, and 4 hundredths remainder, and so on as in division of decimals. Hence,

Rule.

Annex one or more ciphers to the numerator and divide the result by the denominator, continuing the operation until there is no remainder, or as far as desirable. Point off as in division of decimals.

65. Reduce $\frac{3}{8}$ to a decimal.

66. Reduce $\frac{1}{3}$ to a decimal.

67. Reduce $\frac{3}{7}$ to a decimal.

68. Reduce $\frac{7}{16}$ to a decimal.

69. Reduce $\frac{7}{64}$ to a decimal.

70. Reduce $\frac{77}{32}$ to a decimal.

71. Reduce $\frac{7}{12}$ to a decimal.

72. Reduce $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{13}{16}$, $\frac{7}{64}$, $\frac{7}{25}$, and $\frac{17}{50}$ to decimals.

164. A *decimal is a fraction*, and, if its denominator is written, it will *appear* as a *common fraction.*

73. Reduce 0.48 to the form of a common fraction and then to its lowest terms. $0.48 = \frac{48}{100} = \frac{12}{25}$. Ans.

74. Reduce 0.125 to its lowest terms.

75. Reduce 0.17 to the form of a common fraction.

76. Reduce 2.8 to the form of a common fraction.

77. Reduce 1.5, 3.75, 8.25, 9.125, and 2.0125.

78. Reduce 0.275, 0.325, 0.00025, and 0.00625.

79. Reduce 0.225, 0.075, 0.815, and 0.0085.

80. Reduce 0.175, 0.0945, 0.0015, and 0.0045.

81. Reduce $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{5}{12}$, $\frac{4}{25}$, and $\frac{4}{75}$ to decimals.

82. Add as decimals $21\frac{1}{2}$, $75\frac{3}{4}$, 8.075, and 125.4.

83. From $746\frac{1}{16}$ subtract 353.65.

84. From 853.078 subtract $563\frac{5}{16}$.

85. Multiply $67\frac{5}{8}$ by 14.5.

86. Multiply 514.85 by $15\frac{15}{64}$.

87. Divide 146.25 by $8\frac{3}{5}$.

88. Divide $553\frac{7}{20}$ by 9.25.

89. Divide 1720.8 by $8\frac{1}{4}$.

90. Divide $653\frac{3}{40}$ by 17.3.

91. Divide $787\frac{7}{8}$ by 7.65.

(For Circulating Decimals, see Appendix, page 340.)

DECIMAL SYSTEM OF MONEY, WEIGHTS, AND MEASURES.

166. UNITED STATES MONEY.

10 mills (m.)	=	1 cent (c.).
10 cents	=	1 dime (d.).
10 dimes	=	1 dollar ($).
10 dollars	=	1 eagle (e.).

167. In this currency, the *dollar* is the *unit,* cents and mills being *decimals* of a dollar. Thus, $ 3.62 represents three dollars and sixty-two cents; $ 4.085 represents four dollars, eight cents, and five mills; etc. Figures at the right of the third decimal place represent *parts of a mill;* thus, $ 5.3627 = 5 dollars, 36 cents, 2 mills, and $\frac{7}{10}$ of a mill. The terms eagle and dime are not much used. Eagles and dollars are read together as dollars, and dimes and cents as

cents; thus, $47.843 is read 47 dollars, 84 cents, 3 mills. Mills are often written and read as fractions of a cent; thus, $0.12½, twelve cents and a half.

168. Reduction is the changing of a quantity to units of greater or less value without changing the value of the quantity expressed.

169. *Dollars are reduced to cents by moving the decimal point two places to the right, and to mills by moving the decimal point three places to the right. Thus,* $6 = 600 *cents* = 6000 *mills.*

170. *Cents are reduced to dollars by moving the decimal point two places to the left. Mills are reduced to dollars by moving the decimal point three places to the left. Thus,* 3789 *cents* = $37.89; *and* 3789 *mills* = $3.789.

171. Oral Exercises.

1. Reduce $47 to cents.
2. Reduce $34.56 to cents.
3. Reduce $3.456 to mills.
4. Reduce $483 to cents; to mills.
5. Reduce $6.84 to cents; to mills.
6. Reduce $1.876 to mills.
7. Reduce 564 cents to dollars.
8. Reduce 3692 mills to dollars.
9. Reduce 87694 cents to dollars.
10. Reduce 76843 mills to dollars.

Write the answers to these ten examples.

11. If to pay for a knife at $2, a pocket-book at $3, a lot of paper and envelopes at $1.25, I have only a ten-dollar bill, how much change ought I to receive back?

12. If I had \$ 75, and paid out of it three 10's, four 5's, two 2's, and 62 cents, how much should I have left?

172. When a decimal can be reduced to a common fraction with not more than one figure as its denominator, instead of multiplying by the decimal, it is often easier to use this common fraction as a multiplier.

Thus, in United States currency, instead of multiplying by the number expressing the number of cents, in practice we often use the common fractions in the following

TABLE.

\$ 0.50 $= \frac{1}{2}$ of a dollar.	\$ 0.80 $= \frac{4}{5}$ of a dollar.
$0.33\frac{1}{3} = \frac{1}{3}$ of a dollar.	$0.16\frac{2}{3} = \frac{1}{6}$ of a dollar.
$0.66\frac{2}{3} = \frac{2}{3}$ of a dollar.	$0.83\frac{1}{3} = \frac{5}{6}$ of a dollar.
$0.25 = \frac{1}{4}$ of a dollar.	$0.12\frac{1}{2} = \frac{1}{8}$ of a dollar.
$0.75 = \frac{3}{4}$ of a dollar.	$0.37\frac{1}{2} = \frac{3}{8}$ of a dollar.
$0.20 = \frac{1}{5}$ of a dollar.	$0.62\frac{1}{2} = \frac{5}{8}$ of a dollar.
$0.40 = \frac{2}{5}$ of a dollar.	$0.87\frac{1}{2} = \frac{7}{8}$ of a dollar.
$0.60 = \frac{3}{5}$ of a dollar.	

13. What cost 64 yards of cloth at \$ $0.87\frac{1}{2}$ a yard?

Solution. \$ $0.87\frac{1}{2} = \frac{7}{8}$ of a dollar. If a yard is worth $\frac{7}{8}$ of a dollar, 64 yards would be worth 64 times $\frac{7}{8}$ of a dollar, or $\frac{7}{8}$ of 64 dollars, or \$ 56, Ans.

173. **Price** is the assigned value of a unit of any article; *cost* is the assigned value of the whole, or of a number of units, of the quantity.

174. The rules given for operations in decimals apply to operations in United States Money.

14. Bought a hat for \$ 4.50, a coat for \$ 18.75, a vest for \$ 5.25, and a pair of boots for \$ 5; what did I pay for all?

15. If apples are worth \$ 2.50 a barrel, what are 3 barrels worth?

16. When 8 cords of wood are worth \$ 44, what is the value of 1 cord?

17. At \$ 6 a ton how many tons of coal can I buy for \$ 24?

Solution. I can buy as many tons as \$6 is contained times in \$24; \$24 ÷ \$6 = 4; hence, I can buy 4 tons.

18. What cost 48 barrels of apples at \$ 1.37½ a barrel?

Solution. \$1.37½ = \$1⅜. 48 barrels at \$1 a barrel would cost \$48. ⅜ of a dollar more on each barrel would be ⅜ of 48 dollars, or \$18; \$48 + \$18 = \$66, Ans.

19. What cost 48 pounds of raisins at 12½ cents a pound?

20. What cost 54 yards of muslin at 16⅔ cents a yard?

21. What cost 74 bushels of apples at 33⅓ cents a bushel?

22. What cost 40 pairs of gloves at 50 cents a pair?

23. How many pounds of cheese at 12½ cents a pound may be bought for 3 bushels of corn at 87½ cents a bushel?

(First find how many pounds for 1 bushel.)

175. Written Exercises.

24. Paid \$ 87.50 for a horse, \$ 145.25 for a pair of oxen, \$ 14.25 for a wagon, and \$ 45.75 for a cart; what did I pay for all?

25. A man who owed \$ 699.60, paid \$ 164.67; how much did he still owe?

26. Bought a farm for \$ 3684.75, and stock and tools for the farm for \$ 1476.25; how much more did I pay for the farm than for the stock and tools?

27. If 12 gentlemen have \$ 7497.84 apiece, what sum have they all?

28. If 45 persons deposit \$ 346.25 each in a savings-bank, how many dollars do they deposit?

29. Divide \$ 69345.36 equally among 18 men.

30. Divide \$4832.40 into 24 equal parts.

31. What is the cost of 9 barrels of flour at \$7.75 a barrel?

32. Bought 25 sheep at \$6.25 each; what was the cost of the flock?

33. Bought 18 yards of broadcloth at \$3.875 a yard; what was the cost of the piece?

34. What is the value of 75 acres of land at \$37.50 an acre?

35. If 24 yards of broadcloth cost \$93, what is the price a yard? Ans. \$3.87$\frac{1}{2}$.

36. Bought 37 pounds of butter for \$8.51; what was the price? Ans. 23 c.

37. Bought 356 barrels of sugar for \$3026; what was the price?

38. Bought a farm containing 125 acres for \$6843.75; what was the price an acre?

39. At \$3 a yard how many yards of cloth can be bought for \$546? Ans. 182.

40. At \$22.50 an acre how many acres of land can be bought for \$1822.50?

41. At 56 cents a pound how many pounds of tea may be bought for \$25.20?

42. A drover bought oxen at \$62.50 each; how many oxen did he buy for \$1562.50?

43. What cost 36 sheep at \$5.66$\frac{2}{3}$ each? (\$5.66$\frac{2}{3}$ = \$5$\frac{2}{3}$.)

44. How many cords of wood at \$5.50 a cord must be given in exchange for a barrel of flour at \$7.50 and 5 yards of cloth at \$2.35 a yard?

45. What cost 3$\frac{1}{2}$ yards of ribbon at 56 cents a yard?

46. What cost 3 barrels of flour at \$7.62$\frac{1}{2}$ a barrel?

47. If 4 cords of wood cost \$22.50, what is the price of a cord?

48. If 15 yards of silk cost \$16.87$\frac{1}{2}$, what is the price of a yard?

49. If a merchant deposits $ 375.50 in a bank at one time, and $ 487.75 at another, how much will remain after he has withdrawn $ 176.37 and $ 346.83?

50. A merchant bought 75 barrels of cranberries for $ 650, and sold 25 barrels at $ 9.50 a barrel, and the remainder at $ 9.25 a barrel; did he gain or lose? How much?

51. What cost 87½ rods of wall at 75 cents a rod?

52. What cost 75 cords of wood at $ 5.50 a cord?

53. What cost 14 horses at $ 144.44 apiece?

54. If a ride of 46 miles cost $ 5.98, what is the price a mile?

55. If 27 bushels of rye cost $ 14.31, what is the price of a bushel?

56. My farm cost $ 3725 and my house cost $ 1862.75; how much more did the farm cost than the house?

57. A gentleman bequeathed $ 750 to each of his 3 sons, and $ 500 to each of his 4 daughters; how much did he bequeath to his children?

58. Paid $ 16.50 for a coat, $ 4.25 for a vest, $ 5.75 for a pair of trousers, $ 3.50 for a hat, $ 4.37½ for a pair of boots, and $ 12.62½ for other articles. What did I pay for all?

59. Divide $ 113,75 equally among 7 men.

60. Paid $ 68.75 for flour at $ 6.25 a barrel; how many barrels did I buy?

61. How many yards of lace at 62½ cents a yard may be bought for $ 3.75?

62. What cost 8725 feet of boards at $ 12.50 a thousand feet?

63. What cost 82.48 tons of coal at $ 6 a ton?

64. How many pounds of cheese at 12½ cents a pound will pay for 12 dozen eggs at 16⅔ cents a dozen?

65. My real estate is worth $ 4756.75, my personal estate $ 4562.75, and I owe $ 2468.50. What am I worth?

66. At 25 cents a mile for a horse and carriage how far can I ride for $ 3.37½?

67. A drover bought sheep at \$ 3.37½ a head and sold them at \$ 3.87½ a head, and gained \$ 37.50 by the transactions; how many sheep did he buy?

68. Bought 100 sheep at \$ 3.375 a head, and sold them again at \$ 3.875; what was the gain on each and what the total gain?

69. Bought 20.5 tons of hay at \$ 12.375 a ton; what was the cost of the whole?

70. What is the value of 67.75 acres of land, at \$ 62.50 an acre?

71. Paid \$ 4234.375 for 67.75 acres of land; what was the price an acre?

72. Paid \$ 4234.375 for a piece of land at \$ 62.50 an acre; how many acres were bought?

73. Bought land at \$ 62.50 an acre, and sold it again at \$ 75 an acre, thereby making \$ 846.875; how many acres were bought?

74. Bought 67.75 acres of land at \$ 62.50 an acre, and sold the lot for \$ 5081.25; was there a gain or loss? how much on the whole, and how much an acre?

75. Bought 356.25 pounds of wool at 37½ cents a pound, which was manufactured into cloth at an expense of \$ 62.50; for what sum must it be sold to gain \$ 37.50?

76. Bought 14.75 yards of gingham at 14 cents a yard; what was the cost of the piece?

77. What would 7¾ bales of cotton cost, each bale weighing 537.5 pounds, at \$ 0.11¾ a pound?

78. Bought 1 barrel beans at \$ 8.50, 3 bushels corn at 87½ c. a bushel, 24.5 pounds raisins at 8½ c. a pound, 3 gallons molasses at 37½ c. a gallon, 2 pounds tea at 62½ c. a pound, 6 pounds coffee at 35 c. a pound, 15 pounds rice at 8 c. a pound, and 4 pounds butter at 22 c. a pound. What was the cost of the whole?

BILLS.

176. A **Bill of Goods** is a written statement of articles sold, containing the date of the transaction, the names of the purchaser and the seller, the quantity and price of each item, and the cost of the whole.

177. A **Debtor** (Dr.) is a person who owes a debt; and a **Creditor** (Cr.) a person to whom a debt is owed.

178. A bill when paid is receipted by the creditor, or some person authorized by him, acknowledging the payment in writing.

179. Find the cost of the several articles, and **the amount or footing of each of the following bills.**

(79.)

Boston, Jan. 1, 1895.

Mr. C. E. Hammett,

1894. To THOMPSON, BROWN & CO. Dr.

Aug. 17.	*60 Bradbury's Algebras @ $0.90*	*$*	
" "	*24 Cushing's Manuals " 0.60*		
Oct. 5.	*10 Philbrick's Speakers " 1.50*		
Dec. 17.	*15 Reams Note Paper " 1.25*		
" "	*25 " Letter " " 1.75*		

$145.90

Received Payment,

Thompson, Brown & Co.

(80.)

Thetford, Vt., Apr. 4, 1895.

Mr. James Hosford,

Bought of JOSEPH PIERCE.

	27 Sheep .... @ $ 3.75	*$*	
	13 Oxen .... " 53.50		
	15 Cows .... " 23.25		

(81.)

New York, Jan. 7, 1895.

Mr. A. J. Palmer,

Bought of JOHN FOX & CO

25 lb. Ham	@ 10 c.	$	
40 " Maple Sugar	" 12 "		
6 " Cheese	" 13 "		
8 " Butter	" 28 "		
4 " Raisins	" 11 "		
2 " Cream Tartar	" 40 "		

Received Payment,

(82.)

Cincinnati, March 1, 1895.

Mr. J. S. Simpson,

To MAYHEW, JONES & CO. **Dr.**

1895.				
Jan. 4.	3 yds. Flannel	@ $0.42	$	
" "	2 prs. Cuffs	" 0.33		
" "	2 " Hose	" 0.28		
" 11.	4 yds. Ribbon	" 0.25		
" "	2 " Hamburg	" 0.12		
" "	3 " Lace	" 0.20		
Feb. 8.	7 " Crash	" 0.10		
" "	5 " Cotton	" 0.15		
	Cr.			
" 19.	Allowance on Cotton	$0.45		
" "	Cash	3.00		
		$		
	Balance due M., J. & Co.			

Received Payment,

83. Feb. 15, 1895, Samuel Farwell bought the following items of H. J. Thompson & Co.: $9\frac{1}{2}$ pounds of paint at 12 cents a pound; 10 rolls of paper at 20 cents a roll; 28 rolls at 8 cents a roll; 44 rolls at 10 cents a roll; 4 rolls at 25 cents a roll; 34 yards of border at 6 cents a yard; 58 yards at 2 cents a yard; 66 yards at 1 cent a yard. March 7, 1895, Mr. Farwell returned 3 rolls of paper at 8 cents a roll, 5 rolls at 10 cents a roll, 5 yards of border at 6 cents a yard. Make out Mr. Farwell's account.

84. July 2, 1895, James Fox bought of M. J. Hopkins 8 tons of furnace coal at $5.25 a ton; 6 tons of stove coal at $5.50 a ton; $\frac{1}{2}$ cord of pine wood at $6.50 a cord; $\frac{1}{2}$ cord of hard wood at $8.50 a cord. Mr. H. also charged Mr. F. $0.25 a ton for putting the coal into the cellar. Make out Mr. F.'s bill.

85. Sept. 27, 1895, William Malloy bought of Stephen Leavitt 10 pounds of Paris white at 5 cents a pound; 2 pounds of glue at 25 cents a pound; 1 whitewash brush for $5; 12 pounds of tenpenny nails at 5 cents a pound; 5 pounds of eightpenny finishing nails at 5 cents a pound; 3 dozen screws at 7 cents a dozen; 3 pairs of brass butts at 20 cents a pair; 16 yards of border at 12 cents a yard; $\frac{1}{2}$ gallon of spirits at 50 cents a gallon; $12\frac{1}{2}$ pounds of Beymer lead at $11\frac{3}{4}$ cents a pound. Mr. Leavitt afterward deducted 2 cents a yard on the border. Prepare this bill for settlement?

86. Jan. 3, 1895, John Robinson sold to George Hosmer 25 pairs of laced shoes at $3.50 a pair; 15 pairs of kid slippers at $1.25 a pair; 20 pairs of ladies' boots at $3.25 a pair; 30 pairs of men's boots at $2.25 a pair; 35 pairs of boy's boots at $1.75 a pair; 20 pairs of ladies' slippers at $2.50 a pair; 20 pairs of men's slippers at $1.65 a pair; 25 pairs of children's rubbers at $0.50 a pair; 45 pairs of ladies' rubbers at $0.65 a pair; 50 pairs of men's overshoes at $1.37½ a pair. There was also a charge for truckage of $1. For cash Mr. Robinson deducted $5. Make out a receipted bill.

WEIGHTS AND MEASURES.

180. The Decimal System of Weights and Measures is often called the **Metric* System** from its linear unit, the *Meter*. It is used in France, Germany, Spain, Portugal, Belgium, Holland, Switzerland, Italy, Austria, Sweden, Denmark, Greece, Mexico, Brazil, and most of the other States of South America, and in most of these countries its use is compulsory. In Great Britain and the United States its use is legalized.

1 DECIMETER = 10 CENTIMETERS = 100 MILLIMETERS.

181. LINEAR MEASURE.

10 millimeters $(^{mm})$	=	1 **centimeter**	$(^{cm})$.
10 centimeters	=	1 decimeter	$(^{dm})$.
10 decimeters	=	1 METER	$(^{m})$.
10 meters	=	1 dekameter	$(^{Dm})$.
10 dekameters	=	1 hektometer	$(^{Hm})$.
10 hektometers	=	1 **kilometer**	$(^{Km})$.

182. The standard unit of linear measure is the meter, which is one ten-millionth of the distance from the equator to the pole, measured on the meridian. Measures below the meter are generally written and read as centimeters.

183. Like the eagle and dime in United States Money, the terms dekameter and decimeter are not much used. Thus, 37.48^{m} is 37, and 48 hundredths meters, or 37 meters 48 centimeters.

* Pronounced *mĕt-ric;* and meter, *mee-ter.* All the metric names are accented on the first syllable. For equivalents in the old system of weights and measures, see pages 156–7.

184. The names of the divisions of the unit are formed by prefixing to the unit of the table *milli* for 0.001, *centi* for 0.01, *deci* for 0.1 of the unit; and the multiples by prefixing *deka* for 10, *hekto* for 100, *kilo* for 1000, *myria* for 10000 units of the table.

185. *Deci, centi, milli,* correspond to *dimes, cents, mills,* in United States Money.

186. Any one of the denominations of the table can be taken as the unit. Thus, 5.756^{Km} is 5, and 756 thousandths kilometers; 25.5^{mm} is 25, and 5 tenths millimeters.

187. Read the following metric quantities.

1. 15.5^{m}.	6. 187^{mm}.	11. 2.48^{mm}.
2. 77.6^{cm}.	7. 5.63^{cm}.	12. 91.23^{m}.
3. 17.45^{mm}.	8. 144.175^{m}.	13. 843.2^{Km}.
4. 4.875^{Km}.	9. 173.18^{Km}.	14. 76.3^{cm}.
5. 18.75^{cm}.	10. 876.5^{m}.	15. 841^{mm}.

188. Write the following metric numbers as meters and decimals.

16. 177 millimeters. Ans. 0.177^{m}.
17. 3, and 3 tenths centimeters. Ans. 0.033^{m}.
18. 14, and 5 hundredths meters.
19. 73 meters and 7 millimeters.
20. 184 meters and 87 centimeters.
21. 718 kilometers and 4 millimeters.
22. 84, and 7 hundredths kilometers.
23. 217, and 15 thousandths meters.
24. 3527 meters and 15 millimeters as kilometers and decimals.
25. $4^{\text{Km}}\ 7^{\text{Hm}}\ 8^{\text{m}}\ 4^{\text{mm}}$ as hektometers and decimals.

189. To reduce a higher denomination to a lower, or a lower to a higher,

Rule.

Move the decimal point one, two, three, or more places to the right, or left, as the case may require.

190. Oral Exercises.

26. Reduce 37 meters to millimeters.

Solution. As 1000 millimeters = 1 meter, to reduce meters to millimeters we must move the decimal point three places to the right (Art. 63); that is, $37^{m} = 37000^{mm}$, Ans.

27. Reduce 25700 meters to kilometers. Ans. 25.7^{Km}.

28. Reduce 18.7^{Km} to meters; to millimeters.

29. Reduce 2750^{mm} to meters.

30. Reduce 1856^{Hm} to centimeters.

31. Reduce 1756^{mm} to centimeters; to meters; to kilometers.

32. Reduce 6278.5^{m} to centimeters; to kilometers; to millimeters.

33. How many meters will a hoop 3^{m} in circumference pass over in 12 revolutions on the ground?

34. $1.2^{Km} \times 7 =$ how many meters?

35. $0.4^{cm} \times 25 =$ how many meters?

36. $5.5^{mm} \times 9 =$ how many meters?

191. Written Exercises.

37. Add 7.25^{m}, 814^{Km}, 14^{mm}, 744^{cm}.

$$\begin{array}{rr} & 7.25 \\ & 814000.014 \\ & 7.44 \\ \hline \text{Ans.} & 814014.704^{m} \end{array}$$

38. Add 724^{cm}, 185^{mm}, 34^{Km}, 17.35^{m}.

39. Add 17^{Km}, 18^{m}, 417^{cm}, 180^{mm}, 44.3^{Hm}.

40. From 7.18^{m} take 417^{mm}.

41. From 18^{Km} take 18^{m}.

42. From 217^{cm} take 1.13^{m}.

43. The lengths of the principal buildings at the Centennial Exhibition were as follows: Main Building, 426.72^{m}; Machinery Hall, 426.72^{m}; Agricultural Hall, 249.936^{m}; Horticultural Hall, 116.738^{m}; Art Gallery, 111.25^{m}. How far would these buildings have extended if placed end to end?

44. How long would it take a man to walk the length of the buildings placed as named above, if he walked 5.2^{Km} an hour?

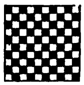
45. If a wheel is 6.3^{m} in circumference, how many times must it turn in going 173.88^{Km}?

46. How many kilometers will a man walk in 18 days, if he walks 8 hours each day and 4.8^{Km} an hour?

192. SQUARE MEASURE.

The units of this table are obtained by squaring (taking twice as a factor) the units of the table of Linear Measure.

100 sq. millimeters ($^{sq\ mm}$)	=	1 sq. centimeter ($^{sq\ cm}$).	
100 sq. centimeters	=	1 sq. decimeter ($^{sq\ dm}$).	
100 sq. decimeters	=	1 sq. meter ($^{sq\ m}$),	or centar (^{ca}).
100 sq. meters	=	1 sq. dekameter ($^{sq\ Dm}$),	or ar (^{a}).
100 sq. dekameters	=	1 sq. hektometer ($^{sq\ Hm}$),	or hektar (^{Ha}).
100 sq. hektometers	=	1 sq. kilometer ($^{sq\ Km}$).	

1 square centi-  meter, divided into 100 square millimeters.

193. The hektar, ar, and centar are used only in land measure. The hektar, the unit for land measure, is a square whose side is a hektometer; hence it is equal to 10000 square meters.

194. Since the scale in square measure is 100 (10 × 10), there will be two places for each denomination. Thus, 25

hektars, 7 ars, 17 centars, and 20 square decimeters, is written 25.07172 hektars, or 250717.2 square meters.

195. Write the following metric quantities as square meters and decimals.

47. $3^{\text{sq Km}}$, $3^{\text{sq Hm}}$, $3^{\text{sq Dm}}$, $3^{\text{sq m}}$. Ans. $3030303^{\text{sq m}}$.

48. $2^{\text{sq m}}$, $2^{\text{sq dm}}$, $2^{\text{sq cm}}$. Ans. $2.0202^{\text{sq m}}$.

49. $137^{\text{sq Km}}$, $9^{\text{sq m}}$.

50. $43^{\text{sq Km}}$, $25^{\text{sq mm}}$.

51. 13^{Ha}, 14^{a}, 2^{ca} in ars and decimals.

196. Oral Exercises.

52. In $8^{\text{sq m}}$ how many square centimeters?

Solution. As 10000 square centimeters = 1 square meter, to reduce square meters to square centimeters we must move the decimal point four places to the right (Art. 63); that is, $8^{\text{sq m}} = 80000^{\text{sq cm}}$, Ans.

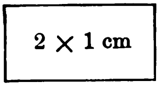
53. In $78456^{\text{sq mm}}$ how many square decimeters? how many square meters?

54. In $75^{\text{sq Km}}$ how many hektars? how many square dekameters? square meters?

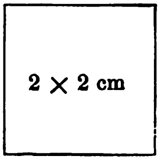
55. Reduce 7.875^{Ha} to square meters.

56. Reduce $843.21^{\text{sq m}}$ to ars; to hektars.

197. A **Rectangle** is a plane (or flat) surface bounded by four straight lines and having all its angles equal.



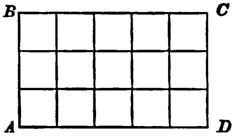
198. The figure is called *rectangular,* and the angles *right angles.*



199. A **Square** is a rectangle whose sides are equal.

200. To find the area of a rectangle.

Let $A\,B\,C\,D$ be a rectangle whose base $A\,D$ is 5 centimeters in length, and whose altitude $A\,B$ is 3 centimeters. (The figure is drawn on a scale of 1 to 2.) If $A\,D$ is divided into 5 equal parts and $A\,B$ into 3, and lines are drawn through the points of division, the rectangle will be divided into squares, each containing 1 square centimeter; and the rectangle will evidently contain 5×3, or 15, of these squares; that is, its area $= (5 \times 3)$ square centimeters $= 15$ square centimeters. Therefore,



The area of a rectangle is the product of its length and breadth.

201. The area of a rectangle divided by the length will give the breadth, and the area divided by the breadth will give the length.

202. Written Exercises.

57. In a rectangular field 200 meters long and 56 meters broad how many ars? $11200^{\text{sq m}} = 112$ ars, Ans.

58. How many sq. meters in a floor 6.8^{m} long and 4.5^{m} wide?

59. How many meters of carpeting 0.6^{m} wide will be needed to cover this floor?

60. At 5 mills a square decimeter of sheet tin what will $5^{\text{sq m}}$ cost?

61. If a square meter of land cost \$ 8, what will an ar cost?

62. At 10 cents a square meter what will it cost to paint a rectangular surface 25.5^{m} long and 10.7^{m} wide?

63. How many bricks, each 20^{cm} long and 10^{cm} wide, will it take to pave a sidewalk 3.3^{m} wide, 1.7^{Km} long?

64. How many shingles will cover a roof 28$^{\text{m}}$ by 15$^{\text{m}}$, if each shingle covers a space 15$^{\text{cm}}$ by 10$^{\text{cm}}$?

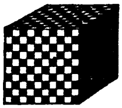
65. What will it cost to cement a cellar 13.5$^{\text{m}}$ by 9$^{\text{m}}$ at $0.65 a square meter?

203. CUBIC MEASURE.

The units of this table are obtained by cubing (taking three times as a factor) the units of the table of Linear Measure.

1000 cubic millimeters ($^{\text{cu mm}}$)	=	1	cubic centimeter	($^{\text{cu cm}}$).
1000 cubic centimeters	=	1	cubic decimeter	($^{\text{cu dm}}$).
1000 cubic decimeters	=	1	cubic meter	($^{\text{cu m}}$).

1 cubic centimeter, divided into 1000 cubic millimeters.



204. In measuring wood a cubic meter is sometimes called a *ster* ($^{\text{st}}$).

205. Since the scale is 1000 ($10 \times 10 \times 10$), three places are required for each denomination. Thus, 2$^{\text{cu m}}$, 2$^{\text{cu dm}}$, 2$^{\text{cu cm}}$ is written 2.002002$^{\text{cu m}}$.

206. Write the following metric quantities as cubic meters and decimals.

66. 4$^{\text{cu m}}$, 4$^{\text{cu dm}}$, 4$^{\text{cu cm}}$. Ans. 4.004004$^{\text{cu m}}$.

67. 55$^{\text{cu m}}$, 55$^{\text{cu dm}}$, 4$^{\text{cu cm}}$.

68. 78786$^{\text{cu cm}}$ 65$^{\text{cu mm}}$.

69. 18478$^{\text{cu dm}}$ 15$^{\text{cu cm}}$.

70. 9167$^{\text{cu m}}$, 8$^{\text{cu cm}}$.

207. Oral Exercises.

71. Reduce 12.87 $^{cu\ m}$ to cubic decimeters.

Solution. As 1000 cubic decimeters = 1 cubic meter, to reduce cubic meters to cubic decimeters we must move the decimal point three places to the right (Art. 63); that is, 12.87 $^{cu\ m}$ = 12870 $^{cu\ dm}$, Ans.

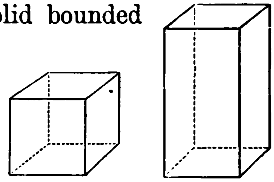
72. Reduce 32500 $^{cu\ dm}$ to cubic meters.

73. Reduce 8 $^{cu\ m}$ to cubic centimeters.

74. Reduce 875 $^{cu\ cm}$ to cubic decimeters; to cubic meters.

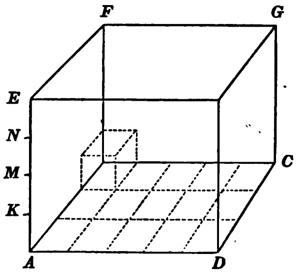
208. A **Rectangular Prism** is a solid bounded by six rectangles.

209. A **Cube** is a rectangular prism bounded by squares.



Cube. Prism.

210. To find the volume of a rectangular prism.



Let $ABCD$, $EFGH$ be the rectangular prism whose base is a rectangle 5 centimeters from A to D, and 3 from A to B, and whose altitude AE is 4 centimeters. (The figure is drawn on a scale of 1 to 2.)

The rectangle $ABCD$ contains $(5 \times 3)^{sq\ cm}$ (Art. 200). If parallel to the base $ABCD$ a plane is passed through the point K, one centimeter from A, it will evidently cut off 15 cubic centimeters, that is, $(5 \times 3 \times 1)$ cubic centimeters. In like manner, if a plane passes through M parallel to the base, it will cut off 15 more; and so on. That is, the rectangular prism contains $(5 \times 3 \times 4)$ cubic centimeters = 60 cubic centimeters. Therefore,

The volume of a rectangular prism is equal to the product of its three dimensions.

211. The volume of a rectangular prism divided by the area of its base will give the altitude; the volume divided by the area of one end will give the length; and the volume divided by the area of one side will give the breadth or width.

212. Written Exercises.

75. How many cubic meters in a box 1.25 ^{m} long, 1.13 ^{m} wide, and 0.8 ^{m} deep?

76. How deep must a box be to contain 6 $^{cu\,m}$ if the top is 2.13 × 2.13 ^{m}?

77. How many cubic meters in a wall 25 ^{m} long, 3.4 ^{m} high, and 268 ^{mm} thick?

78. How many sters in a pile of wood 25 ^{m} long, 1.25 ^{m} wide, and 2 ^{m} high?

79. How many cubic meters of earth must be removed to dig a ditch 75.4 ^{m} long, 83 ^{cm} wide, and 44 ^{cm} deep?

80. At $ 0.85 a $^{cu\,m}$ what would it cost to dig a cellar 13.3 ^{m} long, 11.5 ^{m} wide, and 2.3 ^{m} deep?

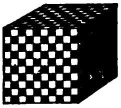
81. How many bricks 20 by 10 by 5 ^{cm} will it take to build a solid wall 75 ^{m} long, 2.5 ^{m} high, and 55 ^{cm} thick?

82. What will it cost for the bricks to build this wall if bricks are $ 8.50 a thousand?

213. CAPACITY MEASURE.

		1 milliliter (^{ml})	= 1 cubic centimeter.
10 milliliters	=	1 centiliter (^{cl}).	
10 centiliters	=	1 deciliter (^{dl}).	
10 deciliters	=	1 LITER (^{l})	= 1 cubic decimeter.
10 liters *	=	1 dekaliter (^{Dl}).	
10 dekaliters	=	1 hektoliter (^{Hl}).	
10 hektoliters	=	1 kiloliter (^{Kl})	= 1 cubic meter.

* Pronounced *lee-ter*.



214. These measures are usually written as liters and decimal parts; or as dekaliters and decimal parts. Thus, 2 kiloliters, 7 hektoliters, 7 liters, and 5 deciliters are written 2707.5^{l}, or 270.75^{Dl}.

215. Oral Exercises.

83. Read 5.6^{l}; 17.5^{Hl}; 41.83^{cl}; 84.6^{Hl}.

84. In 75^{l} how many dekaliters? deciliters? centiliters?

85. In 4789^{cl} how many deciliters? liters? dekaliters?

86. In 78768^{l} how many dekaliters? hektoliters?

87. At $ 3 a hektoliter what is the price of a liter?

88. At 5 cents a centiliter what is the price of a liter?

89. At $ $0.16\frac{2}{3}$ a liter what must I pay for a kiloliter?

216. Written Exercises.

90. What must I pay for 17.3^{Hl} of wheat at $0.28 a dekaliter?

91. What did I pay a liter, if 3.45^{Hl} of wine cost me $ 179.40?

92. If I pay $ 7.30 for 4.25^{Hl} of oats, how much are 4^{l} worth?

93. How many liters in a tank 4.3^{m} long, 3.75^{m} wide, and 1.6^{m} deep?

$(4.3 \times 3.75 \times 1.6)^{cu\,m} = 25.8^{cu\,m} = 25800^{cu\,dm} = 25800^{l}$, Ans.

94. How many hektoliters in a bin 3^{m} long, 2^{m} wide, and 1.5^{m} deep?

95. What is to be the depth of a bin 12^{m} long and 8^{m} wide, in order to hold 2500 hektoliters of grain?

217. WEIGHTS.

10 milligrams (^{mg})	=	1 centigram	(^{cg}).
10 centigrams	=	1 decigram	(^{dg}).
10 decigrams	=	1 GRAM	(^{g}).
10 grams	=	1 dekagram	(^{Dg}).
10 dekagrams	=	1 hektogram	(^{Hg}).
10 hektograms	=	1 **kilogram**, or **kilo**	(^{K}).
1000 kilograms	=	1 **ton**	(^{T}).

218. These measures are usually written as grams and decimal parts; or as kilos and decimal parts. Thus, 32 grams, 17 centigrams are written 32.17^{g}. 2 tons, 7 kilos, 25 grams are written 2007.025^{K}.

219. EQUIVALENTS.

1^{g} = weight of 1 $^{cu\,cm}$, or milliliter, of pure water (at 4° C.).
1^{K} = " of 1 $^{cu\,dm}$, or liter, " " "
1^{T} = " of 1 $^{cu\,m}$, or kiloliter, " " "

 = weight of 

Of the coins of the United States,

The silver half-dollar	weighs	$12\frac{1}{2}$ grams.
" " quarter-dollar	"	$6\frac{1}{4}$ "
" " twenty-cent piece	"	5 "
" " ten-cent "	"	$2\frac{1}{2}$ "
" nickel five-cent "	"	5 "

NOTE. The limit of the weight of a letter for a single postage is 15 grams.

220. Oral Exercises.

96. Read 78.6^{g}; 21.4^{K}; 144^{cg}; 25.4^{T}.

97. In 2.5^{T} how many kilograms? how many grams?

98. In 14.4^{K} how many grams?

99. In 414.7^{g} how many centigrams?

100. In 1215^{mg} how many grams?

101. In 2787^{g} how many kilograms?

102. In 846.8^{K} how many tons?

103. What is the weight of 27^{Hl} of water?

104. How many liters in 55^{dg} of water?

105. How many cubic meters in 757 kilograms of water?

106. If I pay \$85 a ton for sugar, what do I pay a kilogram?

221. Written Exercises.

107. How many kilos of water will a cistern 1.75^{m} long, 1.3^{m} broad, and 0.8^{m} deep, contain?

108. At \$0.75 a kilogram what will 1.17^{T} of coffee cost?

109. At \$7 a ton for coal what will it cost to keep an engine running 6 days if it takes 60^{K} of coal a day?

110. What is the capacity of a bottle that weighs when empty 630^{g}, and when filled with water 7.75^{K}?

WEIGHT OF BODIES.

222. The *specific gravity* of a body is its weight compared with the weight of the same bulk of water. Thus, if a cubic foot of cast iron weighs $7\frac{1}{5}$ times as much as a cubic foot of water, the specific gravity of the iron is 7.2. If cork is one fourth as heavy as water, its specific gravity is 0.25.

223. The specific gravity of a few substances is, approximately, as follows:

Gold	19.3	Copper	8.8	Marble	2.7
Mercury	13.5	Bar Iron	7.8	Milk	1.03
Lead	11.4	Cast Iron	7.2	Alcohol	0.835
Silver	10.5	Granite	2.7	Maple	0.8

224. When the specific gravity of a body and its magnitude in metric measures are known, its weight is easily computed.

225. Written Exercises.

111. What is the weight of 44 liters of mercury?

Solution. 44 liters of water weigh 44 kilos (Art. 219); and 44 liters of mercury weigh 13.5 times as much. $44 \times 13.5^{K} = 594^{K}$, Ans. Hence,

Rule.

Multiply the weight of an equal bulk of water by the specific gravity.

112. Find the weight of a block of marble 1.6^{m} long, 0.75^{m} wide, and 0.4^{m} thick.

113. What is the weight of a bar of gold 25 centimeters long, 3 centimeters wide, and 16 millimeters thick?

114. What is the weight of a bar of iron 3 meters long and 2 centimeters square?

115. What is the weight of a stick of maple timber 4^{m} long, 0.3^{m} wide, and 0.25^{m} thick?

116. What is the value at 30 cents a kilo of a piece of sheet lead 3.25^{m} long, 2^{m} wide, and 3^{mm} thick?

117. What is the weight of 12 liters of milk?

118. If I buy 25 liters of milk and it weighs only 25.5^{K}, is it pure?

119. If a barrel full of alcohol weighs 154^{K} and the barrel alone weighs 11.2^{K}, how many liters of alcohol are there in the barrel?

226. The *names* of the decimal weights and measures are formed according to a simple law, as will be seen by inspection of the following scheme:

MONEY.	LENGTHS.	CAPACITIES.	WEIGHTS.	RATIOS.
Mill.	Milli - meter.	Milli - liter.	Milli - gram.	0.001
Cent.	Centi - meter.	Centi - liter.	Centi - gram.	0.01
(Dime.)	Deci - meter.	Deci - liter.	Deci - gram.	0.1
DOLLAR.	METER.	LITER.	GRAM.	1
	Deka - meter.	Deka - liter.	Deka - gram.	10
	Hekto - meter.	Hekto - liter.	Hekto - gram.	100
	Kilo - meter.	Kilo - liter.	Kilo - gram.	1000

227. Miscellaneous Written Examples.

120. Reduce 12 kilometers to meters.

121. Reduce 1256.7 kilos to tons.

122. Add 125^{m}, 17.06^{Hm}, $.071^{Km}$, and 3000^{mm}.

123. Add 25.06^{T}, 18315^{g}, 16.05^{K}, and 7^{Dg}.

124. From 20 hectoliters subtract 1862.15 liters.

125. From 91 cubic meters subtract 280.31 hektoliters.

126. Multiply 17.28^{g} by 312500.

127. Multiply 51.2^{mm} by 687500.

128. Divide 120^{Km} by 15^{m}.

129. Divide 12.04^{T} by 17200.

130. Divide 2700^{Hl} by 90^{l}.

131. Divide 32^{Km} by 47621.

132. How many coins, each weighing 5 grams, can be coined from a cubic decimeter of silver?

133. In a board 4^m long and 0.4^m wide, how many square decimeters?

134. If a stone 1.25^m long, 0.72^m wide, and 0.4^m thick, weighs 1 ton, what is its specific gravity?

135. Find the cost of 75.6 meters of velvet at $2.50 a meter.

136. Find the weight of 31.5 hektoliters of grain at 45 grams a liter.

137. How many kilos of pure water will it take to fill a rectangular cistern 2 meters deep, 1.5 by 1.75 meters in its other two dimensions?

138. How many kilos do 115 hektoliters of water weigh? Of milk?

139. At $0.17 a dekaliter what must be paid for 25.3^{kl} of corn?

140. What is the capacity of a jar that weighs when empty 275^g, and when filled with water weighs 8.25^k?

141. What is the weight in grams of $253.2^{cu\,cm}$ of milk?

142. If I buy 25 liters of alcohol, and it weighs 22 kilos, is it pure?

143. How many cubic decimeters in a mass of cast-iron that weighs 108 tons?

144. What will a rectangular prism of copper whose dimensions are 8 by 5 by 3^{cm} weigh?

145. If a cistern is 2.5^m long, 15^{dm} wide, and 75^{cm} deep, how many liters will it hold? How many metric tons of water?

146. Brass contains 4 parts of copper to 3 of zinc. How much of each metal is there in 1723.4 grams of brass?

147. What is the weight of a bar of iron 5 meters long, and 3 by 4 centimeters in the other two dimensions?

148. How many liters in 327 kilograms of pure milk?

(For Metric Equivalents see Appendix, p. 374.)

COMPOUND NUMBERS.

229. A **Simple Number** consists of but *one kind* or *denomination;* as 2, $4, 8 books, 5 men, 6 days, 10 miles.

230. A **Compound Number** is composed of *two or more denominations of the same kind;* as 4 days and 7 hours; 3 dollars, 2 cents, and 5 mills; 5 rods, 4 feet, and 6 inches.

NOTE 1. Although the quantities in the tables of the decimal system of money, weights, and measures are of different denominations, yet, as their relations are in the decimal scale, they can be written and treated as simple numbers, any denomination of the table being taken as the unit.

NOTE 2. The several parts of a compound number, though of *different denominations*, are yet of the *same general nature;* thus, 2 weeks, 3 days, and 6 hours are of like nature, *and constitute a compound number;* but 2 weeks, 3 miles, and 6 quarts are unlike in their nature, *and do not constitute a compound number.*

231. The following tables of weights and measures are now generally used in the United States.

232. LINEAR MEASURE.

12	inches (in.)	=	1 foot (ft.).
3	feet	=	1 yard (yd.).
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet	=	1 rod (rd.).
320	rods, or 5280 feet	=	1 mile (m.).

233. Oral Exercises.

1. How many inches in 1 yard?
2. How many yards in 144 inches?
3. How many feet in 3 rods?
4. What is the cost of making a mile of road at $1.50 a rod?
5. How many feet is it round a room that is $16\frac{1}{2}$ by $12\frac{1}{2}$ feet?
6. How many feet in 6 rods?
7. How many yards in 48 feet?

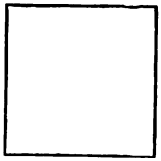
8. How many miles in 960 rods?
9. What part of a foot is 3 inches?
10. What part of a mile is 40 rods?
11. How many inches in $\frac{3}{4}$ of a yard?

234. SQUARE MEASURE.

144	square inches (sq. in.)	=	1 square foot	(sq. ft.).
9	square feet	=	1 square yard	(sq. yd.).
$30\frac{1}{4}$ / $272\frac{1}{4}$	square yards, or / square feet	=	1 square rod	(sq. rd.).
160	sq. rds., or 43560 sq. ft.	=	1 acre	(a.).
640	acres	=	1 square mile	(sq. m.).

NOTE. The units of square measure, so far as they are the same in name, are obtained by squaring (that is, taking twice as a factor) the units of the table of linear measure.

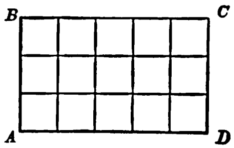
235. A **Rectangle** is a plane (or flat) surface bounded by four straight lines and having all its angles equal.



236. The figure is called *rectangular*, and the angles *right angles.*

237. A **Square** is a rectangle whose sides are equal.

238. To find the area of a rectangle.



Let $A\,B\,C\,D$ be a rectangle whose base $A\,D$ is 5 inches in length, and whose altitude $A\,B$ is 3 inches. If $A\,D$ is divided into 5 equal parts and $A\,B$ into 3, and lines are drawn through the points of division, the rectangle will be divided into squares, each containing 1 square inch; and the rectangle will evidently contain 5×3, or 15, of these squares; that is, its area $= (5 \times 3)$ square inches $= 15$ square inches. Therefore,

The area of a rectangle is the product of its length and breadth.

239. The area of a rectangle divided by the length will give the breadth, and the area divided by the breadth will give the length.

240. Oral Exercises.

12. How many square inches in a rectangle 9 in. long and 7 in. wide? (See Art. 238.)

13. How many square feet in 9 square yards?

14. How many square rods in 4 acres?

15. How many acres in 400 square rods?

16. How many square yards in 72 square feet?

17. What part of an acre is 40 square rods? 80 square rods? 120 square rods?

18. How many square rods in $\frac{3}{8}$ of an acre?

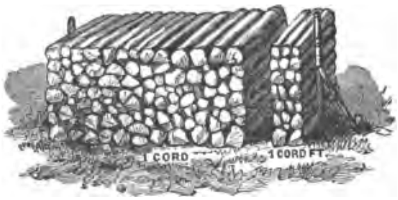
241. SOLID OR CUBIC MEASURE.

1728 cubic inches (cu. in.)	=	1 cubic foot	(cu. ft.).
27 cubic feet	=	1 cubic yard	(cu. yd.).

NOTE. The units of this table are obtained by cubing (that is, taking three times as a factor) the units of the table of linear measure.

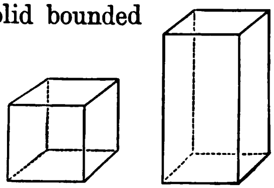
242. WOOD MEASURE.

16 cubic feet	=	1 cord foot	(cd. ft.).
8 cord feet, or 128 cubic feet	=	1 cord	(cd.).



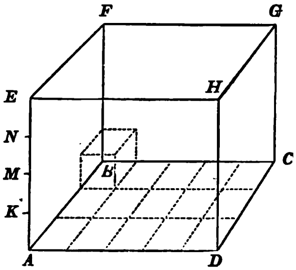
243. A **Rectangular Prism** is a solid bounded by six rectangles.

244. A **Cube** is a rectangular prism bounded by squares.



Cube. Prism.

245. To find the volume of a rectangular prism.



Let $A\,B\,C\,D$, $E\,F\,G\,H$ be the rectangular prism whose base is a rectangle 5 inches from A to D, and 3 from A to B, and whose altitude $A\,E$ is 4 inches.

The rectangle $A\,B\,C\,D$ contains (5 × 3) square inches (Art. 238). If parallel to the base $ABCD$ a plane is passed through the point K, one inch from A, it will evidently cut off 15 cubic inches, that is, (5 × 3 × 1) cubic inches. If, in like manner, a plane passes through M, it will cut off 15 more; and so on. That is, the rectangular prism contains (5 × 3 × 4) cubic inches = 60 cubic inches. Therefore,

The volume of a rectangular prism is equal to the product of its three dimensions.

246. The volume of a rectangular prism divided by the area of its base will give the altitude; the volume divided by the area of one end will give the length; and the volume divided by the area of one side will give the breadth or width.

247. Oral Exercises.

19. How many cubic feet in a cubical block whose edge is 2 feet?

20. How many cord feet in 48 cubic feet of wood?

21. How many cord feet in 3 cords of wood?

22. What will 4 cords of wood cost if 4 cord feet cost $4?

23. How many cubic feet in 3 cd. ft. 8 cu. ft.?

24. How many cubic feet in $\frac{2}{3}$ of a cubic yard?

25. What part of a cord foot is 3 cubic feet?

248. LIQUID MEASURE.

4 gills (gi.)	=	1 pint	(pt.).
2 pints	=	1 quart	(qt.).
4 quarts	=	1 gallon	(gal.).

249. DRY MEASURE.

2 pints (pt.)	=	1 quart	(qt.).
8 quarts	=	1 peck	(pk.).
4 pecks	=	1 bushel	(bush.).

4 quarts liquid measure	=	231	cubic inches.
4 quarts dry measure	=	$268\frac{4}{5}$	cubic inches.
The standard bushel	=	2150.4	cubic inches.

250. Oral Exercises.

26. How many gills in 6 quarts?

27. How many pints in 8 gallons?

28. At 8 cents a quart what will 2 gal. 3 qt. of milk cost?

29. How many gallons in 32 pints?

30. At 25 cents a pint what will 1 gal. 2 qt. 1 pt. of wine cost?

31. At 60 cents a gallon what will 2 gal. 2 qt. 1 pt. of oil cost?

32. At 4 cents a quart what will 4 gal. 3 qt. 1 pt. of vinegar cost?

33. What part of a gallon is 2 qt. 1 pt.?

34. How many gills in $\frac{3}{8}$ of a gallon?

35. What part of a pint is $\frac{1}{7}$ of a quart?

36. What part of a gallon is 3 quarts?

37. How many pecks in 6 bushels?

38. How many quarts in 5 bushels?

39. At 10 cents a quart what will a bushel of cherries cost?

40. At $1 a bushel what will 3 pecks of corn cost?

41. At $2 a bushel what must I pay for 6 quarts of chestnuts?

42. At $0.50 a peck what will 3 bush. 1 pk. of grass-seed cost?

43. If I pay $1.60 for a bushel of walnuts and sell them at $6\frac{1}{4}$ cents a quart, how much do I gain?

44. How many quarts in $\frac{3}{8}$ of a bushel?

45. What part of a bushel is $\frac{1}{2}$ a peck?

46. What part of a bushel is 1 pk. 3 qt. 1 pt.?

251. AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	=	1 pound	(lb.).
2000 pounds	=	1 ton	(t.).

NOTE. In some kinds of business a ton means 2240 pounds. This is called the *long* ton.

252. Oral Exercises.

47. How many ounces in 5 pounds?

48. How many pounds in 48 ounces?

49. At 9 cents a pound what will 100 pounds of sugar cost?

50. At $20 a ton what will 3 t. 100 lb. cost?

51. What part of a ton is 600 pounds?

52. What part of a pound is 10 ounces?

53. What part of an ounce is $\frac{1}{17}$ of a pound?

54. What part of a pound is $\frac{2}{3}$ of an ounce?

253. TIME MEASURE.

60 seconds (sec.)	=	1 minute	(m.).
60 minutes	=	1 hour	(h.).
24 hours	=	1 day	(d.).
7 days	=	1 week	(wk.).
365 days	=	1 common year	(c. yr.).
366 days	=	1 leap year	(l. yr.).
100 years	=	1 century	(C.).

The names of the months, called calendar months, and the number of days in each are:

1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.)	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

NOTE 1. The number of days in each month may be easily remembered by committing the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine."

NOTE 2. A solar year, that is, a year by the sun, is, very nearly, 365 days, 5 hours, 48 minutes, and 50 seconds.

254. According to the Gregorian Calendar, by which we reckon time, there is a leap year whenever the number representing the year (A. D.) is divisible by 4 and not by 100, and also when it is divisible by 400. Thus, 1892 was a leap year, 1894 not, 1600 was a leap year, 1700 and 1800 not.

255. Oral Exercises.

55. How many minutes in 12 hours?
56. How many days in 4 weeks?
57. How many hours in $\frac{2}{3}$ of a day?
58. What part of a day is 9 hours?
59. What part of a minute is 35 seconds?
60. How many weeks in 75 days?
61. How many days in March, April, and May?
62. How many days from July 4 to November 4?

63. What date is it 27 days after May 17?

64. What date was it 27 days before May 17, 1878?

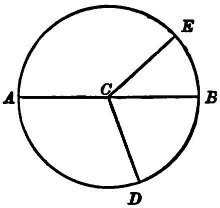
65. Which are leap years? 1854? 1876? 1882? 1700? 1900? 2000?

256. CIRCULAR MEASURE.

60 seconds (″)	=	1 minute	(′).
60 minutes	=	1 degree	(°).
360°	=	1 circumference (circ.).	

NOTE. A degree of the circumference of the earth at the equator is 60 geographical miles (minutes), equal to about $69\frac{1}{8}$ common miles.

257. A **Circle** is a figure bounded by a curved line called the *circumference*, of which every point is equally distant from a point within called the *centre*.



258. An **Arc** is any portion of the circumference, as *A E* or *B D*. An arc equal to a quarter of the circumference is called a *quadrant*.

259. A **Radius** is a line drawn from the centre to the circumference, as *C A* or *C B*.

260. A **Diameter** is a line drawn *through* the centre and limited by the curve, as *A B*.

261. The circumference of a circle is divided into 360 equal parts, called *degrees*, and these are subdivided as shown in the table.

262. Oral Exercises.

66. How many degrees in a semicircumference? in a quadrant, or quarter of a circumference?

67. How many degrees in the four angles of a rectangle? How many in each angle?

68. How many minutes in 5 degrees?

69. How many degrees in 360 minutes?

70. What part of a circumference is 60°?

263. MISCELLANEOUS TABLE.

12 units	=	1 dozen.
12 dozen	=	1 gross.
12 gross	=	1 great gross.
20 units	=	1 score.
24 sheets	=	1 quire.
20 quires	=	1 ream.

(For other tables, see Appendix.)

264. Oral Exercises.

71. How many screws in a gross?

72. How many dozen in 42 units?

73. How many sheets in a ream of paper?

74. How many years in "threescore and ten"?

REDUCTION OF COMPOUND NUMBERS.

265. As there is no common ratio running through these tables of weights and measures, reduction of compound numbers, though the same in principle as in the decimal system, is much more difficult.

266. Written Exercises.

75. In 6 gal. 3 qt. 1 pt. 2 gi. how many gills?

```
 6 gal. 3 qt. 1 pt. 2 gi.
 4
 ――
 27 qt.
  2
 ――
 55 pt.
  4
 ――
222 gi., Ans.
```

As 4 qt. = 1 gal., 6 gal. = 6 × 4 qt.; adding the 3 qt. we have 27 qt. As 2 pt. = 1 qt., 27 qt. = 27 × 2 pt.; adding the 1 pt. we have 55 pt. As 4 gi. = 1 pt., 55 pt. = 55 × 4 gi.; adding the 2 gi. we have 222 gi. Therefore, 6 gal. 3 qt. 1 pt. 2 gi. = 222 gi. Hence,

267. Reducing numbers of higher denominations to a number of a lower is called *Reduction Descending.*

268. To reduce numbers of higher denominations of a compound number to a number of a lower denomination,

Rule.

Multiply the number of the highest denomination given by the number of units it takes of the next lower to make one of this higher, and to the product add the given number (if any) of the lower denomination; multiply this result by the number of units it takes of the next lower denomination to make one of this; add as before, and so proceed till the number is brought to the denomination required.

76. Reduce 222 gills to gallons, quarts, etc.

$$\begin{array}{r l}
4\,)\,\underline{222} & \text{gi.} \\
2\,)\,\underline{55} & \text{pt.} + 2 \text{ gi.} \\
4\,)\,\underline{27} & \text{qt.} + 1 \text{ pt.} \\
6 & \text{gal.} + 3 \text{ qt.}
\end{array}$$

Ans. 6 gal. 3 qt. 1 pt. 2 gi.

As 4 gi. = 1 pt., we divide the 222 gi. by 4 and obtain 55 pt. and 2 gi. remainder. As 2 pt. = 1 qt., we divide the 55 pt. by 2 and obtain 27 qt. and 1 pt. remainder. As 4 qt. = 1 gal., we divide the 27 qt. by 4 and obtain 6 gal. and 3 qt. remainder. Therefore, 222 gi. = 6 gal. 3 qt. 1 pt. 2 gi. Hence,

269. Reducing a number of a lower denomination to numbers of higher is called *Reduction Ascending.*

270. To reduce a number of a lower denomination to numbers of higher denominations,

Rule.

Divide the given number by the number of units it takes of that denomination to make one of the next higher; divide the quotient in the same way, and so proceed till the number is brought to the denomination required. The last quotient and the remainders (if any) will be the result required.

271. Reduction Ascending and Reduction Descending *prove* each other.

77. Reduce 18 bush. 3 pk. 7 qt. 1 pt. to pints.

78. Reduce 1577048 seconds to minutes, hours, etc.

79. Reduce 20° 25′ 30″ to seconds.

80. Reduce 4 t. 1473 lb. 7 oz. to ounces.

81. Reduce 2548 square inches to higher denominations.

82. Reduce 4 sq. m. 25 a. 154 sq. rd. to square rods.

83. Reduce 8 cu. yd. 1727 cu. in. to cubic inches.

84. Reduce 4 sq. yd. to square inches.

85. Reduce 4 gal. 1 pt. to gills.

86. Reduce 2 wk. 6 d. 8 h. 16 sec. to seconds.

87. Reduce 4 m. 319 rd. to rods.

88. Reduce 3795 rods to miles, etc.

89. Reduce 6598 pints to quarts, pecks, etc.

90. Reduce 368294″ to higher denominations.

91. Reduce 4680 gills to higher denominations.

92. Reduce 195261 cubic inches to feet and yards.

93. Reduce 310556 square rods to higher denominations.

NOTE 1. The rules given in Arts. 268 and 270 are equally applicable when the number given is a fraction.

94. Reduce $\frac{10}{21}$ rd. to inches.

$$\frac{10 \times 16\frac{1}{2} \times 12}{21}\text{ in.} = \frac{10 \times \overset{11}{\cancel{33}} \times \overset{6}{\cancel{12}}}{\underset{7}{\cancel{21}} \times \cancel{2}}\text{ in.} = \frac{660}{7}\text{ in.} = 94\tfrac{2}{7}\text{ in., Ans.}$$

NOTE 2. In the first statement of Ex. 8, the $16\frac{1}{2}$, in the numerator, is equal to $\frac{33}{2}$, and, in the second statement, the 33 is retained in the numerator as a factor in the dividend, and the 2 is put in the denominator as a factor in the divisor. In practice the first statement should not be written at all.

95. Reduce $\frac{11}{16}$ of a gallon to integers of lower denominations.

$$\frac{11}{16}\text{ gal.} = \frac{11}{16} \times 4\text{ qt.} = \frac{11}{4}\text{ qt.} = 2\tfrac{3}{4}\text{ qt.}$$

$$\frac{3}{4}\text{ qt.} = \frac{3}{4} \times 2\text{ pt.} = \frac{3}{2}\text{ pt.} = 1\tfrac{1}{2}\text{ pt.}$$

$$\frac{1}{2}\text{ pt.} = \frac{1}{2} \times 4\text{ gi.} = 2\text{ gi.}$$

Ans. 2 qt. 1 pt. 2 gi.

NOTE 3. When the given fraction is a decimal the operation is much more simple.

96. Reduce 0.713 m. to integers of lower denominations.

$$\begin{array}{r}
0.713\text{ m.} \\
320 \\ \hline
14260 \\
2139 \\ \hline
228.16\cancel{0}\text{ rd.} \\
16\tfrac{1}{2} \\ \hline
96 \\
168 \\ \hline
2.64\text{ ft.} \\
12 \\ \hline
7.68\text{ in.}
\end{array}$$

0.713 m. × 320 = 228.16 rd.
0.16 rd. × $16\frac{1}{2}$ = 2.64 ft.
0.64 ft. × 12 = 7.68 in.

Ans. 228 rd. 2 ft. 7.68 in.

97. What part of a bushel is 3 pk. 5 qt.?

$$3\text{ pk. }5\text{ qt.} = 3\tfrac{5}{8}\text{ pk.} = (3\tfrac{5}{8} \div 4)\text{ bush.} = \tfrac{29}{32}\text{ bush.},\ \text{Ans.}$$

NOTE 4. The principle is the same as in the Rule in Art. 268, the lowest denomination being reduced to the next higher.

98. Reduce 2 qt. 1 pt. 3 gi. to the decimal of a gallon.

$$\begin{array}{r|l} 4 & 3. \\ \hline 2 & 1.75 \\ \hline 4 & 2.875 \\ \hline & 0.71875 \text{ gal., Ans.} \end{array}$$

The given units are first written in a column with the lowest denomination at the top. 3 gi. are $\frac{3}{4}$ of a pt.; $\frac{3}{4}$ (or $3 \div 4$) pt. = 0.75 pt.; this is written at the right of the 1 pt., making 1.75 pt. The 1.75 pt. are reduced to quarts by dividing by 2 (2 pints = 1 quart), giving 0.875 qt., which is written at the right of the 2 qt., making 2.875 qt.; and in like manner we continue dividing on the same principle as in Arts. 268 – 9.

99. Reduce $\frac{11}{12}$ of a gallon to lower denominations.

100. Reduce $\frac{9}{25}$ of an acre to lower denominations.

101. Reduce $\frac{7}{24}$ of a ton to lower denominations.

102. Reduce 0.375° to lower denominations.

103. Reduce 7654.825 rods to miles, rods, feet, and inches.

104. Reduce 0.984375 of a bushel to lower denominations.

105. Reduce 0.40625 of a gallon to lower denominations.

106. Reduce 6 rd. 5 ft. 9 in. to the fraction of a mile.

107. Reduce 42 sq. rd. 181 sq. ft. 72 sq. in. to the fraction of an acre.

108. Reduce 875 lb. 10 oz. to the fraction of a ton.

109. Reduce 175 ft. 6 in. to the decimal of a mile.

110. Reduce 145 sq. rd. 68 sq. ft. 9 sq. in. to the decimal of an acre.

111. Reduce 3 pk. 4 qt. 1 pt. to the decimal of a bushel.

272. ADDITION OF COMPOUND NUMBERS.

The difference between operations with the compound numbers of these tables and with simple numbers is, that in these compound numbers the scale is not decimal, nor regular, but varies; the principles, however, are the same.

Therefore no special rules are needed for addition, subtraction, multiplication, or division of compound numbers.

112. Find the sum of 3 gal. 3 qt. 1 pt. 3 gi., 4 gal. 2 qt. 1 pt. 2 gi., 7 gal. 2 qt. 3 gi., 5 gal. 3 qt. 1 pt., 9 gal. 2 gi.

	gal.	qt.	pt.	gi.
	3	3	1	3
	4	2	1	2
	7	2	0	3
	5	3	1	0
	9	0	0	2
Sum,	31	0	1	2

Arrange the units of the same denomination in the same column. The sum of the right-hand column is 10 gi. = 2 pt. 2 gi. Writing the 2 gi. under the column of gills, add the column of pints, including the 2 pt. from the gills. The sum is 5 pt. = 2 qt. 1 pt. Write the 1 pt. under the column of pints and add the 2 qt. to the sum of the quarts in the next column. Proceed in this manner until all the columns are added.

(113.)

wk.	d.	h.	min.	sec.
3	4	18	40	30
3	6	6	20	30
1	2	20	30	15
3	0	2	28	45

(114.)

rd.	yd.	ft.	in.
5	3	2	10
4	4	2	5
6	5	0	6
3	4	2	7
21	$1\frac{1}{2}$	2	4
21	2	0	10

NOTE. A fraction occurring in the amount should be reduced to integers of lower denominations; or, if possible, enough taken from the lower to cause the fraction to disappear. Thus, in Ex. 114, $\frac{1}{2}$ yd., or 1 ft. 6 in., taken from the 2 ft. 4 in. increases the $1\frac{1}{2}$ yd. to 2 yd. and leaves 0 ft. 10 in. in the columns of feet and inches.

115. A farmer raised in one field 21 bush. 3 pk. 7 qt. 1 pt. of wheat; in another, 48 bush. 2 pk. 1 pt.; in another, 28 bush. 6 qt.; and in another, 75 bush. 1 pk. 5 qt. 1 pt.: how much wheat did he raise in the four fields?

116. Add 29° 59′ 59″, 25° 20′ 30″, 8° 25′ 55″, and 10° 10′ 10″ together.

273. SUBTRACTION OF COMPOUND NUMBERS.

117. From 14 gal. 2 qt. 1 pt. 2 gi. take 5 gal. 3 qt. 0 pt. 3 gi.

	gal.	qt.	pt.	gi.
	14	2	1	2
	5	3	0	3
Remainder,	8	3	0	3
Proof,	14	2	1	2

As 3 gi. cannot be taken from 2 gi., take 1 pt. = 4 gi. from the 1 pt. and add it to the 2 gi.; this gives 6 gi. 3 gi. from 6 gi. leaves 3 gi., which is written under the column of gills. As we have used the 1 pt. of the minuend, we have 0 pt. from 0 pt. = 0 pt. as the next term in the remainder; and so on.

(118.)

	bush.	pk.	qt.	pt.
	125	1	5	1
	24	3	7	1
Ans.	100	1	6	0

(119.)

wk.	d.	h.	min.	sec.
3	4	23	45	30
1	6	16	30	45

120. If of the month of December 17 days, 9 hours, 43 minutes, 27 seconds have passed, how many days, hours, etc., remain?

121. If from a lot of land containing 7 a. 80 sq. rd. 5 sq. ft. there are sold 1 a. 157 sq. rd. 78 sq. ft., how much remains?

274. To find the difference of time between two dates.

122. What is the difference of time between July 15, 1892, and Nov. 8, 1894?

Solution. From July 15, 1892, to July 15, 1894, is 2 years; from July 15 to Oct. 15 is 3 months; from Oct. 15 to Oct. 31 is 16 days; and from Oct. 31 to Nov. 8 is 8 days. Hence, from July 15, 1892, to Nov. 8, 1894, is 2 yr. 3 m. 24 d., Ans. Hence,

Rule.

Find the number of complete years, then the number of complete calendar months, and then the remaining days.

123. How long from the battle of Waterloo, June 18, 1815, to the death of Napoleon, May 5, 1821? Ans. 5 yr. 10 m. 17 d.

124. How long from the battle of Lexington, Apr. 19, 1775, to the surrender of Cornwallis, Oct. 19, 1781?

125. How long from the inauguration of Washington, Apr. 30, 1789, to the battle of New Orleans, Jan. 8, 1815?

126. How long from the Declaration of Independence, July 4, 1776, to the present time?

127. Daniel Webster was born Jan. 18, 1782, and died Oct. 24, 1852; at what age did he die?

128. A note given July 6, 1891, was paid Sept. 9, 1895. How long was it on interest?

129. Find the exact number of days from April 7, 1891, 9 A.M., to June 13, 1893, 4 P.M.

	days.	hours.
April 7, 1891, 9 A.M., to April 7, 1893, 9 A.M....	731	
April 7, 1893, " " June 7, 1893, " ...	61	
June 7, " " " " 13, " " ...	6	
" " 9 A.M. to 4 P.M....	0	7
Ans.	798	7

From Apr. 7, 1891, 9 A.M., to Apr. 7, 1893, 9 A.M., there are 2 years, of which one is a leap year; that is, 366 d. + 365 d., or 731 d. From April 7, 1893, 9 A.M., to May 7, 1893, 9 A.M., there are 30 days; and from May 7, 9 A.M., to June 7, 9 A.M., 31 days.

130. Find the exact number of days from Feb. 26, 1888, 7 P.M., to Dec. 23, 1895, 3 P.M.

275. MULTIPLICATION OF COMPOUND NUMBERS.

131. Multiply 4 gal. 3 qt. 1 pt. 3 gi. by 7.

	gal.	qt.	pt.	gi.
Multiply	4	3	1	3
By				7
Product,	34	3	0	1

7 times 3 gi. = 21 gi. = 5 pt. and 1 gi. Write the 1 gi. under the gills, and reserve the 5 pt. to be added to the 7 times 1 pt. 7 times 1 pt. plus 5 pt. = 12 pt. = 6 qt. Write 0 under the pints, and reserve the 6 qt. to be added to the 7 times 3 qt.; and so on.

(132.)

	a.	sq. rd.	sq. yd.	sq. ft.	
	3	104	25	9	
				12	
	43	138	$7\frac{1}{2}$	6	sq. in.
or,	43	138	8	1	72

(133.)

m.	rd.	yd.	ft.	in.
2	123	4	1	6
				7

134. Bought 5 loads of wood, each measuring 1 cd. 5 cd. ft. 8 cu. ft., at $6 a cord; what was the quantity bought and the cost of the whole? Ans. 8 cd. 3 cd. ft. 8 cu. ft.; $50.62½.

135. If a ship sail 2° 30′ 20″ a day, how far will she sail in 8 days?

276. DIVISION OF COMPOUND NUMBERS.

136. Divide 30 gal. 3 qt. 1 pt. 3 gi. by 7.

	gal.	qt.	pt.	gi.
7)	30	3	1	3
Ans.	4	1	1	$1\frac{4}{7}$
				7
Proof,	30	3	1	3

30 gal. ÷ 7 gives 4 gal. and a remainder of 2 gal. 2 gal. = 8 qt. which added to the 3 qt. gives 11 qt. 11 qt. ÷ 7 gives 1 gal. and a remainder of 4 qt. 4 qt. = 8 pt. which added to the 1 pt. gives 9 pt.; and so on.

137. Divide 3 wk. 6 d. 14 h. 17 min. 57 sec. by 3.

138. Divide 25° 4′ 10″ by 10.

139. Divide 107 gal. 1 qt. by 12.

140. Divide 98 bush. 3 pk. 2 qt. 1 pt. by 11.

LONGITUDE AND TIME.

277. As the earth turns on its axis from west to east once in 24 hours, the sun appears to go from east to west round the earth, 360°, in 24 hours; consequently in 1 hour it appears to go $\frac{1}{24}$ of 360°, or 15°; in 1 minute, $\frac{1}{60}$ of 15°, or 15′; in 1 second. $\frac{1}{60}$ of 15′, or 15″. Hence the following

278. TABLE OF LONGITUDE AND TIME.

360° of longitude	correspond to	24 hours, or 1 day of time.
15° of longitude	"	1 hour of time.
15′ of longitude	"	1 minute of time.
15″ of longitude	"	1 second of time.

279. To obviate the inconvenience arising from a constant change of time in going east or west, what is called STANDARD TIME was adopted in the United States, Nov. 18, 1883. The country was divided into four time-belts, each 15° wide. These belts are called, respectively, Eastern, Central, Mountain, and Pacific. The local time of the central meridian of each belt is the Standard Time of the entire belt. The Central Meridians are the 75th, 90th, 105th, and 120th, west from Greenwich. As the central meridians are 15° apart, it is evident that the difference of time of these belts is marked by exact hours. Thus, when it is 12 o'clock, M., in the Eastern belt, throughout the Central belt it is 11 A. M., throughout the Mountain belt it is 10 A. M., and throughout the Pacific belt it is 9 A. M. (See Map, page 377.)

141. What is the absolute difference of time between Greenwich and Washington, Washington being 77° 2′ 48″ west of Greenwich?

$$15\overline{)\,77^\circ \quad 2' \quad 48''}$$
$$5\text{ h. } 8\text{ m. } 11\tfrac{1}{5}\text{ sec., Ans.}$$

Since 15° of longitude make a difference of 1 hour in time, and the scale for degrees, minutes, and seconds is the same as that for hours, minutes, and seconds, the difference in longitude in degrees, minutes, and seconds divided by 15 will give the absolute difference in time in hours, minutes, and seconds. Hence, 77° 2′ 48″ divided by 15 gives the absolute difference in time, viz. 5 h. 8 m. $11\frac{1}{5}$ sec. between Greenwich and Washington. Hence,

280. To find the absolute difference of time between two places when their longitude is known,

Rule.

Divide the difference of longitude in degrees, minutes, and seconds by 15, *and the quotient will represent the absolute difference in time in hours, minutes, and seconds.*

NOTE. At the place most easterly the sun rises first; that is, the day begins first, and hence mid-day comes first, and the day closes first. In using the expression *absolute difference of time,* we refer to the time of the day as it relates to sunrise, or mid-day, or sunset.

Find the absolute difference in time between

142. Boston, 71° 4′ 9″ W., and Washington 77° 2′ 48″ W.

143. Paris, 2° 20′ 15″ E., and New York, 74° 0′ 3″ W.

144. Philadelphia, 75° 10′ W., and Chicago, 87° 35′ W.

145. New Orleans, 90° 7′ W., and St. Petersburg, 30° 19′ E.

146. What is the absolute difference in time for 90° in longitude?

281. To find the difference in the longitude of two places, when the absolute difference in time is known.

147. The absolute difference in time between Washington and Boston is 23 m. $54\frac{3}{5}$ sec.; what is the difference in the longitude of the two places?

	m.	sec.
	23	$54\frac{3}{5}$
		15
5°	58′	39″, Ans.

As 15° of longitude make a difference of 1 hour in time, and the scale for degrees, minutes, and seconds is the same as that for hours, minutes, and seconds, the difference in time in hours, minutes, and seconds multiplied by 15 will give the difference in longitude in degrees, minutes, and seconds. Hence, 23 m. $54\frac{3}{5}$ sec. multiplied by 15 gives the difference in longi-

tude in minutes and seconds, viz. 358′ 39″ = 5° 58′ 39″ ; that is, the difference in longitude between Boston and Washington is 5° 58′ 39″, Ans. Hence,

Rule.

Multiply the absolute difference in time, expressed in hours, minutes, and seconds, by 15, *and the product will represent the difference in longitude in degrees, minutes, and seconds.*

148. How many degrees west of Greenwich is New York, the absolute difference in time between Paris and New York being 5 h. 5 m. 21⅕ sec.?

NOTE. The difference in longitude between Paris and New York is found to be 76° 20′ 18″, and this diminished by 2° 20′ 15″, the *east* longitude of Paris, gives 74° 0′ 3″ for the *west* longitude of New York. Ans. 74° 0′ 3″.

149. The sun rises at Springfield 6 m. 6⅕ sec. later by the clock (Standard Time) than in Boston. What is the longitude of Springfield?

150. The absolute difference in time between the Bermudas (which are east of New York) and New York is 37 m. Find the longitude of the Bermudas.

282. Miscellaneous Oral Exercises.

151. In 33 feet how many yards? How many rods?

152. If I hire a horse and carriage at 16⅔ cents a mile, what must I pay for a ride of 15 miles?

153. A certain rectangular garden is 5 rods long and 3 rods wide; how many square rods does it contain?

154. How many rods round a rectangular garden which is 5 rods long and 3 rods wide?

155. How many square rods in a garden that is 6 rods square?

156. How many rods round a square garden, each side being 6 rods long?

157. How many square rods in a rectangular garden which is

10 rods long and 6 rods wide? How many rods round this garden? What will it cost to fence it at $ 1.25 a rod?

158. At $ 2 a square rod what will half an acre of land cost?

159. How many cubic inches in a rectangular prism or block which is 4 inches long, 3 inches wide, and 2 inches thick? How many square inches in one of its greatest faces? How many in one of its smallest faces? How many in one of the other faces? How many in its six faces?

160. How many square feet in the surface of a cubical box whose edges are 2 feet in length?

161. How many cubic feet in a cube, each edge of which is 4 feet long?

162. What will 3 cords of wood cost, if 4 cord feet cost $ 3?

163. At 12½ cents a pound what will 12 pounds of beef cost?

164. At 33⅓ cents a pound what will 15 pounds of butter cost?

165. How many seconds in 1 m. 30 sec.?

166. How many minutes between half past 9 o'clock and noon?

167. If a man earns $ 1½ a day, how many dollars will he earn in the six working-days of a week?

168. How many days in leap year from the 1st of January to the 10th of March inclusive? How many in a common year?

169. How many days in June, July, and August?

170. At 10 cents a dozen what will 2 gross of buttons cost?

171. At 3 cents apiece what will 2 dozen oranges cost?

172. What will 2 reams of paper cost at 10 cents a quire?

283. Miscellaneous Written Examples.

173. What cost 43 a. 140 sq. rd. of land at $ 40 an acre?

174. What cost 3 t. 1562½ lb. of coal at $ 6 a ton?

175. What will it cost to build 24 m. 140 rd. of railroad at $ 5775 a mile?

176. A rectangular field is 40.5 rods long, and 30.5 rods wide; what will it cost to build a wall around it at $ 1 a rod?

177. How much land in a rectangular field that is 40.5 rods long and 25.75 rods wide?

178. What cost 82.5 lb. of coal at $ 7 a ton?

179. If 365¼ days make a year, how many days, hours, etc. are there in 0.785 of a year?

180. What is the cost of 3 pieces of cloth, the first containing 15 yards at $ 2.25 a yard; the second, 12.5 yards at $ 3.50 a yard; and the third, 8.8 yards at $ 3.25 a yard?

181. A three-sided plat of ground is enclosed by a railroad on one side and highways on the other two sides; the side next the railroad is 4.1 rods long, and the other two sides are respectively 4 rods and 0.9 of a rod in length; what is the cost of fencing this plat at $ 3.75 a rod?

182. How many square feet in a board which is 18.25 feet long and 2.8 feet wide?

183. Bought a load of straw that weighed 1 t. 287½ lb. at $ 8 a ton; what shall I pay for the load?

184. Paid $ 7.175 for 35 gal. 3 qt. 1 pt. of vinegar; what was the price a gallon?

185. What is the cost of carpeting a room that is 16.5 feet long and 15 feet wide, the carpet costing $ 1.25 a square yard?

186. If a family use 29 gal. 3 qt. 2 gi. of molasses in 6 months, what is the average a month?

187. If 10 t. 1825 lb. of hay is harvested from 5 acres, what is the crop on one acre?

188. If 8 boxes of sugar weigh 2 t. 760 lb., what is the weight of a box?

189. If 9 grain-bins contain 143 bush. 2 pk. 2 qt. 1 pt. of grain, what does 1 bin contain?

190. If a man travel 212 m. 66 rd. in 7 days, what distance does he travel a day?

191. How many years, months, and days from Apr. 19, 1894, to Feb. 7, 1899?

192. Find the exact number of days, hours, and minutes from Dec. 7, 1895, 4 o'clock P. M., to June 21, 1896, 9 h. 30 m. A. M.

193. Find the absolute difference in time, stating in which it is earlier, between San Francisco, 122° 26′ 15″ W., and Oahu, 157° 37′ W.

194. Find the absolute difference in time, stating in which it is earlier, or later, between Boston and Rome, 12° 27′ 14″ E.; between Boston and San Francisco; between Boston and Oahu; between Boston and Calcutta, 88° 19′ 2″ E.

195. Two travellers, A and B, one from Maine, the other from the West, meet in Boston. A finds that the sun sets $10\frac{1}{2}$ minutes earlier than at his home, and B that it sets $3\frac{1}{2}$ minutes later than at his home. Which one came from Maine, and how many degrees apart are their homes?

196. Find the absolute difference in time, stating in which it is later, between London, 0° 5′ 38″ W., and New York.

197. How many cords in a pile of wood 40 feet long, 4 feet wide, and $6\frac{1}{2}$ feet high? What is it worth at $6.50 a cord?

198. If a bale of hay weighs 425 pounds, how many bales are there in 2 tons and 675 pounds?

199. A owns three fifths of a wood lot and B the rest. Two thirds of the difference between their shares is 3 acres and 26 square rods. How many acres in each man's share?

200. What will it cost to dig a cellar 35 feet long, 22 feet wide, and 6 feet deep at $0.25 for removing each cubic yard of earth?

201. How many bushels of grain will it take to fill a bin that is 10 ft. 5 in. long, 3 ft. 4 in. wide, and 5 ft. deep?

202. How many acres in a rectangular field 35 rods 10 feet by 47 rods 6 feet?

203. What will it cost to lay a pavement 55 feet long, 8 feet 6 inches wide, at $37\frac{1}{2}$ cents a square yard?

204. A rectangular lot of land 65 rods by 34 rods is enclosed by a fence of 3 lines of wire. If seven yards of this wire weigh a pound, and it costs $3\frac{1}{2}$ cents a pound, what does the wire for the fence cost?

205. On a railroad 45 m. 217 rd. $13\frac{1}{2}$ ft. long there are 11 stations, including those at the two ends of the road. What is the average distance between the stations?

206. How deep is a cellar 53 feet long and 45 feet wide, if 265 cubic yards of earth are removed in digging it?

207. A rectangular piece of land 1320 yards long and 2 rods wide was taken to lay out a street. At $160 an acre what ought to be paid to the owner of the land?

208. In digging a well 600 tubfuls of earth, each tubful equal to a bushel, were removed. At $1.50 a cubic yard, what ought to be paid for digging the well?

209. If a man takes 3 steps of 28 inches each in 2 seconds, how long will it take him to walk 5 miles?

210. How much must I pay for 3 tons 357 pounds of hay at $11 a ton?

211. How many pickets $1\frac{1}{2}$ inches wide will it take to fence from the streets a corner lot running on one street 312 feet 6 inches and on the other 162 feet 6 inches, if the distance between the pickets is the same as the width of the pickets?

212. How many posts and rails will it take for the fence named in Ex. 211, if the posts are 12 feet 6 inches apart, and two rails are used in each length of fence, and the rails so put on as to touch end to end?

213. The Manufacturers and Liberal Arts Building of the Columbian Exposition at Chicago was 1687 feet long and 787 feet wide. How many acres were there in the ground-area of the building?

214. In constructing the building named in Ex. 213, 17000000 square feet of lumber were used. What was the cost of this lumber at $20 a thousand square feet?

215. How many gallons of water will fill a tank 6 feet long, $2\frac{1}{2}$ feet wide, and 5 feet deep?

216. A coal dealer buys 550 (long) tons of coal at $3.50 a ton and sells it at $4.50 a (short) ton. If the average expense of delivery is $0.75 a (short) ton, what does the dealer make on the 550 (long) tons?

217. How many fence pickets $1\frac{1}{2}$ inches wide and 3 feet 5 inches long can be cut from a board 15 inches wide and 14 feet 6 inches long, if the saw-cut is $\frac{1}{8}$ of an inch in width?

218. How long is the piece of board that is left after cutting off the pickets named in Ex. 217? How much wider must the board be to get in all four more pickets?

219. How many slates will it take to cover a roof of which each side is 45 feet 8 inches long and 24 feet wide, if it takes 4 slates to a square foot? What will the slates cost at $4 a hundred?

220. A rectangular lot of land 300 feet wide and 400 feet long, already surrounded by streets, is to be cut up into house lots. A street 50 feet wide is to be cut midway from side to side and parallel to the 300 feet ends. If the lots each have a frontage of 75 feet on the streets that are 300 feet long, and the corner lots are sold for 0.37\frac{1}{2}$ a square foot, and the others for 0.33\frac{1}{3}$ a square foot. What is received for the entire lot of land?

221. How many bricks, laid on the 8 by 4 face, will it take to pave a sidewalk 320 feet long and $6\frac{1}{2}$ feet wide? (Bricks are usually 8 by 4 by 2 inches.)

222. How much will the bricks for the sidewalk mentioned in Ex. 221 cost at $6 a thousand, if we allow 100 bricks for waste?

223. At $0.75 for laying a square yard of the sidewalk named above, what will it cost to lay the bricks?

224. If there are two driveways across the sidewalk named above, each covering 12 feet of the length of the walk, where the bricks are laid with the edge 8 by 2 inches as the face, how many more bricks must be bought?

225. If double price is paid for laying these driveways, what will the bricks and the laying for the whole walk cost?

226. A room is 22 feet long, 18 feet wide, and 8 feet from the base-board to the ceiling. There are 2 doors, 7 by $3\frac{1}{2}$ feet, and 3 windows, 6 by $3\frac{1}{2}$ feet. Paper for the room costs 25 cents a roll of 8 yards in length and 18 inches in width. If a yard loss is allowed in each roll for matching and waste, and 25 cents a roll is paid for laying, what does it cost to paper the side-walls of the room? The base-board is 2 feet wide.

227. If picture-moulding, passing over the doors but not over the windows, is put round the room named above, and the moulding costs 5 cents a foot (including putting on), what is the additional cost?

228. The room described in Ex. 226 is to be carpeted with a carpet $\frac{3}{4}$ of a yard wide, and the breadths are to run lengthwise of the room. How many strips will be required?

229. If 12 inches is allowed for matching and turning under at the ends, what will it cost to carpet the room described in Ex. 226, if the carpet ($\frac{3}{4}$ of a yard wide) costs $1.50 a yard, and 10 cents a yard for making and laying it?

230. If the carpet for the room named above were laid with the strips running widthwise of the room, how much more or less would it cost to carpet the room?

231. If 10 square yards are deducted for doors, windows, and base-boards, what will it cost to plaster a room 21 feet 6 inches long, 18 feet wide, and 10 feet high, at $0.30 a square yard?

PERCENTAGE.

284. Oral Exercises.

1. What is $\frac{1}{100}$ of \$100? $\frac{3}{100}$? $\frac{15}{100}$? $\frac{45}{100}$?

2. What is $\frac{7}{100}$ of \$400? $\frac{11}{100}$? $\frac{25}{100}$?

3. What is $\frac{5}{100}$ of \$500? of \$800? of \$1200?

4. How many hundredths of \$100 is \$1? \$6? \$21?

5. How many hundredths of anything is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{3}$ of it? $\frac{1}{5}$ of it?

285. Percentage is the term applied to numerical operations in which the unit of computation is *one hundred.* It is so called from **Per Cent,** which means *by the hundred. Ten* per cent of a bushel of corn means ten hundredths of it, or ten parts out of every hundred parts; *six* per cent of a sum of money is six hundredths of the sum, or \$6 out of every \$100. Instead of the words *per cent* it is customary to use the sign, %; thus, 6 per cent is written 6%.

286. The **Rate per cent,** or the **Rate,** is the number of hundredths. Thus, 6% is $\frac{6}{100}$, or 0.06, that is, 6 parts of each hundred parts.

287. The **Percentage** is such part of the base as is represented by the rate. Thus, the percentage on \$200 at 6 per cent is \$12.

288. The **Base** is the number on which the percentage is computed. Thus, \$200 is the base on which the percentage is computed in Art. 292; a bushel of corn is the first base mentioned in Art. 290.

289. The **Rate**, being a certain number of hundredths, may be expressed either *decimally*, or by a *common fraction*, as in the following

TABLE.

1	per cent	or	1 %	is	0.01	=	$\frac{1}{100}$.
2	"	"	2 %	"	0.02	=	$\frac{1}{50}$.
5	"	"	5 %	"	0.05	=	$\frac{1}{20}$.
$6\frac{1}{4}$	"	"	$6\frac{1}{4}$ %	"	0.0625	=	$\frac{1}{16}$.
$8\frac{1}{3}$	"	"	$8\frac{1}{3}$ %	"	$0.08\frac{1}{3}$	=	$\frac{1}{12}$.
$12\frac{1}{2}$	"	"	$12\frac{1}{2}$ %	"	0.125	=	$\frac{1}{8}$.
$16\frac{2}{3}$	"	"	$16\frac{2}{3}$ %	"	$0.16\frac{2}{3}$	=	$\frac{1}{6}$.
20	"	"	20 %	"	0.20	=	$\frac{1}{5}$.
25	"	"	25 %	"	0.25	=	$\frac{1}{4}$.
$33\frac{1}{3}$	"	"	$33\frac{1}{3}$ %	"	$0.33\frac{1}{3}$	=	$\frac{1}{3}$.
50	"	"	50 %	"	0.50	=	$\frac{1}{2}$.
$66\frac{2}{3}$	"	"	$66\frac{2}{3}$ %	"	$0.66\frac{2}{3}$	=	$\frac{2}{3}$.

NOTE. When the per cent is expressed by a decimal of more than two places, the figures after the second decimal place are parts of 1 per cent. Thus, 0.125 is $12\frac{5}{10}$ or $12\frac{1}{2}$ per cent.

290. Exercises.

6. Write the decimal for 4 per cent. Ans. 0.04.

7. Write the decimal for 8 %; 12 %; $16\frac{1}{2}$ %; 25 %; 72 %.

8. Write the common fraction for $16\frac{2}{3}$ %; 20 %; $33\frac{1}{3}$ %; 75 %; $66\frac{2}{3}$ %. 1st Ans. $\frac{1}{6}$.

9. Write the decimal for $\frac{1}{5}$ %; for $\frac{1}{8}$ %; for $\frac{1}{10}$ %; for $\frac{1}{2}$ %; for $\frac{3}{8}$ %? 1st Ans. 0.002.

The Base and the Rate given to find the Percentage.

291. Oral Exercises.

10. What is 5 % of \$4?

Solution. 5 % of \$4 is $\frac{5}{100}$ of \$4, or $\frac{20}{100}$ of \$1 = \$0.20, Ans. Or, 5 % of \$1, that is, $\frac{5}{100}$ of \$1, is 5 cents; and therefore 5 % of \$4 is 4 times 5 cents, or 20 cents, Ans.

11. What is 5 % of \$5? of \$8? of \$12?

12. What is 6 % of \$8? of \$10? of \$30?

13. What is 10 % of \$100? 20 %? 50 %?

14. A farmer having 48 sheep lost 25 % of them; how many did he lose?

Solution. $25\ \% = 0.25 = \frac{1}{4}$; $\frac{1}{4}$ of $48 = 12$, Ans.

15. What is 6 % of \$250? 7 %? 8 %?

16. What is $8\frac{1}{3}$ % of 600?

17. What is $16\frac{2}{3}$ % of 120?

18. What is $12\frac{1}{2}$ % of \$500?

19. What is $8\frac{1}{3}$ % of 600 bush. of wheat?

20. What is $16\frac{2}{3}$ % of 1200 lb. of cheese?

21. A farmer cultivates 25 acres of corn this year, and intends to cultivate 20 % more next year; how many acres does he intend to cultivate next year?

22. In an orchard of 900 trees, $33\frac{1}{3}$ % are peach-trees; how many peach-trees are there in the orchard?

23. A teacher pronounced 56 words for his pupils to spell, but $14\frac{2}{7}$ % were misspelled; how many words were misspelled?

24. Only $66\frac{2}{3}$ % of a class of 27 pupils solved a problem given them for a lesson; how many of the class failed?

25. One acre of corn yields 80 bushels, and another acre 20 % more; how many bushels does the second acre yield?

292. Written Exercises.

26. B had $ 3776 and spent $12\frac{1}{2}$ % of it; how many dollars did he spend?

$$\begin{array}{r} \$\,3776 \\ 0.12\frac{1}{2} \\ \hline 45312 \\ 1888 \\ \hline \$\,472.00, \text{ Ans.} \end{array}$$

Or,

$\$\,3776 \times \frac{1}{8} = \$\,472$, Ans.

As $12\frac{1}{2}$ % is $0.12\frac{1}{2} = \frac{1}{8}$, the amount spent is found by multiplying $ 3776 by $0.12\frac{1}{2}$, or by taking $\frac{1}{8}$ of it. Hence,

293. The Base and the Rate given to find the Percentage,

Rule.

Multiply the base by the rate, written decimally; or, find such part of the base as the rate is of unity.

If p represents the percentage, b the base, r the rate (in hundredths), then

$$p = b \times r.$$

NOTE 1. Rate always means hundredths.

27. What is 43 % of $ 917.84?

28. What is 18 % of $ 756.13?

29. Find 17 % of $ 973.64.

30. The population of a certain city is 18775; what will it be in one year from this time if it gains 8 %?

NOTE 2. If it *gains* 8 %, it will be 108 % of itself.

31. The population of a certain State is 1376875; what will it be in one year if it loses 12 %?

32. A and B commenced business, each with $ 3456. A gained 25 % and B lost 12 %; how much was A then worth more than B?

33. A speculator paid \$56895 for a lot of flour, and lost 9 %; for what sum did he sell the flour?

34. If a man owes \$3564, and pays 30 %, how many dollars does he pay?

35. If 5 % is deducted from a bill of \$755.44, how much will pay the bill?

36. If a man who owes me \$876.65 fails and pays 65 %, how much shall I lose? How much shall I receive of the debt?

37. How much paper currency could have been bought for \$300 gold, when gold was at 125 %?

Solution. \$1 gold will buy \$1.25 currency, and \$300 will buy \$1.25 $\times$ 300 = \$375, Ans.

38. How much paper currency could have been bought for \$638 gold, gold being at 114 %?

39. When gold was worth $112\frac{1}{2}$, how much currency did it take to buy \$2500 in gold?

The Base and the Percentage given to find the Rate.

294. Oral Exercises.

40. What per cent of \$8 is \$2?

Solution. \$2 is $\frac{2}{8}$, or $\frac{1}{4}$, of \$8; $\frac{1}{4} = 0.25$, or 25 %, Ans. Or, $2 \div 8 = \frac{1}{4} = 0.25 = 25$ %, Ans.

The question is the same as, "\$2 is what part of \$8?", and the answer expressed decimally is the rate written decimally.

\$8 is the Base, \$2 is the Percentage, 25 % is the Rate.

41. What per cent of \$64 is \$16?

42. What per cent of \$100 is \$14?

43. What per cent of \$0.25 is \$$0.06\frac{1}{4}$?

44. What per cent of \$100 is \$5? of \$1000?

45. What per cent of \$340 is \$34?

46. B inherited \$3500, and in 6 months spent \$875; what per cent of his inheritance did he spend? What per cent had he remaining?

47. Out of a cask of syrup containing 96 gallons, 32 gallons were drawn; what per cent of the whole remained in the cask?

48. A merchant having \$ 1000, deposited \$ 650 in a bank; what per cent of his money did he deposit?

49. A teacher having a salary of \$ 2400, spends \$ 2000 annually; what per cent of his salary does he save?

295. Written Exercises.

50. What per cent of \$ 745 is \$ 149?

$$\$\,149 \div \$\,745 = \frac{149}{745} = \frac{1}{5} = 0.20, \text{ or } 20\,\%, \text{ Ans.}\quad \text{Hence,}$$

296. The Base and the Percentage given to find the Rate,

Rule.

Divide the percentage by the base.

As (Art. 293) $p = b \times r$,

then $r = \frac{p}{b}$.

51. What per cent of \$ 168 is \$ 18?

52. What per cent of \$ 300 is \$ 19?

53. What per cent of \$ 350 is \$ 43.75?

54. What per cent of \$ 43.75 is \$ 350?

55. If of 550 words a boy misspells 44, what per cent does he misspell?

56. If of 500 scholars an average of 15 are absent each day, what is the per cent of attendance?

The Percentage and the Rate given to find the Base.

297. Oral Exercises.

57. 12 is 3 % of what number?

Solution. If 12 is 3 %, that is, $\frac{3}{100}$ of some number, $\frac{1}{100}$ is $\frac{1}{3}$ of 12, or 4; and $\frac{100}{100}$ is 100 times 4, or 400; that is, 12 is 3 % of 400, Ans.

58. $16 is 25 % of what sum of money?

Solution. $16 is 25 %, that is, 0.25, or $\frac{1}{4}$, of 4 times $16, or $64, Ans.

59. What number increased by 10 % of itself gives 440?

NOTE. A number increased by 10 % of itself gives 110 % of itself. The question then is, 440 is 110 %, or $\frac{110}{100}$, of what number.

60. A farmer bought a farm for $2500, which was 25 % of his property; what was his property?

61. A man sold 56 geese, which was 28 % of his flock; how many geese had he?

62. A merchant, having a quantity of flour, bought 600 barrels more, when he found that the quantity bought was 75 % of all he then had; how many barrels had he before he bought the last lot?

63. A teacher saves $400 annually, which is $16\frac{2}{3}$ % of his salary; what is his salary?

298. Written Exercises.

64. $111.90 is 15 % of what sum of money?

$$\begin{array}{r} 0.15)\ \$111.90\ (\$746,\ \text{Ans.} \\ \underline{105\phantom{.90\ (\$746,\ \text{Ans.}}} \\ 69\phantom{0\ (\$746,\ \text{Ans.}} \\ \underline{60\phantom{0\ (\$746,\ \text{Ans.}}} \\ 90\phantom{\ (\$746,\ \text{Ans.}} \\ \underline{90\phantom{\ (\$746,\ \text{Ans.}}} \\ 0\phantom{\ (\$746,\ \text{Ans.}} \end{array}$$

If $111.90 is 15%, 100% is ($111.90 ÷ 15) × 100 = (Art. 79 d) $111.90 ÷ 0.15 = $746. Or, since, by Art. 298, $111.90 is the *product* obtained by multiplying the Base by the Rate, therefore $111.90 ÷ 0.15, or $746, is the Base. Hence,

299. The Percentage and the Rate given to find the Base,

Rule.

Divide the percentage by the rate.

As (Art. 298) $p = b \times r$,

then $b = \frac{p}{r}$.

65. $9 is 4 % of what sum?

66. $37.50 is 3 % of what sum?

67. $12 is 7 % of what sum?

68. $8 is 16 % of what sum?

69. $37\frac{1}{2}$ is 6 % of what number?

70. 33 is $1\frac{3}{8}$ % of what number?

71. The population of a town was 769 more in 1890 than in 1880, and this was an increase of 20 % on the population of 1880; what was the population of 1880?

72. What number diminished 16 % of itself gives 420?

NOTE. A number diminished 16 % of itself gives 84 % of the number. The question then is, 420 is 84 % of what number?

73. A farmer bought 500 bushels of corn for $475, paying $16\frac{2}{3}$ % less than he paid the preceding year. What did he pay a bushel the preceding year?

74. The number of pupils belonging to a certain school is 486, which is 8 % more than belonged a year ago; how many belonged to the school a year ago?

75. When the premium on gold was 25 %, how much gold would $2.50 currency purchase? How much would $1 currency purchase?

Solution. When the gold dollar was worth $1.25 currency, $2.50 currency would purchase as many gold dollars as $1.25 is contained times in $2.50; that is, it would purchase $2, Ans. $1 currency would purchase as many gold dollars as $1.25 is contained times in $1; that is, it would purchase $0.80, Ans.

This second question is equivalent to, What was the gold value of $1 currency, when gold was 25 % premium?

76. What was the gold value of $1 currency, when gold was worth 175? when it was worth 250?

PROFIT AND LOSS.

300. **Profit and Loss,** as a commercial term, signifies the gain or loss in business transactions.

301. *The cost is the base.*
The profit, or loss, is the percentage.

302. Oral Exercises.

77. If I buy a knife for \$0.50 and sell it at a loss of 10 %, how much do I lose, and how much do I get for it?

78. If I sell a horse for which I paid \$150 so as to gain 6 %, how much shall I get for him?

79. If I pay \$0.20 a pound for fish, at what price must I sell to gain 15 %?

80. If I buy boots at \$4 a pair and sell for \$5, what per cent shall I gain?

(What per cent is \$1 of \$4?)

81. If I buy boots at \$5 a pair and sell at \$4, what per cent do I lose?

82. Sold wheat at \$1.50 a bushel, gaining 25 %; what did it cost?

(\$1.50 is $\frac{125}{100}$, or $\frac{5}{4}$, of what?)

83. If I lose 10 % by selling apples at \$1.80 a barrel, what did I pay?

84. If I lose 20 % by selling 25 pounds of fish for \$4, what was the cost a pound?

85. If I sell 10 shares of railroad stock for \$1090 and gain 9 % on the cost, what was the cost?

86. A custom-made coat, failing to suit the customer for whom it was made, was sold at cost \$27, which was 10 % below the custom price; what was the custom price?

(\$27 is 0.90, or $\frac{9}{10}$, of what?)

87. Bought a piece of broadcloth at $5 a yard, but, it being afterwards damaged, I sold it 25 % below the marked price, and thus lost 20 % on the cost. What was the marked price?

88. Paid $4 a pair for a case of boots; on one pair that was damaged I fell 10 % from the marked price and yet made $12\frac{1}{2}$ % on the cost. What was the marked price?

89. Paid $8 each for a case of bonnets; to close out the case, I sold the last 3 bonnets 16 % below the marked price and yet made 5 % on the cost. What was the marked price?

90. What per cent do I gain if I buy boots at $3 a pair and sell them at $\$3.37\frac{1}{2}$?

91. Paid $3 a yard for a piece of lace; but, the lace getting injured, I fell 10 % from the marked price and yet gained 20 % on the cost. What was the marked price?

92. Bought hats at $3 apiece and sold them at $2.50; what per cent on the cost was lost?

93. Sold a watch for $42 and lost $12\frac{1}{2}$ % on the cost; what was the cost?

303. Written Exercises.

94. If I pay $87.50 for a horse and sell it at an advance of 12 %, what shall I receive for it?

95. If sugar that cost $7.75 a barrel is sold for $8.50 a barrel, what is the gain per cent?

96. If Mr. James buys a farm for $2576 and sells it for $2485.84, what per cent does he lose?

97. If 18 % is lost on a horse for which $205 was received, what did the horse cost?

98. Bought 164 yards of broadcloth and 287 yards of cassimere for $1107; sold the broadcloth at $3 and the cassimere at $2.25 a yard; did I gain or lose? How much per cent?

99. Bought 50 pounds of wool for $20 and sold it at 34 cents a pound; did I gain or lose? How much per cent?

100. Bought goods for \$2000 and sold them for \$2155, out of which paid \$95 for storage, etc.; how much per cent on the cost was gained?

101. Bought a farm for \$4848; for what shall I sell it to gain 5 %?

102. Bought a house for \$3500, expended \$750 in repairing it, and then sold it so as to lose 15 % on the whole cost; what did I receive for it?

103. Sold 6 yards of cloth for \$26.88 and gained 12 % on the cost; what was the purchase price a yard?

104. Sold flour at \$7 a barrel and thereby gained 12 %; what per cent should I have gained if I had sold it at \$7.25?

(First find the cost.)

105. Sold beef at \$0.06 a pound and thereby lost 4 %; should I have gained or lost, and how much per cent, had I sold it at \$0.06½?

106. Sold a watch for \$21 and gained 5 % on the cost; had I sold it for \$18, should I have gained or lost, and how much per cent?

107. Bought a case of watches at \$23.50 each; finding one of the watches damaged, I abated 6 % from the selling price, and yet sold it at cost. What was my selling price?

108. Bought 75 pounds of tea for \$37.50, and sold ½ of it at 48 cents a pound and the remainder at 56 cents a pound; did I gain or lose? How much per cent?

109. Sold sugar at \$7.50 a barrel and lost 6¼ % on the cost; for what should it be sold to gain 12½ %?

110. Sold cloth at \$2 a yard and lost 10 %; should I have gained or lost, and how much per cent, if I had sold it at \$2.12½?

111. If I buy tea at the rate of 7 pounds for \$5 and sell it at the rate of 5 pounds for \$7, what per cent do I gain?

112. If a merchant buys a hogshead of molasses containing 75 gallons for $0.50 a gallon and loses by leakage 15 gallons, what per cent on the cost per gallon must he gain in order to get back the first cost of the molasses?

113. If I sell a pair of horses for $175 and gain 5%, what would be my gain per cent if I had sold them for $200?

114. If a merchant sells sugar at $8.50 a barrel and gains 10%, what per cent would he gain if he sold the sugar at $9.50 a barrel?

115. If I sold my farm for $5000 and made 25%, what per cent should I have gained, or lost, if I had sold it for $3500?

116. If a merchant sells goods at retail 15% above cost, and at wholesale for 8% less than the retail price, what per cent does he gain on the goods sold at wholesale?

117. If Mr. Fox buys one fifth of an acre of land for $2178, for how much a square foot must he sell it to gain 20%?

118. If I sell from an acre of land a rectangular lot 363 feet long and 75 feet wide for what the whole acre cost me, what per cent do I gain on the part sold?

INSURANCE.

304. **Insurance** is a contract of indemnity against loss.

305. The **Premium** is the sum paid for the insurance, and is usually computed at a certain per cent on the sum insured.

306. The **Policy** is the writing or record of the contract given by the insurer to the insured.

307. *The sum insured is the base.*
The premium is the percentage.

308. Written Exercises.

119. What is the cost of insuring \$ 2500 on my house for 5 years at 2 %, including \$ 1 to pay for the policy?

$$
\begin{array}{rl}
\$\,2500 \times 0.02 = \$\,50, & \text{Premium.} \\
\underline{1,} & \text{Policy.} \\
\$\,51, & \text{Ans.}
\end{array}
$$

120. What is the annual premium for insuring a manufacturing establishment in the sum of \$ 75000 at 3 %? Ans. \$ 2250.

121. In a certain house the furniture, worth \$ 2400, is insured for $\frac{2}{3}$ its value at $1\frac{3}{4}$ %; what is the premium?

122. The Middlesex Mutual Fire Insurance Company have insured \$ 4000 on my house for a period of 5 years, at $1\frac{3}{8}$ %; what is the cost, including \$ 1 to pay for the policy?

123. I buy a house for \$ 8000, and get it insured for $\frac{3}{4}$ of its value at $\frac{2}{3}$ %. If the house is burned, what is my loss? What is the loss of the insurers?

124. What is the premium, at $5\frac{1}{2}$ %, for insuring \$ 75000 on a steamboat and cargo from Boston to Havre?

125. A cotton factory worth \$ 25000, and the machinery and stock worth \$ 35000, are insured for $\frac{1}{2}$ their value at 3 %; what is the premium?

126. A man 25 years of age has his life insured for \$ 6000 at \$ 19.85 on \$ 1000 annually; what annual premium does he pay?

127. A man 30 years of age took out a life policy for \$ 8500 at \$ 22.70 on \$ 1000 annually: he died at the age of 60; how much greater was the amount insured than the sum of the annual premiums?

128. If a man 35 years of age takes out a life policy for \$ 5000 at \$ 26.40 on \$ 1000, and dies at the age of 90, how much greater is the sum of the annual premiums than the amount insured?

COMMISSION AND BROKERAGE.

309. **Commission** or **Brokerage** is the compensation received by an agent for transacting certain kinds of business. It is usually a percentage of the money involved.

310. The agent is variously styled as factor, broker, collector, correspondent, commission merchant, etc.

311. *The sum on which commission is paid is the base.*
The commission is the percentage.

312. Oral Exercises.

129. If I sell $300 worth of goods on 3 % commission, how much shall I receive?

130. If my agent sells $800 worth of goods on 5 % commission, what must I pay him?

131. A commission merchant sells farm produce to the amount of $1892; what is his commission at 2 %?

132. The taxes in the town of B for 1896 are $15000; what is the cost of collecting them at $\frac{1}{2}$ %? Ans. $75.

133. My agent has lent for me $2124. His commission is $\frac{1}{4}$ %; what shall I pay him?

134. Sent my agent $105, to expend in goods after deducting his commission of 5% on the sum expended. What sum does he expend for me, and what is his commission?

NOTE. In this case the $105 includes the commission and the sum expended; that is, $105 is 105% of the sum expended.

135. If I pay $10 for collecting a bill of $200, what per cent do I pay?

313. Written Exercises.

136. What shall I pay my agent for selling $4786 worth of goods, his commission being 4 %?

137. My correspondent in Paris has bought for me 6 bales of French calico, each bale containing 50 pieces of 30 meters each, at 25 cents a meter; what is his commission at $\frac{3}{8}$ %?

138. My agent in New Orleans has sold for me 400 pairs of boots at \$ 1.50 a pair, 400 pairs of shoes at 75 cents, and 500 pairs at \$ 1; what is his commission at 3 %, and what ought he to remit to me?

139. Sent my agent in London \$ 5100, out of which he is to take a commission, and invest the balance in goods. What sum will he invest, his commission being 2 % on the purchase, and what is his commission?

140. I intrust \$ 10000 to my factor in New Orleans for the purchase of cotton. What sum does he invest after deducting ½ % commission for the purchase, and what are his fees?

314. Commercial Discount is a deduction from the nominal price of goods.

141. A merchant bought a lot of goods amounting to \$ 987. By paying cash down he was allowed a discount on the lot of 10 %. What did the goods cost him?

142. Mr. Fox bought a bill of books amounting to \$ 475.58, on which he was allowed a discount of 12½ %. For cash he was allowed a farther discount of 5 %. What was the net cost of the books to Mr. Fox?

143. Find the net amount of a bill of \$ 753, if a discount of 15 %, and afterwards of 10 % is made.

144. What is the discount on a bill of goods, if 20 %, 15 %, and 5 % are successively made? Ans. 35.4 %.

145. What is the net amount of a bill of \$ 735, if discounts of 15 %, 10 %, and 5 % are successively made?

315. The profits from the business of companies, distributed from time to time among the stockholders, are called *Dividends.* The sums of money occasionally required of the stockholders to meet the losses or expenses of the company are called *Assessments.* Assessments, dividends, discounts, premiums, and brokerage are percentages on the par value of the stock as a base.

The *par value* of stock is its nominal value a share, usually $100; if it sells for *more*, it is *at a premium*, or *above par*; if it sells for *less*, it is *at a discount*, or *below par*. Thus, in the quotation below, shares of the Maine Central Railroad are 11¼ % *above par*, the *nominal value* being $100, while the Baltimore and Ohio Railroad is 28 % *below par*.

NOTE. In selling and buying stocks the brokerage is usually ⅛ % on the par value. In this work $100 is considered the par unless stated otherwise.

NEW YORK AND BOSTON STOCK SALES.

165	Chicago and Northwestern Railroad	104½
10	Lake Erie and Western R. R., preferred	67½
10	Maine Central Railroad	111¼
10	Shawmut National Bank	118¼
25	American Express Co.	114
50	Central Massachusetts Railroad	11
4	New York Central Railroad	101
10	New York & New England R. R., preferred	64
7	Baltimore and Ohio Railroad	72
2	Pullman Palace Car Co.	173

At the quotations above,

146. What would 17 Lake Erie and Western preferred cost?

147. What is the cost of 12 Chicago and Northwestern Railroad?

148. What is the premium on 35 Maine Central Railroad?

149. What is the cost of 18 New York Central?

150. What is the cost of 29 New York and New England, preferred?

151. What is the discount on 17 shares Central Massachusetts?

152. What must I pay a broker for buying for me 25 shares American Express Co., including his brokerage?

153. If a broker sells for me 17 Maine Central Railroad, and buys 17 New York Central, what balance, after allowing his brokerage on both the sale and the purchase, ought he to give me credit for in these transactions?

154. A broker sold for a merchant 19 shares Shawmut National Bank, and 27 shares American Express Co., and with the proceeds, after deducting his commission on the sale, bought as many shares as possible of New York Central Railroad. Allowing the broker commission also on the purchase, how many shares New York Central Railroad did he buy, and what balance of cash did he have left to the merchant's credit?

155. The directors of a manufacturing company, wishing to enlarge their works, call for an assessment of 5 % on the capital stock of the company; what will be the assessment on $15000 worth of the stock?

156. A certain bank declared a dividend of $4\frac{1}{2}$ %. What was paid on 37 shares?

157. The Atlantic National Bank paid a dividend of 3 % Apr. 1, 1895. How much did a man who owns 57 shares of the stock receive?

158. How many shares of the Pullman Palace Car Co. (see quotation above) can be bought for $2941?

159. How many shares of Baltimore and Ohio Railroad can be bought for $3757 including the broker's commission?

160. If a man exchanges 13 shares of Shawmut National Bank for 13 of Maine Central Railroad, what balance of cash ought he to receive?

161. If the exchange in Ex. 160 is effected through a broker who takes brokerage on the shares given up and on those received, what balance of cash will be left?

TAXES.

316. A **Tax** is a sum of money assessed upon the person, property, or income of individuals for public purposes.

317. A tax on property is called a *property tax.* It is assessed at a certain per cent on the estimated value of the property. A tax on income is called an *income tax.* A tax on the person is called a *poll tax.*

318. **Real Estate** is *immovable* property; as lands, houses, mills, etc.

319. **Personal Estate** is *movable* property; as money, notes, cattle, tools, bank stocks, etc.

320. An **Inventory** is a list of articles of property, with their estimated value.

321. **Assessors** are persons appointed to estimate the value of property and assign the amount to be paid by each tax-payer.

322. Written Exercises.

162. The town of A is to be taxed \$ 10999. The real estate of the town is valued at \$ 500000 and the personal at \$ 300000. There are 666 taxable polls, each assessed at \$ 1.50. What is the tax of B, whose real estate is valued at \$ 4000 and his personal property at \$ 8000, and who pays one poll tax?

$\$1.50 \times 666 = \$999$, sum assessed on the polls.
$\$10999 - \$999 = \$10000$, sum to be assessed on the property.
$\$500000 + \$300000 = \$800000$, amount of taxable property.
$\$10000 \div 800000 = 12\frac{1}{2}$ mills, tax on \$ 1, or the rate.
$\$4000 + \$8000 = \$12000$, B's taxable property.
$\$12000 \times 0.012\frac{1}{2} = \$150$, tax on B's property.
$\$150 + \$1.50 = \$151.50$, B's entire tax, Ans. Hence,

323. To find the *rate* of taxation,

From the entire tax to be raised subtract the amount of the poll taxes, and divide the remainder by the value of the taxable property.

To find each individual's tax,

Multiply the value of each individual's taxable property by the rate, and to the product add his poll tax.

163. A certain town proposes to raise \$ 20228 by taxation. The valuation of both real estate and personal property is \$ 975000, and there are 416 polls, each assessed at \$ 1.75. What is the tax of Mr. A., whose property is valued at \$ 2578, and who pays one poll tax?

164. If \$ 6256.32 taxes must be raised, and 2 % is paid for collecting, and 5 % of the amount assessed cannot be collected, and 4 % is deducted from the tax bill of each person who pays the tax before a given date, what sum must be assessed, on the supposition that all who pay, pay before the given date?

0.98) \$ 6256.32
0.96) \$ 6384, tax collected.
0.95) \$ 6650, tax assessed and collectible.
\$ 7000, whole tax assessed.

324. When the rate of taxation has been determined, the labor of preparing a tax list is lessened by using a table giving the tax on \$ 1 to \$ 9 inclusive.

Table for a Rate of \$ 0.015, or \$ 15 on \$ 1000.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
\$ 1...........	\$ 0.015	\$ 4...........	\$ 0.06	\$ 7...........	\$ 0.105
2...........	0.03	5...........	0.075	8...........	0.12
3...........	0.045	6...........	0.09	9...........	0.135

165. Using this table, find the tax on $973.60.

Tax on	$900	......$13.50
" "	70	...... 1.05
" "	3	...... 0.045
" "	0.60	...... 0.009
" "	$973.60	......$14.604 Ans.

As the tax on $9 is by the table $0.135, on $900 it is $0.135 × 100 = $13.50. In like manner we find the tax on the other parts of the given amount by moving the decimal point of the tax in the table to the right or left as the given part requires.

Using this table, find the tax of

166. Mr. A on $809.

167. Mr. B on $1245.

168. Mr. C on $2500.

169. Mr. D on $8755.

170. Mr. E on $9753.

171. Mr. F on $10108.

172. Mr. G on $15875.

173. Mr. H on $55684.

325. Miscellaneous Examples.

174. The population of Cambridge, Mass., in 1880, was 52669, and in 1890 it was 70028. If it increases at the same rate, what will it be in 1900? In 1910? In 1920?

175. What is the per cent of deduction if 25 % is first deducted, then 10 %, then 5 %?

176. If a school with 365 pupils has 10 absent Monday, 8 Tuesday, 7 Wednesday, 6 Thursday, and 4 Friday, what is its average per cent of attendance for the week?

177. If a man gains 15 % in a year on his stock in trade, and has $5685 worth at the end of the year, what was it worth at the beginning of the year?

178. If I buy 10 shares of stock at 20 % below par, and get $40 dividends a year, what per cent do I get on my investment?

179. Which pays the greater per cent on the investment, stock at 118 paying $5 a share a year dividends, or stock at 83 paying $4 a share a year?

INTEREST.

326. **Interest** is money paid for the use of money.

327 The **Principal** is the sum for whose use interest is paid.

328. The **Amount** is the *sum* of the *principal* and *interest.*

329. The **Rate** is the number of hundredths of the principal paid for its use for a year, or other specified time.

NOTE. When no time is specified a year is meant.

330. An example in interest is only a question in *percentage.*

The principal is the base of percentage (Art. 288).

The interest is the percentage (Art. 287).

The interest on $1 for the specified time is the *rate* (Art. 289).

331. The rate is usually fixed by law, and then is called the *legal* rate.

NOTE 1. When no rate is mentioned 6 % is understood.

NOTE 2. For the legal rates in the various States, see Appendix, Page 349.

332. Oral Exercises.

1. What is the interest of $ 50 at 6 % for 1 year? for 2 years and 6 months?

Solution. At 6 % the interest of $1 for 1 year is 6 cents; the interest of $50 is 50 times 6 cents = 300 cents, or $3, 1st Ans. For 1 year at 6 % the interest of $50 is $3; for 2 years and 6 months, or $2\frac{1}{2}$ years, it must be $2\frac{1}{2}$ times $3, or $7.50, 2d Ans.

2. What is the interest of $50 for 1 year at 7 %? at 8 %? at 10 %?

3. What is the interest of \$75 for 2 years at 10 %? at 5 %?

4. What is the interest of \$500 for 1 year at 5 %? at 6 %? at 7 %?

5. What is the interest of \$1000 for 1 year at 6 %? at 8 %? at 9 %? at 10 %?

6. What is the interest of \$1 at 6 % for 1 month? for 2 months? 3 months? 6 months? 9 months?

7. What is the interest of \$1 at 6 % for 6 days? for 1 day? for 12 days? for 18 days?

Solution. As the interest of \$1 for 1 year, or 12 months, is 6 cents, for 1 month the interest is $\frac{1}{12}$ of 6 cents, that is, is 5 mills; and the interest for 6 days is $\frac{6}{30}$, or $\frac{1}{5}$ of 5 mills, that is, is 1 mill; and therefore for one day the interest of \$1 is $\frac{1}{6}$ of a mill; for 12 days, 2 mills; and for 18 days, 3 mills.

8. What is the amount of \$100 for 1 year at 6 %? at 8 %? at 10 %?

9. What is the amount of \$200 for 2 years and 6 months at 6 %? at 10 %?

10. What is the amount of \$500 for 2 years at 8 %?

11. What is the amount of \$1000 for 3 years, 6 months at 5 %?

333. Written Exercises.

12. What is the interest of \$825 for 3 years and 4 months at 8 %?

$$\begin{array}{r} \$825 \\ \underline{0.08} \\ \$66.00 \\ \underline{3\frac{1}{3}} \\ 198 \\ \underline{22} \\ \$220, \text{ Ans.} \end{array}$$

The interest of \$1 for 1 year at 8 % is \$0.08; and the interest of \$825 for 1 year at 8 % is 825 times \$0.08 = \$825 × 0.08, or \$66. For 3 years and 4 months, or $3\frac{1}{3}$ years, it must be $3\frac{1}{3}$ times \$66, or \$220, Ans. Hence,

334. To find the interest on any sum for any given time,

Rule.

Multiply the principal by the rate, and this product by the time expressed in years.

To find the amount,

Add the interest to the principal.

335. If i represents the interest, p the principal, t the time in years, and r the rate (in hundredths), then

$$i = p \times r \times t.$$

If a represents the amount, then

$$a = p + p \times r \times t = p\,(1 + r \times t).$$

336. In computing interest it is the custom to reckon 30 days a month, and 12 months, or 360 days, a year.

13. What is the interest of \$ 7845 for 2 yr. 8 m. at 8 %?
14. What is the interest of \$ 1617.43 for 4 yr. 2 m. at 7 %?
15. What is the interest of \$ 847.53 for 5 yr. 3 m. at 6 %?
16. What is the amount of \$ 63.54 for 3 yr. 2 m. at 6 %?

To find the interest on any sum of money, at 6 %, for any given time.

337. At 6 %, any sum of money in 2 months, or $\frac{1}{6}$ of a year, will gain 1 %, or 0.01 of itself; and in $\frac{1}{10}$ of 2 months, that is, $\frac{1}{10}$ of 60 days, or 6 days, it will gain $\frac{1}{10}$ of 1 %, or 0.001 of itself.

17. Find the interest of \$ 456 for 3 months and 6 days.

Interest for 2 months..................	\$ 4.56
" " 1 "	2.28
" " 6 days	0.456
" " 3 m. 6 d.	\$ 7.296, Ans.

For 2 months we take $\frac{1}{100}$ of the principal, or $ 4.56 ; for 1 month $\frac{1}{2}$ of $ 4.56 ; and for 6 days $\frac{1}{1000}$ of the principal. The sum of these, or $ 7.296, is the interest of $ 456 for 3 months and 6 days.

18. What is the interest of $ 324 for 3 months and 3 days?

Interest of $ 324	for	2 months	....	$ 3.24
" "	"	1 "	....	1.62
" "	"	3 days	....	0.162
" "	"	3 m. 3 d.	....	$ 5.022, Ans.

19. What is the interest of $ 720 for 7 months and 3 days?

Interest of $ 720	for	2 months	....	$ 7.20
" "	"	6 "	....	$ 21.60
" "	"	1 "	....	3.60
" "	"	3 days	....	0.36
" "	"	7 m. 3 d.	....	$ 25.56, Ans.

20. What is the interest of $ 1260 for 2 months and 15 days?

Interest of $ 1260	for	2 months	....	$ 12.60
" "	"	15 days ($\frac{1}{4}$ of 2 m.)	...	3.15
" "	"	2 m. 15 d.	....	$ 15.75, Ans.

21. What is the interest of $ 348 for 22 days?

Interest of $ 348	for	60 days	....	$ 3.48
" "	"	20 "	....	$ 1.16
" "	"	2 "	....	0.116
" "	"	22 days	....	$ 1.276, Ans.

22. What is the interest of $ 1848.50 for 1 year, 5 months and 19 days?

Int. of $ 1848.50	for 2 m., $ 18.485 ;	for 6 d., $ 1.8485.		
"	"	" 16 months	....	$ 147.88
"	"	" 1 "	....	9.2425
"	"	" 18 days (3 × 6 d.)	......	5.5455
"	"	" 1 "	....	0.3081
"	"	" 1 y. 5 m. 19 d.	......	$ 162.976, Ans.

338. Hence, to compute interest at 6 % for any given time,

Rule.

Move the decimal point in the principal two places to the left, and the result will be the interest for two months, or sixty days. Move the point three places to the left, and the result will be the interest for six days. Then take such multiples and parts of these results as the given time requires; the sum of these will be the interest.

To find the amount,

Add the interest to the principal.

23. What is the interest of $ 412 for 5 m.? Ans. $ 10.30.

24. What is the interest of $ 42 for 2 m. 22 d.? Ans. $ 0.574.

25. What is the interest of $ 54 for 22 d.? Ans. $ 0.198.

26. What is the interest of $ 2148 for 3 m. 10 d.?

27. What is the interest of $ 75 for 10 m. 6 d.?

28. What is the interest of $ 173 for 1 m. 8 d.?

29. What is the interest of $ 16.50 for 5 m. 9 d.?

30. What is the interest of $ 300 for 3 m. 24 d.?

31. What is the interest of $ 700 for 4 m. 12 d.?

32. What is the interest of $ 400 for 5 m. 15 d.?

33. What is the interest of $ 350 for 2 m. 24 d.?

34. What is the amount of $ 356 for 3 m. 17 d.?

Principal		....	$ 356
Interest for	2 months	....	3.56
" "	1 "	....	1.78
" "	15 days	....	0.89
" "	2 "	....	0.119
Amount "	3 m. 17 d.	....	$ 362.349, Ans.

NOTE 1. The interest for the two days is \$ $0.118\frac{2}{3}$ and we write $ 0.119, as the fraction is *more than half* a mill. In paying the amount in this example $ 362.35 would be paid.

In business transactions mills in the *result* are omitted; but if they are more than 5, the cents are increased by 1.

35. What is the amount of $ 48.50 for 2 m. 21 d.?

36. What is the interest of $ 248 for 3 m. 18 d.?

37. What is the interest of $ 965.188 for 3 m. 11 d.?

38. What is the interest of $ 225.87 for 3 m. 15 d.?

39. What is the amount of $ 35.40 for 6 m. 9 d.?

40. What is the interest of $ 450.87 for 3 m. 9 d.?

41. What is the interest of $ 375.50 for 1 m. 8 d.?

42. What is the interest of $ 225.75 for 5 m. 12 d.?

43. What is the interest of $ 84.82 for 4 m. 18 d.?

44. What is the interest of $ 125.16 for 11 m. 25 d.?

45. What is the interest of $ 658.25 for 2 m. 13 d.?

46. What is the amount of $ 325.75 for 4 m. 24 d.?

47. What is the amount of $ 224.48 for 6 m. 15 d.?

48. What is the amount of $ 48.33 for 1 yr. 6 m.?

49. What is the amount of $ 365.25 for 1 yr. 3 m. 9 d.?

50. What is the interest of $845 for 1 yr. 10 m. 6 d.?

Interest for	1 yr. 8 m., or, 20 m.	...	$84.50
" "	$\frac{1}{10}$ of 20 m., or, 2 m.	...	8.45
" "	$\frac{1}{10}$ of 2 m., or, 6 d.	...	0.845
" "	1 yr. 10 m. 6 d	....	$93.795, Ans.

NOTE 2. As the interest for 2 months is $\frac{1}{100}$ of the principal, for 20 months it is $\frac{1}{10}$ of the principal; and for 200 months, or 16 years 8 months, it is exactly equal to the principal.

NOTE 3. In the following examples it is necessary to find the time. This can be done as in Art. 274.

51. What is the interest of $125 from June 7, 1892, to Feb. 11, 1895? Ans. $20.083.

52 Find the interest of $154.25 from April 18, 1893, to Jan. 25, 1895.

53. Find the interest of $172 from Aug. 7, 1894, to Sept. 8, 1896.

54. Find the amount of $254 from Nov. 13, 1893, to Jan. 30, 1895.

55. What is the interest of $132.25 from Nov. 13, 1890, to May 2, 1895?

56. What is the interest of $100 from March 26, 1894, to June 21, 1895?

57. What is the amount of $444 from July 18, 1894, to Sept. 4, 1895?

339. To find the interest of any sum for any time at any other rate than 6 %.

Interest at 1 % is $\frac{1}{6}$ of the interest at 6 %; at 2 %, $\frac{1}{3}$; at 3 %, $\frac{1}{2}$; at 4 %, 6 % − $\frac{1}{3}$ of 6 %; at 5 %, 6 % − $\frac{1}{6}$ of 6 %; at 7 %, 6 % + $\frac{1}{6}$ of 6 %; at 8 %, 6 % + $\frac{1}{3}$ of 6 %; at 9 %, 6 % + $\frac{1}{2}$ of 6 %; etc.

58. What is the interest of $ 124.50 for 4 m. 12 d. at 5 %?

Interest for 4 months at 6 %	......	$ 2.49	
" " 12 days " "	......	0.249	
" " 4 m. 12 d. " "	......	2.739	
" " " " " 1 %	......	0.4565	
" " " " " 5 %	......	$ 2.2825,	Ans.

Rule.

First find the interest at 6 %; then increase, or decrease, this interest by such part of itself as will give the interest at the required rate.

59. What is the amount of $ 476 for 3 months and 13 days at 9 %?

Interest for 2 months at 6 %	....	$ 4.76	
" " 1 " " "	....	2.38	
" " 12 days " "	....	0.952	
" " 1 " " "	....	0.0793	
" " 3 m. 13 d. " "	....	$ 8.1713	
" " " " " 3 %	....	4.0856	
Principal	....	476.	
Amount for 3 m. 13 d. at 9 %	....	$ 488.2569,	Ans.

60. What is the interest of $ 342.25 for 2 m. 18 d. at 8 %?

61. What is the interest of $ 256.84 for 3 m. 15 d. at 8 %?

62. What is the interest of $ 79.84 for 5 m. 8 d. at 10 %?

63. What is the amount of $ 343.17 for 3 m. 10 d. at 7 %?

64. What is the amount of $ 817.57 for 2 m. 18 d. at 4 %?

65. What is the amount of $ 32.25 from Nov. 15, 1893, to July 25, 1895, at 7½ %?

66. What is the interest of $ 67.43 from Nov. 25, 1894, to March 7, 1895, at 5 %?

340. Those who prefer can use the following method of casting interest.

When the rate is 6 %,

The interest of	$ 1	for	1 year	is	$ 0.06.
"	"	"	1 month (or $\frac{1}{12}$ of a year)	"	0.005.
"	"	"	6 days (or $\frac{1}{5}$ of a month)	"	0.001.
"	"	"	1 "	"	0.000$\frac{1}{6}$.

341. Hence, to find the interest of $ 1 at 6 % for any time,

Take 6 *cents for each year,* 1 *cent for each* 2 *months in the part of a year,* 5 *mills for the odd month, and* 1 *mill for each* 6 *days in the part of a month.*

342. Oral Exercises.

67. What is the interest of $ 1 for 3 yr. 9 m. 18 d.?

Solution. The interest of $ 1 for 3 years is 18 cents; for 9 months is 4 cents and 5 mills; for 3 yr. 9 m., therefore, it is 22 cents and 5 mills; for 18 days it is 3 mills; therefore, for 3 yr. 9 m. 18 d. it is 22 cents and 8 mills, or $ 0.228, Ans.

68. What is the interest of $ 1 for 2 yr. 5 m. 20 d.?

69. What is the interest of $ 1 for 3 yr. 1 m. 15 d.?

70. What is the interest of $ 1 for 1 yr. 3 m. 29 d.?

71. What is the interest of $ 1 for 4 yr. 2 m. 4 d.?

72. What is the interest of $ 1 for 4 yr. 3 m. 17 d.?

73. What is the interest of $ 1 for 4 yr. 9 m. 12 d.?

74. What is the interest of $ 1 for 10 yr. 11 m. 7 d.?

75. What is the interest of $ 1 for 2 yr. 11 m. 5 d.?

76. What is the interest of $ 1 for 1 yr. 8 m. 3 d.?

343. Written Exercises.

77. What is the interest of \$ 256 for 1 yr. 5 m. 12 d.? What is the amount?

$$\begin{array}{r l} \$\,256 & \\ 0.087 & \\ \hline 1792 & \\ 2048 & \\ \hline \$\,22.272, & \text{1st Ans.} \\ 256. & \\ \hline \$\,278.272, & \text{2d Ans.} \end{array}$$

The interest of \$ 1 for 1 yr. 5 m. 12 d. is \$ 0.087. Therefore the interest of \$ 256 will be 256 times \$ 0.087 = \$ 256 × 0.087 = \$ 22.272. The amount is \$ 22.272 + \$ 256 = \$ 278.272.

Or, to find the amount,

$$\begin{array}{r l} \$\,256 & \\ 1.087 & \\ \hline 1792 & \\ 2048 & \\ 256 & \\ \hline \$\,278.272, & \text{Ans.} \end{array}$$

The amount of \$ 1 for 1 yr. 5 m. 12 d. is \$ 1.087. Therefore the amount of \$ 256 will be 256 times \$ 1.087 = \$ 256 × 1.087 = \$ 278.272. Hence,

344. To find the interest of any sum at 6 % for any time,

Rule.

Multiply the principal by the decimal which expresses the interest of \$ 1 at 6 % for the given time.

345. To find the amount,

Add the principal to the interest. Or,

Multiply the principal by the decimal which expresses the amount of \$ 1 at 6 % for the given time.

For examples, let this method be applied to those already given, pages 180–185.

EXACT INTEREST.

346. In the methods given for finding the interest for months and days, 30 days have been counted a month, and 12 months, or 360 days, a year (Art. 336). By these methods the computed interest of 360 days, that is, of $\frac{360}{365}$, or $\frac{72}{73}$ of a year, is the interest of a *full* year. Thus, the computed interest for any number of days is $\frac{1}{73}$ of itself too great.

347. To find the *exact* interest for any number of days,

Rule.

Find the interest for the exact number of days as in the common method, and decrease it one seventy-third of itself. Or,

Multiply one year's interest of the principal at the given rate by the exact number of days, and divide the product by 365.

NOTE 1. In England and in dealings with the United States Government *exact* interest is computed.

78. What is the exact interest of $438 from Feb. 12, 1894, to May 8, 1894?

Feb.	12 to March	1	17	days
March	1 to April	1	31	"
April	1 to May	1	30	"
May	1 to May	8	 7	"
			85	days.

Int. of $438 for 60 days	....	$4.38
" " " 20 "	....	1.46
" " " 5 "	....	0.365
" " " 85 "	....	$6.205
$\frac{1}{73}$ of $6.205	....	0.085
		$6.12, Ans.

Or,

$$\begin{array}{r} \$\,438 \\ 0.06 \\ \hline \text{Int. of } \$\,438 \text{ for 1 year, } \$\,26.28 \end{array}$$

$$\frac{\overset{0.36}{\$\,\cancel{26.28}}}{1} \times \frac{\overset{17}{\cancel{85}}}{\underset{73}{\cancel{365}}} = \$\,6.12, \text{ Ans.}$$

79. What is the exact interest of \$ 256.15 from May 17 to July 31, 1895?

80. Find the amount at exact interest of \$57.38 at 7 % from Sept. 18 to Dec. 13, 1895.

81. What is the amount of \$187.44 from May 25, 1893, to April 19, 1895, at $7\frac{3}{10}$ %, exact interest?

NOTE 2. The exact interest at $7\frac{3}{10}$ % is 2 cents a day for every \$ 100.

PROBLEMS IN INTEREST.

348. In every example in interest there are four elements, viz. *Principal*, *Rate*, *Time*, and *Interest*, any three of which being given, the other can be found.

349. Principal, Interest, and Time given, to find the Rate.

82. At what rate must \$ 300 be put on interest to gain \$ 18 in 2 years?

Solution. \$ 300 at 1 % will gain \$ 6 in 2 years. Therefore, to gain \$ 18, the rate must be the quotient of \$ 18 ÷ \$ 6 = 3; **3 %, Ans.** Hence,

Rule.

Divide the given interest by the interest of the principal, for the given time, at 1 *per cent, and the quotient will be the rate*

Or, as (Art. 335), $i = p \times r \times t,$

therefore $r = \dfrac{i}{p \times t}.$

83. At what rate must \$ 142 be put on interest to gain \$ 21.30 in 3 years? Ans. 5 %.

84. If \$ 36 gain \$ 7.56 in 3 years, what is the rate?

85. If \$ 300 gain \$ 43.80 in 2 years, what is the rate?

350. Amount, Time, and Rate given, to find the Principal.

86. What principal, at 5 %, will amount to \$ 110 in 2 years?

Solution. \$ 1 in 2 years at 5 % amounts to \$ 1.10. Therefore, the principal must be the quotient of \$ 110 ÷ 1.10 = \$ 100, Ans. Hence,

Rule.

Divide the given amount by the amount of \$ 1 for the given rate and time, and the quotient will be the principal.

Or, as (Art. 335) $a = p\ (1 + r \times t)$,

therefore $$p = \frac{a}{1 + r \times t}.$$

87. What principal at 6 % will amount to \$ 130.39 in 8 months? Ans. \$ 125.375.

88. What principal at 8 % for 3 years will amount to \$ 74.40?

89. What is the interest for 2 yr. 6 m. at 8 % of that sum which will, at the given rate and time, amount to \$ 240?

351. The **Present Worth** of a debt, payable at a future time without interest, is a sum of money which, put at legal interest, will amount to the debt at the time of its becoming due.

352. The *debt*, then, is an *amount*, the *present worth* is the *principal*, that is, we have the amount, time, and rate given to find the principal. The interest on the principal is called the *discount*.

353. *The* **Discount** *can be found by subtracting the present worth from the face of the debt.*

NOTE. The discount on a sum of money obtained in this way is sometimes called *true discount.*

90. What is the present worth of $ 37.44 due in 8 months? What is the discount?

Amount of $ 1 for 8 m....$ 1.04)	$ 37.44 ($ 36, Present worth.	
	31 2	
$ 37.44, Given sum,	6 24	
36.00, Present worth,	6 24	
$ 1.44, Discount.	0	

91. What is the present worth of $ 346.87 due in 2 yr. 4 m. 12 d.? Ans. $ 303.74—.

92. What is the discount on $ 456.25 due in 9 m. 12 d?

93. What is the present worth of $ 490.50 due in 1 yr. 6 m.? What is the discount?

94. What is the discount on $ 315 due in 1 yr. at 5 %?

95. What is the present worth of a note for $ 350 due 6 months hence?

96. I have a note for $ 436 payable June 21, 1894. What is the worth of the note May 12, 1894, money being worth 8 % per annum?

354. Principal, Interest, and Rate given, to find the Time.

97. For what time must $ 200 be on interest at 6 % to gain $ 36?

Solution. $ 200 in 1 year, at 6 %, will gain $ 12; therefore, to gain $ 36, the time in years must be the quotient of $ 36 ÷ $ 12 = 3, Ans. Hence,

Rule.

Divide the given interest by the interest of the principal for one year at the given rate, and the quotient will be the time.

Or, as (Art. 335), $i = p \times r \times t$,

therefore $t = \dfrac{i}{p \times r}$.

98. How long must \$ 254 be on interest at 5 % to gain \$ 44.45?

99. How long must \$ 75 be on interest at 8 % to gain \$ 15.80?

100. How long must \$ 200 be on interest at 6 % to amount to \$ 236?

101. For what time must \$ 72 be on interest at $8\frac{1}{2}$ % to amount to \$ 87.30?

102. For what time must \$ 1000 be on interest at 9 % to gain \$ 247.50?

103. How long must \$ 100 be on interest at 5 % to gain \$ 100?

Solution. \$ 100 in 1 year at 5 % will gain \$ 5; hence, to gain \$ 100, the time in years must be the quotient of \$ 100 ÷ \$ 5 = 20; that is,

To find the time in which any sum will double itself, at any rate per cent, divide 100 *by the rate, and the quotient will be the number of years.*

104. In how many years will \$ 50 amount to \$ 100 at 8 %?

105. How long will it take any sum of money to double itself on interest at 6 %?

106. In what time will a sum of money triple itself on interest at 5 %?

355. Interest, Time, and Rate given, to find the Principal.

107. What principal at 6 % will gain \$ 18 in 1 yr. 6 m.?

Solution. \$ 1, in 1 yr. 6 m., at 6 %, will gain \$ 0.09. Therefore the principal must be the quotient of \$ 18 ÷ 0.09 = \$ 200, Ans. Hence,

Rule.

Divide the given interest by the interest of $ 1 for the given rate and time, and the quotient will be the principal.

Or, as (Art. 335), $i = p \times r \times t$,

therefore $p = \dfrac{i}{r \times t}$.

108. What principal at 6 % will gain $ 13 in 8 months?

109. What principal on interest at 8 % per annum will gain $ 150 semi-annually?

110. B endowed a professorship with a salary of $ 2000 per annum; what sum must he invest at 6 % to provide this salary?

PARTIAL PAYMENTS.

356. **Partial Payments** are payments in part of a note or other obligation.

357. A **Promissory Note,** usually called a **Note,** is a written promise to pay on demand, or at a specified time, a certain sum of money for value received.

358. The **Principal,** or **Face of the Note,** is the sum named in the note.

359. The **Promisor,** or **Maker of the Note,** is the person who signs the note.

360. The **Promisee,** or **Payee,** is the person to whom, or to whose order, the money is to be paid.

361. The **Indorser of a Note** is the person who writes his name on the back of a note, and thus becomes responsible for its payment.

362. Statements on the back of the note of the payments with the dates are called **Indorsements.**

$653.57. *Boston, Feb. 17, 1891.*

On demand, I promise to pay James E. Torrey, or order, six hundred fifty-three dollars and fifty-seven cents, with interest at 7 %. Value received.

Matthew H. Shields.

(Back of the note.)

Jan. 8, 1892.
Rec'd on within
one hundred fifty
dollars ($150).
Feb. 27, 1893.
Rec'd on within
one hundred twenty-
five dollars ($125).
Aug. 10, 1894.
Rec'd on within
two hundred fifteen
dollars ($215).

Principal .... $ 653.57.
Promisor .... Matthew H. Shields.
Promisee .... James E. Torrey.

Indorsements	Jan. 8, 1892...	$ 150.
	Feb. 27, 1893...	125.
	Aug. 10, 1894...	215.

363. For finding the sum due on a note on which there are indorsements the following is

The United States Rule.

Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due; then subtract the payment, or the sum of the payments, from the amount of the principal, and the remainder is a new principal, with which proceed as before.

(111.)

$346.36. BOSTON, March 26, 1892.

On demand, we promise to pay Stephen C. Jones, or bearer, three hundred forty-six and $\frac{36}{100}$ dollars, with interest. Value received. BRUCE & DAVIS.

INDORSEMENTS: July 20, 1892, $54.75; April 8, 1893, $10; Sept. 26, 1893, $5.50; Jan. 6, 1894, $150.46.

What was due May 2, 1894?

Principal	$346.36
Interest to July 20, 1892 (3 m. 24 d.)	6.581
Amount	$352.941
1st payment	54.75
New principal	$298.191

(The payment April 8, 1893, is less than the interest then due; and the sum of the payments April 8, 1893, and Sept. 26, 1893, is less than the interest due Sept. 26, 1893.)

Interest to Jan. 6, 1894 (1 yr. 5 m. 17 d.)	26.191
Amount	$324.382
Sum of 2d, 3d, and 4th payments	165.96
New principal	$158.422
Interest to May 2, 1894 (3 m. 26 d.)	3.063
Amount due May 2, 1894	$161.485, Ans.

NOTE. Unless the payment and the interest are very nearly equal, one can calculate mentally whether the payment exceeds the interest or not. In the example above the interest of $298.191 (the amount due July 20, 1892) is almost $18 a year, while the payment April 8, 1893, is only $10; and the sum of the payments made April 8 and Sept. 26, 1893, is only $15.50.

(112.)

$525. CAMBRIDGE, MASS., June 4, 1889.

On demand, I promise to pay John Davis, or order, five hundred twenty-five dollars, with interest. Value received.

DANIEL FOX.

On this note are the following indorsements: Sept. 9, 1890, $114.20; May 15, 1891, $78.28; Aug. 6, 1892, $244.38.

What was due Feb. 7, 1894? Ans. $191.02.

(113.)

$586.96. ANDOVER, May 12, 1892.

On demand, we, jointly and severally, promise to pay Abel Stevens, or order, five hundred eighty-six dollars and ninety-six cents, with interest. Value received. JAMES CARTER.
JOHN DAVIS.

INDORSEMENTS: Jan. 24, 1893, $154.87; Dec. 6, 1893, $75.18; Aug. 15, 1894, $124.87; Dec. 6, 1894, $100.

What was due April 24, 1895?

(114.)

$163.42. PROVIDENCE, April 15, 1891.

Six months after date, I promise to pay Asa Brewer, or order, one hundred sixty-three and $\frac{42}{100}$ dollars, with interest. Value received. CALVIN DUNSTER.

INDORSEMENTS: May 24, 1892, $42.18; July 17, 1893, $6.25; Sept. 6, 1893, $48.16; Jan. 25, 1894, $27.47.

What was due April 15, 1895?

(115.)

$572.76. NEW YORK, June 4, 1889.

Six months after date, I promise to pay Walter Willis, or his order, five hundred seventy-two dollars and seventy-six cents, with interest at 7 %. Value received.

SAMUEL JOHNSON.

INDORSEMENTS: April 10, 1890, $125.85; Nov. 28, 1890, $133.72; April 15, 1891, $223.08.

What was due Nov. 13, 1891?

(For Connecticut and Vermont Rules, and the rule for Annual Interest, see Appendix, pp. 349–354.)

364. The following is a common rule when settlement is made within a year after the interest begins.

Rule.

From the amount of the note subtract the amount of the payments.

(116.)

$387.75. NEW YORK, May 15, 1893.

On demand, I promise to pay Samuel Adams, three hundred eighty-seven and $\frac{75}{100}$ dollars, with interest from date. Value received. HENRY PHILLIPS.

INDORSEMENTS: July 21, 1893, $75; Oct. 10, 1893, $125; Feb. 24, 1894, $50.

What was due at the time of settlement, May 15, 1894?

Amount of						
$387.75	from	May 15, 1893,	to	May 15, 1894,		$411.015
75	"	July 21	"	" " "	$78.675	
125	"	Oct. 10	"	" " "	129.479	
50	"	Feb.24, 1894,	"	" "	50.675	258.829
Amount due				" "	Ans.	$152.186

(117.)

$1785. BOSTON, April 17, 1894.

On demand, I promise to pay Robert Chase, or order, seventeen hundred eighty-five dollars, with interest. Value received.

SALMON FRENCH.

INDORSEMENTS: July 3, 1894, $300; Sept. 10, 1894, $275; Nov. 6, 1894, $317; Jan. 2, 1895, $453.

What was due March 27, 1895?

(118.)

A note of $2450, dated June 4, 1895, has the following

INDORSEMENTS: Sept. 4, 1895, $562.50; Dec. 24, 1895, $846.37; Feb. 18, 1896, $362.63.

What was due May 12, 1896?

COMPOUND INTEREST.

365. Compound Interest is interest on both *principal and interest.*

366. The interest may become due and be added to the principal at the end of each year, or half-year, or any other *period of time* that may be agreed upon.

119. What is the amount at compound interest of $100 for 3 yr. 3 m. at 6 % per annum?

Principal	$ 100
	1.06
1st amount, or 2d principal	106
	1.06
2d amount, or 3d principal.........	112.36
	1.06
3d amount, or 4th principal	119.1016
	1.015
4th amount	$ 120.888124, Ans.

367. Hence, to find the amount of any sum of money at compound interest,

Rule.

Find the amount of the given principal for the first period of time. With this amount as a principal, find the amount for the next period; and so on to the end of the given time.

368. To find the compound interest,

Subtract the given principal from the last amount.

NOTE 1. In Ex. 119 the compound interest is $120.888 — $100, or $20.888.

NOTE 2. If there is a fraction of a period of time, find the amount for the complete periods, and then the amount of this amount for the remaining fraction of a period.

120. What is the amount at compound interest of $ 300 for 2 yr. 8 m., at 4 % per annum?

121. What is the compound interest of $ 400 for 3 years, at 7 %?

122. What is the amount of $ 6000 at compound interest for 4 yr. 10 m. 12 d.?

123. What is the amount of $ 5000 at compound interest for 2 yr. 6 m. 18 d.?

124. What is the compound interest of $ 12000 for 2 yr. 6 m. 18 d., at 6 %?

125. What is the amount at compound interest of $ 12000 for 2 yr. 6 m. 18 d., at 4 %?

126. What is the amount at compound interest of $ 12000 for 2 yr. 6 m. 18 d., at 8 %?

NOTE 3. *Four* per cent of any number is $\frac{2}{3}$, and 8 % is $\frac{4}{3}$, of 6 % of the same number; but the compound interest of any sum of money at 4 % is *less* than $\frac{2}{3}$ of the compound interest of the same sum for the same time at 6 %, and the compound interest at 8 % is *more* than $\frac{4}{3}$ of the compound interest at 6 %, as may be seen by examples 123, 124, and 125.

The compound interest at 4 % is less than half the compound interest of the same sum at 8 %, because the *base* (that is, the principal), after the 1st year, is *less* in computing interest at 4 % than in computing it at 8 %; thus, in computing interest at 4 % and 8 % the 1st year the base is the *same*, and *one interest is just half of the other;* but the 2d year one base is $ 104 and the other $ 108; hence, the compound interest at 4 % *is less than half of that at* 8 %.

127. What is the amount of $ 350 for 2 yr. 6 m., at 3 % for each 6 m., compounding the interest semi-annually?

128. What is the amount of $ 56 for 1 yr. 9 m., at 2 % a quarter, compounding the interest quarterly?

129. What is the compound interest of $ 824.75 for 3 yr. 8 m. 15 d., at 6 %?

369. Compound interest may be calculated more expeditiously by means of the following

TABLE,

Showing the Amount of $1, interest compounded annually at 4, 5, 6, 7, and 8 per cent, from 1 to 20 years.

Yr.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	Yr.
1	1.040000	1.050000	1.060000	1.070000	1.080000	1
2	1.081600	1.102500	1.123600	1.144900	1.166400	2
3	1.124864	1.157625	1.191016	1.225043	1.259712	3
4	1.169859—	1.215506+	1.262477—	1.310796+	1.360489—	4
5	1.216653—	1.276282—	1.338226—	1.402552—	1.469328+	5
6	1.265319+	1.340096—	1.418519+	1.500730+	1.586874+	6
7	1.315932—	1.407100+	1.503630+	1.605781+	1.713824+	7
8	1.368569+	1.477455+	1.593848+	1.718186+	1.850930+	8
9	1.423312—	1.551328+	1.689479—	1.838459+	1.999005—	9
10	1.480244+	1.628895—	1.790848—	1.967151+	2.158925—	10
11	1.539454+	1.710339+	1.898299—	2.104852—	2.331639—	11
12	1.601032+	1.795856+	2.012196+	2.252192—	2.518170+	12
13	1.665074—	1.885649+	2.132928+	2.409845+	2.719624—	13
14	1.731676+	1.979932—	2.260904—	2.578534+	2.937194—	14
15	1.800944—	2.078928+	2.396558+	2.759032—	3.172169+	15
16	1.872981+	2.182875—	2.540352—	2.952164—	3.425943—	16
17	1.947900+	2.292018+	2.692773—	3.158815+	3.700018+	17
18	2.025817—	2.406619+	2.854339+	3.379932+	3.996019+	18
19	2.106849+	2.526950+	3.025600—	3.616528—	4.315701+	19
20	2.191123+	2.653298—	3.207135+	3.869684+	4.660957+	20

NOTE. The *interest* is $1 *less* than the *amount* in the above table.

130. What is the compound interest of $600 for 20 yr. at 6 %?

Int. of $1 for 20 yr. at 6 % by the table...	$2.207135
	600
Interest of $600 for 20 yr. at 6 %.........	$1324.281, Ans.

131. What is the amount at compound interest of $30 for 5 yr. 6 m. at 4 %?

Amount of $1 for 5 yr. at 4 % by the table...	$1.216653
	30
Amount of $30 for 5 yr. at 4 %	$36.499590
	1.02
Amount of $30 for 5 yrs. 6 m. at 4 %......	$37.2295818, Ans.

132. What is the amount of $60 at 7 % per annum for 12 years at compound interest?

133. What is the amount of $550.50 at 8 % compound interest for 18 years?

134. What is the compound interest of $95 for 20 years at 8 %?

135. What is the interest of $400 for 7 yr. 6 m. at 4 % for each 6 months, compounding the interest semi-annually?

370. Miscellaneous Written Examples.

136. What is the (simple) interest of $555.55 for 5 yr. 5 m. 5 d. at 5 %?

137. What is the amount of $175.25 for 3 yr. 7 m. 8 d. at 6 %?

138. Find the interest of $675 from July 28, 1894, to Nov. 26, 1894, at 4 %.

139. What is the exact interest at $4\frac{1}{2}$ % of $83.35 from May 17, 1895, to Aug. 10, 1895?

140. What principal at 5 % will amount to $621 in 4 yr. 8 m. 12 d.?

141. What is the present worth of $237.53 due in 10 months? What is the discount?

142. How long will it take $525 to gain $85 at 5 %?

143. What sum at 4 % will give an annual income of $576?

144. What was due on the note on page 196 Dec. 12, 1894?

145. What is the compound interest of $875 for 5 yr. 6 mo. at 6 %?

146. What is the amount of $323.50 for 3 yr. 6 m. 10 d. at 5 %?

147. What is the amount at simple interest of $37.45 for 3 yr. 8 m. 19 d. at 5 %?

148. Find the amount at simple interest of $535.58 for 2 yr. 7 m. 8 d. at $4\frac{1}{2}$ %.

BANK DISCOUNT.

$ 275.55. Boston, Aug. 26, 1895

Three months after date I promise to pay to the order of George C. Davis two hundred seventy-five and $\frac{55}{100}$ dollars.

Value received.

William M. Carroll.

371. If the note above is discounted at a bank on the day of its date, Aug. 26, 1895, the bank pays George C. Davis (or his order) $ 275.55 less the interest of $ 275.55 for 3 months, that is, $ 275.55 – $4.13, or $ 271.42.

372. **Bank Discount** is, therefore, only simple interest paid when the money is loaned. $4.13 is the bank discount on the note above.

373. The **Avails,** or **Proceeds,** of a note is the amount received by the borrower, and is equal to the face of the note less the interest. In the note above the avails are $271.42.

374. In some States a note is not *legally* due until *three days* after the time which the note specifies for its payment. These three days are called *days of grace.* A note matures upon the last day of grace. Thus, in the note above, if for Boston we write Newport, R. I., the note would be *nominally* due Nov. 26, 1895, but *legally* Nov. 29, 1895. This is expressed by writing both dates thus, Nov. $^{26}/_{29}$, 1895. Nov. 29 is the date of *maturity* of the note.

375. Hence, to find the bank discount and the avails of a note, payable at a specified future time, without interest,

Rule.

Find the interest on the face of the note, at the given rate, from the time of discounting to maturity, and the result will be the discount. Subtract the discount from the face of the note, and the remainder will be the avails.

NOTE 1. Usually when a note becomes due on a Sunday or legal holiday, it is legally payable on the next secular day; but in a few States, by statute law, it is payable on the preceding secular day.

NOTE 2. A note made payable in a *certain number of days* is due when this number of days has passed. Thus a *thirty days' note* dated Jan. 31 is due March 2 (or in leap year March 1); but a note made payable in a *certain number of months* is due on the same day of the month in which it matures as it is dated; or, if there are not so many days in the month, it is due on the last day of the month. Thus, a *one month note* dated Feb. 28 matures March 28; but a *one month note* dated Jan. 29, or 30, or 31 is due Feb. 28, or in leap year Feb. 29. In States where grace is allowed, these notes mature three days later than the time stated above.

NOTE 3. When the time of discount is less than 2 months many banks count the exact days from discount to maturity. Thus, if a note is discounted Oct. 12, and matures Dec. 5, the time, by Art. 274, is 1 m. 23 d., but the bank computes interest for 54 days.

376. Written Exercises.

1. What is the bank discount on a 90 days' note for $368? What are the avails?

Interest for 60 days..........$3.68
" " 30 " 1.84
$5.52, 1st Ans.

$368 − $5.52 = $362.48 avails, 2d Ans.

With grace the discount is $5.70, and the avails $362.30.

2. I have a 6 months' note for $768, dated May 12. What will be the avails if I get it discounted Sept. 3?

Interest for 2 m.	$7.68	Six months expire Nov. 12. From Sept. 3 to Nov. 12 is 2 m. 9 d., the time for which the note is discounted.
" " 9 d.	1.152	
Discount	$8.832	

$768 − $8.83 = $759.17 avails, Ans.

With grace the discount is $9.216, and the avails $758.78.

3. What will be the bank discount and what the avails of a 4 months' note for $8646?

4. Of a 90 days' note for $1842 at 7%?

5. Of a 6 months' note for $489 at 5%?

6. A 4 months' note for $629, dated Feb. 27, was discounted April 12. What were the avails?

NOTE 4. When a note bearing interest is discounted before its maturity, the *amount* of the note *at maturity*, not its *face*, is the base for discounting.

7. What are the proceeds of a note for $10000, payable in 6 months and bearing interest, if discounted 2 months before its maturity?

Solution. The amount of $10000 for 6 m. is $10300, and the interest of $10300 for 2 m. is $103, which taken from $10300 leaves $10197, Ans. With grace the amount is $10305, and the interest on $10305 for 2 m. is $103.05, and the proceeds are $10201.95.

8. What are the avails of a note for $6844, payable in 4 months with interest, if discounted 1 month after date?

377. To find the sum for which a note must be written that the avails or proceeds may be a specified sum.

9. For what sum must a 45 days' note be written that the avails may be $240.

	$1.0000	The avails of $1 for 45 days are $0.9925, and therefore the face of the note must be as many dollars as $0.9925 is contained times in $240, or $241.81.
Interest of $1 for 45 days	0.0075	
Avails of $1	$0.9925	

$240 ÷ 0.9925 = $241.81, Ans.

With grace the avails of $1 are $0.992, and $240 ÷ 0.992 = $241.94, Ans. Hence,

Rule.

Divide the required proceeds by the proceeds of $ 1 for the given rate and time, and the quotient will be the face of the required note.

10. For what sum must a 3 months' note be given, that the proceeds may be $ 300 ?

11. A farmer sold produce for which he received a 60 days' note, which he immediately had discounted at the bank. The avails of the note were $ 593.70 ; what was its face ?

12. A merchant wishes to borrow $ 1200 at a bank, for 90 days ; what shall be the face of the note, the rate of interest being 7 % ?

378. Miscellaneous Written Examples.

13. What is the bank discount of a note of $ 765.84 due in 3 months at 8 % ?

$ 873.45. BROOKLYN, N. Y., Sept. 17, 1894.

Four months after date I promise to pay John Brown, or order, eight hundred seventy-three dollars and forty-five cents.

Value received. CHARLES STEVENS.

14. Find the proceeds of the above note if discounted on its date at a bank at 6 %.

15. Find the avails if discounted at a bank Oct. 29, 1894, at 7 %.

16. Find the avails if discounted Nov. 26, 1894, at $7\frac{1}{2}$ %.

Find the proceeds of:

17. A 4 months' note for $ 315.70, dated June 9, discounted July 25, at 4 %.

18. A 90 days' note for $ 1125, dated Aug. 15, discounted Sept. 3, at 5 %.

19. A 3 months' note for $ 768.50, dated Dec. 13, 1894, discounted Jan. 22, 1895, at $5\frac{1}{2}$ %.

20. A 75 days' note for \$ 348.56, dated March 9, discounted April 3, at 6 %.

21. A 6 months' note for \$ 2916.80, dated Apr. 19, discounted June 26, at 4 %.

22. A 60 days' note for \$ 285.75, dated Oct. 13, discounted Nov. 5, at 7 %.

23. A 5 months' note for \$ 131.40, dated Sept. 30, discounted Dec. 1, at $7\frac{1}{2}$ %.

24. A 45 days' note for \$ 463.19, dated July 10, discounted Aug. 2, at 6 %.

25. A 3 months' note for \$ 2500, dated July 16, discounted Aug. 22, at 8 %.

26. A 4 months' note for \$ 916.80, dated Mar. 20, discounted May 11, at 7 %.

\$ 987.44. BOSTON, Oct. 25, 1894.

For value received, I promise to pay William Simpson & Co., or order, nine hundred eighty-seven dollars and forty-four cents, six months from date, with interest at 8 %. THOMAS P. FOX.

27. What are the avails of the above note if discounted at date at 6 %?

28. What are the avails if discounted Jan. 1, 1895, at 6 %?

29. What is the value at maturity?

30. What is the value Sept. 13, 1895?

31. What must be the face of a 60 days' note discounted on its date at 6 % in order that the avails may be \$ 150?

32. What must be the face of a 3 months' note discounted on its date at 7 % in order that the avails may be \$ 316?

33. If at 5 % discount \$ 223.75 is received July 12, 1895, on a note dated June 8, 1895, and maturing Sept. $^{2}/_{5}$, 1895, what is its face?

34. If at $7\frac{3}{10}$ % discount \$ 75.15 is received on a 60 days' note 20 days after its date, what is the face?

EQUATION OF PAYMENTS.

379. Equation of Payments is the method of determining when several sums, payable at different times, may be paid at one time without loss to either debtor or creditor.

380. The *equated time* is the *date* of payment.

381. Written Exercises.

1. A owes B \$900, of which \$200 is due June 12, 1895, \$300 Aug. 8, and \$400 Sept. 19. When can A pay the \$900 in one sum without loss to either A or B?

Interest Method.

Due.	Amount.	Time.	Interest.
June 12,	\$200		
Aug. 8,	300	1 m. 27 d.	\$2.85
Sept. 19,	400	3 m. 7 d.	6.47
Int. of	\$900	1 m. = \$4.50)	\$9.32
			2.07 m., or 2 m. 2 d.

June 12, 1895, + 2 m. 2 d. = Aug. 14, 1895, Ans.

If the amounts due were all paid June 12, the debtor would lose the interest on the \$300 from June 12 to Aug. 8, or \$2.85, and also the interest on \$400 from June 12 to Sept. 19, or \$6.47; that is, by paying the \$900 June 12 the debtor would lose \$2.85 + \$6.47 = \$9.32. Therefore, he can equitably retain the \$900 as long after June 12 as it will take it to gain \$9.32. As \$900 gains \$4.50 in one month it will take it as many months to gain \$9.32 as \$4.50 is contained times in \$9.32, or 2 m. 2 d. 2 m. 2 d. after June 12 is Aug. 14.

NOTE 1. In finding the equated time interest may be computed from the date of the first bill or *from any other date*, but it is most convenient to compute the interest from the *first day of the month in which* the first bill is due, because the *time* for which interest is to be computed on the several bills is thereby most easily determined.

NOTE 2. Some accountants compute the interest between the dates for the exact number of days instead of for the calendar months and days. But as interest between merchants is usually computed for months and days, the above method is more equitable as well as expeditious, and will seldom produce a different result, and then will seldom change the equated time more than one day.

382. Hence, to find the equated time for the payment of several sums due at different times,

Rule.

Find the sum of the interests on each amount from the first day of the month in which the first bill is due to the time each bill is due. Divide the sum of these interests by the interest of the sum of the bills for one month, and the quotient will be the number of months from the selected date to the equated time.

NOTE 1. A fraction of a day less than ½ is neglected; otherwise it is counted as a day.

2. Find the equated time for paying $ 2000 due Jan. 17, $ 3000 due Feb. 21, and $ 600 due June 6.

Due.	Amount.	Time.	Interest.		
Jan. 17,	$ 2000,	16 d.	$ 5.00,	int. for	15 d.
			0.33,	"	1 d.
Feb. 21,	3000,	1 m. 20 d.	15.00,	"	1 m.
			10.00,	"	20 d.
June 6,	600,	5 m. 5 d.	12.00,	"	4 m.
			3.00,	"	1 m.
			0.50,	"	5 d.

Int. of $ 5600 for 1 m. = $ 28) $ 45.83
1.63 m. = 1 m. 19 d.

Jan. 1 + 1 m. 19 d. = Feb. 20, Ans.

3. Jan 7, 1895, I purchased of Mr. James $ 75 worth of goods on a credit of 4 months; Jan. 28, $ 155, on a credit of 3 months; and Feb. 15, $ 98, on a credit of 2 months. What is the equitable time for paying the whole amount?

NOTE 2. First find the date at which each bill is due.

4. Jan. 1, A owes B $200 payable in 4 months and $600 payable in 8 months. Find the equated time for settlement.

5. July 6, 1895, I owe to John Smith $4550, payable in 2 m., $5075 in 3 m., and $3500 in 4 m. What is the equated time of payment?

6. $1500, $2100, and $2400 are due in 3, 4, and 5 months, respectively, from Nov. 15, 1895. What is the equated time of payment?

7. Required the equated time of paying the following bills of goods, each bought on a credit of two months. Mar. 12, 1895, $300; Mar. 18, $200; Apr. 6, $600; July 24, $100.

NOTE 3. Find the equated time without regard to the 2 months, and then add 2 months to this result.

8. Required the equated time of paying the following bills, each bought on 3 months' credit. June 5, 1895, $180; June 15, $84; July 12, $240; July 20, $96.

9. Bought the following bills on 2 months' credit: May 14, 1895, $400; June 4, $150; June 6, $80; June 14, $170. What is the equated time of payment?

10. Bought the following bills on 3 months: Feb. 18, 1895, $1200; Mar. 25, $472; Mar. 30, $468; Apr. 1, $500. What is the equated time of payment?

11. Bought the following bills on 2 months: Jan. 8, 1895, $12; Jan. 24, $20; Feb. 18, $1200; Mar. 6, $4000. What is the equated time of payment?

12. Bought the following bills on 30 days: Jan. 8, 1895, $4000; Jan. 14, $1200; Jan. 18, $20; Jan. 26, $12. What is the equated time of payment?

13. If Jan. 1, 1895, A owes B $400 due in 4 months, $600 due in 6 months, and $800 due in 8 months, what is the equated time of payment?

Product Method.

$$\begin{array}{rl} \$400 \times 0 = & \$0000 \\ 600 \times 2 = & 1200 \\ \underline{800} \times 4 = & \underline{3200} \\ \$1800 \quad\quad) & \underline{\$4400} \\ & 2\frac{4}{9}\text{ m.} = 2\text{ m. } 13\text{ d.} \end{array}$$

Jan. 1, 1895, + 4 m. + 2 m. 13 d. = July 14, 1895, Ans.

The \$600 is due 2 months, and the \$800 4 months after the \$400 is due. The use of \$600 for 2 months is equivalent to the use of \$1200 for 1 month; and the use of \$800 for 4 months is equivalent to the use of \$3200 for 1 month. A, then, is entitled to keep, *after the time the* \$400 *is due*, the whole sum, \$1800, till it is equivalent to the use of \$4400 for 1 month; that is, as many months after the \$400 is due as \$1800 is contained times in \$4400, or $2\frac{4}{9}$ m. = 2 m. 13 d. 2 m. 13 d. after Jan. 1, 1895, + 4 m. = July 14, 1895, Ans. Hence,

Rule.

Multiply each debt by the number expressing the time to elapse between its maturity and the earliest maturity of any debt, and divide the sum of these products by the sum of the debts. Add the time thus obtained to the date of the earliest maturity of any debt.

NOTE 4. The examples already given can be performed by the product method if preferred; but the interest method is considered the better.

383. To find the equated time for paying the balance of an account which has both debit and credit entries.

14. What is the equated time for paying the balance of \$500 due on the following account?

Dr. JAMES FOX. **Cr.**

1895.				1895.			
July 18	Merchandise,	\$254	00	Aug. 15	Cash,	\$200	00
Sept. 6	"	475	00	" 30	"	288	00
" 17	"	425	00	Oct. 3	"	412	00
Oct. 9	"	446	00	" 21	"	200	00
		\$1600	00			\$1100	00

Due.	Amounts.	Time.	Interest.		
July 18,	$254,	17 d....	$0.635,	int. for	15 d.
			0.085,	"	2 d.
Sept. 6,	475,	2 m. 5 d....	4.75,	"	2 m.
			0.396,	"	5 d.
Sept. 18,	425,	2 m. 17 d....	4.25,	"	2 m.
			1.062,	"	15 d.
			0.142,	"	2 d.
Oct. 9	446,	3 m. 8 d....	4.46,	"	2 m.
			2.23,	"	1 m.
			0.446,	"	6 d.
			0.149,	"	2 d.
Int. on Dr. side from July 1...			$18.605		

Due.	Amounts.	Time.	Interest.		
Aug. 15,	$200,	1 m. 14 d....	$1.00,	int. for	1 m.
			0.40,	"	12 d.
			0.067,	"	2 d.
Aug. 30,	288,	1 m. 29 d....	1.44,	"	1 m.
			1.152,	"	24 d.
			0.24,	"	5 d.
Oct. 3,	412,	3 m. 2 d....	4.12,	"	2 m.
			2.06,	"	1 m.
			0.137,	"	2 d.
Oct. 21,	200,	3 m. 20 d....	3.00,	"	3 m.
			0.667,	"	20 d.
Interest on Cr. side from July 1...			$14.283		

If Mr. Fox had paid all these bills, amounting to $1600, on the first day of July, he would have lost the interest on them, amounting to $18.605; but he paid only certain amounts, in all $1100, at different dates *after* July 1, on which the interests from July 1 to the date of their payment amount to $14.283. He would therefore have lost in interest $18.605 − 14.283 = $4.322, and should equitably keep the balance of $500 as long after July 1 as it would take for the interest on it to amount to $4.322. $4.322 ÷ $2.50 (the interest of $500 for 1 m.) = 1.73; that is, Mr. Fox should retain the $500 1.73 m., or 1 m. 22 d. July 1, 1895, + 1 m. 22 d. = Aug. 23, 1895, Ans.

15. What is the equated time for paying the balance of $ 196 due on the following account?

Dr. JAMES FOX. **Cr.**

1896.				1896.			
Jan. 13	Merchandise,	$ 275	00	Feb. 10	Cash,	$ 250	00
March 7	"	580	00	March 10	"	780	00
May 13	"	750	00	July 19	"	375	00
June 4	"	146	00	Aug. 25	"	150	00
		$ 1751	00			$ 1555	00

Interest on Dr. side from Jan. 1,		Interest on Cr. side from Jan. 1,	
$ 275 to Jan. 13 ...	$ 0.55	$ 250 to Feb. 10...	$ 1.25
			0.25
580 to March 7 ...	5.80		0.125
	0.58	780 to Mar. 10...	7.80
750 to May 13 ...	15.00		0.78
	1.50		0.39
146 to June 4 ...	2.92	375 to July 19...	11.25
	0.73		1.125
	0.073	150 to Aug. 25...	4.50
	$ 27.153		0.75
			0.60
			$ 28.82

In this case the balance of interest is on the side of the smaller amount. If Mr. Fox had paid all the bills, amounting to $ 1751, on the first day of January, he would have lost the interest on them, amounting to $ 27.153. But he paid $ 1555 only, and at dates so long *after* January 1 that the interest amounts to $ 28.82. He would therefore, if he had settled on January 1, have gained in interest $ 28.82 − $ 27.153 = $ 1.677 ; the balance of the account, $ 196, would therefore be due as long *before* January 1 as it would take for the interest on it to amount to $1.677, viz. 1 m. 21 d. Jan. 1, 1896, − 1 m. 21 d. = Nov. 11, 1895, Ans.

NOTE. It is manifestly impossible to settle an account before the date of the transactions. The advantage of finding an equated date is, that we thus

find a certain date on which, if the account could have been settled, there would have been no loss to either party, and from which interest can be computed to any agreed date of settlement.

384. Hence, to equate accounts when there are both debits and credits,

Rule.

Compute the interest of each item of the account from the first day of the first month in which any transaction matures, to its maturity. Find the sum of the interests on the debit items, also the sum on the credit items, and subtract the less sum from the greater; divide this difference by the interest on the balance of the account for one month, and the quotient will be the time in months from the first day of the first month named above to the equated time of settlement. Count the time forward when the greater interest is on the greater side of the account, and backward when the greater interest is on the smaller side.

16. Find the equated time for paying the balance of the following account.

Dr. JOHN BOWMAN. Cr.

1895.				*1895.*			
Jan. 12	Merchandise,	$634	00	Jan. 18	Cash,	$346	00
" 25	"	896	00	Feb. 1	"	960	00
Feb. 6	"	734	00	" 12	"	454	00
" 18	"	146	00	March 1	"	240	00

17. Find the equated time for paying the balance of the following account.

Dr. WILLIAM FABYAN. Cr.

1896.			*1896.*		
April 24	Mdse., 2 m.	$356	Feb. 6	Mdse., 2 m.	$530
May 17	Mdse., 4 m.	875	May 27	Mdse., 3 m.	652
June 3	Mdse., 3 m.	433	June 16	Cash,	300

Note. Copy off the account, dating each item at its maturity.

18. Find the equated time for paying the balance of the following account.

Dr. GREENE & BARTLETT. **Cr.**

1896.			*1896.*		
March 4	Mdse., 2 m.	$452	March 13	Cash,	$500
" 8	Cash,	224	" 21	Note, 2 m.	100
" 20	Mdse., 2 m.	150	April 18	Cash,	192
" 27	Mdse., 2 m.	496	" 21	Cash,	542
April 19	Mdse., 3 m.	724			
" 30	Mdse., 2 m.	88			

19. The following account was settled by a 3 months' note. When should the note be dated to fall due (with grace) at the equated time of payment?

Dr. ROBERT MARTIN. **Cr.**

1895.			*1895.*		
Dec. 18	Mdse., 4 m.	$800	Nov. 27	Mdse., 3 m.	$1200
1896.			Dec. 12	Mdse., 4 m.	800
Jan. 6	Mdse., 2 m.	350	*1896.*		
Feb. 15	Note, 4 m.	1200	Jan. 15	Mdse., 4 m.	850
March 12	Mdse., 2 m.	200	Feb. 18	Mdse., 3 m.	625

20. The following items are all on 3 months' credit. When should a note, to settle the balance, due in 3 months (with grace), be dated?

Dr. JOHN ROBINSON. **Cr.**

1896.			*1896.*		
Jan. 20	Mdse.,	$986	Jan. 4	Mdse.,	$158
Feb. 17	Mdse.,	152	" 18	Mdse.,	228
" 26	Mdse.,	110	Feb. 5	Mdse.,	450
March 6	Mdse.,	317	" 20	Mdse.,	213
March 20	Mdse.,	175	March 4	Mdse.,	347
			" 13	Mdse.,	116

BONDS.

385. Governments and corporations often borrow money, and as evidence of their indebtedness issue certificates payable at or before some stated time, with interest payable annually, semi-annually, or quarterly. These interest-bearing notes, or government securities, are called **bonds.**

386. Coupons are certificates of interest attached to a bond, which as they fall due are cut off and given up to the payer on receipt of the interest.

387. Bonds are usually named according to their yearly rate of interest and date of maturity; as U. S. 4's 1925, which means United States Bonds bearing 4 % interest annually and payable in 1925; Mass. 3's 1930, which means State of Massachusetts Bonds bearing 3 % interest annually and payable in 1930; Boston and Albany 4's 1913; Northern Pacific 6's 1921. The interest on bonds is generally payable semi-annually. But on most of the U. S. bonds the interest is payable quarterly.

388. Bonds, like stocks, are regularly bought and sold at the Brokers' Boards and at auction. The sum to be paid to the holder of a bond on its maturity is the face value, or 100 %, but bonds are generally bought and sold previous to maturity, at a premium, or discount, which depends upon the value of money, the time to elapse before the principal is due, the credit of the maker of the bond, etc.

389. *The rules of percentage already given apply to bonds and exchange.*

390. Written Exercises.

1. If U. S. 6's are selling at $106\frac{1}{8}$, what will \$5000 in bonds cost?

$$\$5000 \times 1.06\tfrac{1}{8} = \$5306.25, \text{ Ans.}$$

2. If a person invests \$13265.625 in U. S. 6's at $106\frac{1}{8}$, what amount in bonds will he receive?

3. I own 12 U. S. 5's of \$1000 each; what is my annual income?

4. If I purchase a U. S. 6 % bond at 106, what per cent on my investment do I receive annually?

5. What ought I to pay a broker for two \$500 6's at $106\frac{1}{8}$, three \$100 $4\frac{1}{2}$'s at $104\frac{5}{8}$, and five \$500 4's at $99\frac{1}{2}$, with his brokerage of $\frac{1}{4}$ % in addition to the prices named?

NOTE. Brokerage is reckoned on par value (Art. 315, Note 1).

6. What must I pay for \$500 U. S. 4's at $100\frac{1}{2}$, \$2000 New York 6's at $107\frac{1}{2}$, \$1500 St. Louis 7's at 110, \$2500 Union Pacific 6's at 109, \$3000 Cleveland 8's at $109\frac{1}{2}$, including brokerage of $\frac{1}{4}$ %?

7. I invest \$32500 in 8 % bonds, at 30 % premium. What income do I receive?

8. If a broker sells for me five \$500 4's at $99\frac{1}{2}$ and reinvests the proceeds in \$100 6's at $106\frac{1}{8}$, if he takes a commission of $\frac{1}{4}$ % on each transaction, how many bonds does he buy, and what balance of cash will be left to my credit?

9. What sum must I invest in U. S. 5's at $106\frac{1}{8}$ to have an annual income of \$1000?

10. What per cent on the investment do 6 % bonds bought at 90 pay?

11. Bought \$1000 4 % bonds maturing in three years at 95. How much income do I get in all, if I hold the bonds till paid at maturity?

12. Bought $1000 6's at 120 and $1000 7's at 140; what per cent of the investment do I get on each for income?

13. Suppose the bonds in Ex. 12 are both payable at the same time and I hold them till paid, which is the better investment and by how much?

14. If I purchase $15000 Boston 5's, interest payable semi-annually, and $15000 U. S. 5's, interest payable quarterly at the same price, which pays the most annual income and how much, provided the several payments of interest are on receipt put at interest at 6 %?

15. I have $12075 to invest. Which will pay the greater annual income, Missouri 6's at 105, or Chicago 7's at 115? How much greater?

16. How many $1000 U. S. 5's must I buy to have an income each year of $1250? How much must I invest if these bonds cost 104¼?

17. Which yields the greater percentage on the investment, U. S. 4's at 99½, or Union Pacific Railroad 7's at 108¾? How much greater?

18. Which is the better investment if both are redeemed in 2 years, U. S. 6's at 106⅛, or 4's at 99½? How much better?

19. How much shall I send to a broker that he may take out his brokerage of ¼ % and purchase for me 7 Boston 6's, of $100 each, selling at 115?

20. How much must I invest in New York 7's at 120, interest payable semi-annually, to have a semi-annual income of $1400? What would be the brokerage at ¼ % for the purchase of these bonds by a broker?

21. Which pays the greater per cent of interest semi-annually on the investment, New York 6's at 109, or Boston and Albany 7's at 120? How much greater?

EXCHANGE.

391. **Exchange,** in commerce, is a mode of paying debts due in distant places by means of *drafts,* or *bills of exchange.*

To explain the operation of exchange and show its benefits, let us suppose that A of Boston owes B of San Francisco $1000, and C of San Francisco owes D of Boston $1000. Now A and C can each pay his debt by sending $1000 in gold or silver and paying the cost of transportation and insurance; but exchange furnishes a better way. Thus, D of Boston writes a request (bill of exchange) to C of San Francisco to pay A of Boston, or his order, $1000. A buys this bill of exchange of D, indorses the bill, and sends it to B of San Francisco, who presents it to C, and C pays B the $1000; thus A and C have paid their debts and B and D have received their dues without the trouble, cost, or risk of transporting a dollar in money or merchandise.

392. A **Draft,** or **Bill of Exchange,** is a written order, or request, to one person to pay to another a certain sum of money, and charge the same to the account of the person who makes the request.

$500. *Boston, April 8, 1895.*

At sight, pay to the order of James S. Frost, five hundred dollars, and charge to the account of

William P. Mellen.

To Charles J. Rose, Chicago, Ill.

393. The **Maker** or **Drawer** of a draft or bill of exchange is the person who requests another to pay; the **Drawee** is the person who is requested to pay; and the **Payee** is the

person to whom the drawee is requested to pay the money. Thus, in the preceding draft,

The drawer is William P. Mellen.
The drawee is Charles J. Rose.
The payee is James S. Frost.

394. Some bills of exchange are made payable *at sight*, that is, as soon as they are presented to the drawee; others are made payable on a given day or in a specified time, say 30, 60, or 90 days after sight. In some States 3 days of grace (Art. 374) are added to the time specified in the bill.

395. The payee, instead of receiving the money from the drawee, may sell the bill to another, and he in turn may sell it again, and so on indefinitely. Any person who buys the bill is called the **Buyer** or **Remitter**. The person who owns the bill at any given time is the **Holder** or **Possessor.**

396. The payee and the several buyers, by writing their names across the back of the bill, become **Indorsers,** and responsible to the holder for the payment of the bill at *maturity*, that is, at the time when the bill becomes due.

397. Bills payable in a given time after sight are presented to the drawee, and if he agrees to pay, he writes the word "Accepted" and his name, with the date, across the face, or on some other part of the bill, and returns it to the holder. The drawee is then the **Acceptor,** and responsible for the payment of the bill when due.

398. Bills of exchange, payable after sight, like promissory notes, are subject to a discount for the term of credit, the discount being computed on the face of the bill.

399. Written Exercises.

$1000. BOSTON, May 17, 1895.

At sight, pay Sam'l Tyng, or order, one thousand dollars, value received, and charge the same to my account. ALFRED DWIGHT.

To Messrs. Monroe & Dale,
Merchants, Chicago.

1. What is the cost of this draft at $\frac{3}{4}$ % premium?

$$\$1000 \times 0.00\tfrac{3}{4} = \$7.50, \text{ Premium.}$$
$$\$1000 + \$7.50 = \$1007.50, \text{ Ans.}$$

$320. SAN FRANCISCO, May 8, 1895.

Sixty days after sight, pay to W. Pool, or bearer, three hundred twenty dollars, value received, and charge the same to the account of FRANK ADAMS & CO.

To Albert Devoe,
New York.

2. What is the cost of this draft at $\frac{1}{2}$ % discount?

$$\$320 \times 0.005 = \$1.60, \text{ Discount.}$$
$$\$320 - \$1.60 = \$318.40, \text{ Ans.}$$

3. What is the cost of a draft on Omaha for $5328, at $\frac{3}{4}$ % discount?

4. What is the cost of a draft on Sacramento for $8750, at 1 % premium?

5. What ought I to pay for a draft of $640, if exchange is at a discount of $\frac{3}{4}$ %?

6. If the rate of exchange is $\frac{1}{2}$ % premium, what will a draft of $935 cost?

7. My agent in St. Louis bought a draft on New York, at $\frac{1}{4}$ % premium, for $8160; what was the face of the draft?

$$\$1 + \$0.0025 = \$1.0025, \text{ cost of draft of } \$1.$$
$$\$8160 \div 1.0025 = \$8139.65, \text{ Ans.}$$

8. A Philadelphia merchant bought a draft on New Orleans, at $\frac{1}{2}$ % discount, for \$ 2850; what was the face of the draft?

9. How large a draft can be bought for \$ 773.76, if exchange is $\frac{3}{4}$ % premium?

10. How large a draft can be bought for \$ 1876.57, if exchange is $\frac{1}{2}$ % discount?

11. What must be paid for a draft of \$ 1000, payable 60 days after sight, if exchange is at a premium of 1 %, and interest 6 %?

	\$ 1000	
Interest of \$ 1000 for 60 days..........	10	
Worth of the draft at par	\$ 990	
Premium at 1 %	10	
Cost of draft..............................	\$ 1000	Ans.

With grace the interest is \$ 10.50; cost of draft \$ 999.50.

12. If interest is 7.3 %, and the rate of exchange is $\frac{3}{4}$ % premium, what is a draft of \$ 380, payable 60 days after sight, worth?

13. What is the face of a draft, payable 30 days after sight, that cost \$ 876.75, if exchange is $\frac{3}{4}$ % premium and interest 6 %?

Proceeds of \$ 1 discounted for 30 days is $\$1 - \$0.005 = \$0.995$.
Cost of draft of \$ 1 is $\$0.995 + \$0.0075 = \$1.0025$.
Face of draft is $\$876.75 \div 1.0025 = \$874.56$, Ans.

With grace the face of the draft is $\$876.75 \div 1.002 = \$875$.

14. If I pay \$ 325.05 for a draft payable 60 days after sight, what is the face of the draft if exchange is 1 % discount and interest 6 %? Ans. \$ 331.85

15. If a draft, payable 90 days after sight, costs \$ 782, when exchange is $\frac{1}{2}$ % premium and interest 8 %, what is its face?

16. A draft payable 45 days after sight was bought for \$ 1378, when exchange is $\frac{3}{4}$ % discount and interest 6 %. What was its face?

(For foreign exchange, see Appendix).

PARTNERSHIP.

400. Partnership is the association of two or more persons in business.

The association is called a **Company**, a **Firm**, or a **House**, and the members are called **Partners**.

The money or property invested is called the **Capital**, or the **Stock**.

401. Oral Exercises.

1. A and B form a partnership with a capital of $5000, of which A puts in $3000 and B $2000. They gain $1500; how shall they divide this gain?

Solution. A puts in $\frac{3}{5}$ and B $\frac{2}{5}$ of the capital; therefore, A ought to have $\frac{3}{5}$ of the $1500 gain, or $900; B ought to have $\frac{2}{5}$ of $1500, or $600.

2. A and B invest in a venture $25, of which A invests $15 and B $10. How ought a gain of $50 to be divided between A and B?

3. John Fay and his son bought a horse for $250, of which Mr. Fay paid $200 and his son $50. They sell the horse for $300. Find the father's and the son's share of the $300.

4. A, B, and C form a partnership; A puts in $4000, B $5000, and C $6000. They gain $3000; how ought the gain to be divided?

5. A, B, C, and D hire a horse and carriage together for 6 days at the rate of $2 a day. A drives the team alone 40 miles, B 30, C 100, and D 70. How much ought each to pay?

6. Divide $1 between John, Charles, and William so that John shall have 2 cents as often as Charles has 3 and William 5.

7. Mr. Poe owes Mr. Roe $400, Mr. Doe $300, and Mr. Lowe $200, and his whole property sells for $600. How ought the $600 to be divided between Roe, Doe, and Lowe?

8. Hardy and Hill hire a pasture for the season for \$15. Hardy keeps his horse in the pasture 12 weeks and Hill 8 weeks. What part of the \$15 ought each to pay?

9. A and B hire a pasture for \$80. A puts in 10 sheep for 7 weeks, B 30 sheep for 3 weeks. What part of the \$80 ought each to pay?

Solution. Pasturing 10 sheep for 7 weeks is equivalent to pasturing 70 sheep 1 week; pasturing 30 sheep 3 weeks is equivalent to pasturing 90 sheep 1 week. The whole number of sheep pastured is equivalent to 160 for 1 week, of which A had 70 and B 90. Therefore A ought to pay $\frac{70}{160}$, or $\frac{7}{16}$, and B $\frac{90}{160}$, or $\frac{9}{16}$, of the \$80; or A's share is \$35 and B's \$45. Hence,

402. When capital is invested for unequal times,

Each party's share is represented by the product of his capital by the time it is in.

10. If Finney and Brown gain \$1000, and Finney has \$500 invested for 7 months and Brown \$300 for 5 months, how ought the gain to be divided?

11. If I put in \$2000 for 10 months, how much ought my partner to put in for 8 months, that our shares may be equal?

403. Written Exercises.

12. A, B, C, and D freight a ship to Canton; A furnishes \$3500 worth of the cargo, B \$5500, C \$7500, and D \$11000. They gain \$5225; what is each one's share of the gain?

\$3500	$\frac{3500}{27500} = \frac{7}{55}$.
5500	$\frac{5500}{27500} = \frac{1}{5}$.
7500	$\frac{7500}{27500} = \frac{3}{11}$.
11000	$\frac{11000}{27500} = \frac{2}{5}$.
\$27500	

$\frac{7}{55}$ of \$5225 = \$665, A's share
$\frac{1}{5}$ " = 1045, B's "
$\frac{3}{11}$ " = 1425, C's "
$\frac{2}{5}$ " = 2090, D's "
Ans.

Of the cargo, \$27500, A furnishes \$3500, that is, $\frac{3500}{27500} = \frac{7}{55}$; B \$5500, that is, $\frac{5500}{27500} = \frac{1}{5}$; C \$7500, that is, $\frac{7500}{27500} = \frac{3}{11}$; and D \$11000, that is, $\frac{11000}{27500} = \frac{2}{5}$. Hence, of the gain, \$5225, A should have $\frac{7}{55}$, B $\frac{1}{5}$, C $\frac{3}{11}$, and D $\frac{2}{5}$. Hence,

404. To find the gain, or loss, of each partner,

Rule.

Take the same fraction of the gain, or loss, that each partner's capital is of the whole capital.

NOTE 1. The process can often be simplified by omitting a common factor from the several investments. Thus, in Ex. 12, the common factor 500 can be omitted from each partner's share, and the shares will be represented by 7, 11, 15, and 22, and the whole cargo by 55.

13. A man who owes me \$ 310.55 fails. He owes in all \$ 10785, and his whole property realizes in cash \$ 6433, of which it takes \$ 177.70 to pay the expenses of settlement. How much shall I receive? What per cent will each creditor receive?

Ans. \$ 180.12; 58 %.

14. A and B engage in trade; A furnishes \$ 3000 for 10 months, and B \$ 5500 for 8 months. They lose \$ 1480; what is the loss of each? Ans. A's loss, \$ 600; B's, \$ 880.

NOTE 2. For the element of time, see Art. 402.

15. A, B, and C hire a pasture for \$ 91.50. A pastures 4 cows for 7 weeks, B 5 cows for 8 weeks, and C 6 cows for 9 weeks; what part of the rent shall each pay?

16. A, B, and C form a partnership; A furnishes \$ 700 for 11 months, B \$ 1100 for 8 months, and C \$ 900 for 12 months. They gain \$ 1365; what is each one's share of the gain?

17. Jan. 1, 1895, A, B, and C form a partnership for 1 year, and each puts in \$ 2000; May 1, A puts in \$ 500 more; July 1, B withdraws \$ 400, and C adds \$ 500; Oct. 1, A withdraws \$ 1000, and C \$ 800, and B adds \$ 1200. Having gained \$ 3845, at the close of the year the partnership is dissolved. What is each partner's share of the whole?

18. Jan. 1, 1894, A began business with a capital of $4000. Oct. 1, 1894, he took in B as a partner, with a capital of $3000. June 1, 1895, they admit C into the partnership, with a capital of $3500. Jan. 1, 1896, they dissolve partnership, having gained $6620. What should each partner take on retiring from the firm?

19. A, B, and C bought a horse for $100, and sold him for $150, by which A gained $18, and B $19. How much had A, B, and C each paid for the horse?

20. A, B, and C hire a pasture for $87. A pastures 8 horses for 7 weeks, B 15 oxen for 10 weeks, and C 40 cows for 12 weeks. If 5 cows are reckoned as 3 oxen, and 3 oxen as 2 horses, what part of the rent shall each pay? How many dollars?

21. Farley and Howe enter into partnership. Jan. 1, Farley puts in $4000, but Howe puts in nothing until April 1; what shall he then put in that the partners may be entitled to equal shares of the profits at the close of the year?

22. A, B, and C hire a pasture for $252.84. A puts in 10 oxen for 15 weeks, 18 cows for 14 weeks, and 96 sheep for 16 weeks; B puts in 8 oxen for 16 weeks, 15 cows for 12 weeks, and 72 sheep for 15 weeks; C puts in 24 oxen for 8 weeks, 21 cows for 12 weeks, and 48 sheep for 15 weeks. Now, if 8 sheep are reckoned as 1 cow, and 3 cows as 2 oxen, what is the cost a week for a sheep? a cow? an ox? How many dollars does each man pay for sheep? cows? oxen? How many dollars in all does each man pay?

23. A and B form a partnership for a year, A putting in $3000 and B $1800. If A takes out $1000 at the end of 5 months, how much more must B put in at the end of 7 months to receive one half of the profits at the end of the year?

24. X, Y, and Z enter into partnership Jan. 1, 1892, X putting in $5000, Y $3000, and Z $4000. April 1, X puts

in $1000 more, Y $1500, and Z takes out $500. Sept. 1, X takes out $500, Y puts in $1000, and Z $1000. At the end of the year they settle, having gained $6880; what is each partner's share of the gain?

25. A, B, and C traded in company. A at first put in $1200, B $1500, and C $1600; in 4 months A put in $300 more and B $400, and C took out $500; in 8 months from the commencement of business, A withdrew all his stock but $600, B put in as much as he at first put in, and C withdrew $500. At the end of a year they found they had gained $12\frac{1}{2}\%$ on the largest total stock at any one time in trade. How many dollars ought each to take if the firm is dissolved?

26. A, B, and C form a partnership in which A is a silent partner who puts in $50000, B puts in $20000, and C nothing but his services. B is to receive for services $2000 a year, and C, who is the business manager, is to have $5000 a year. If they make $14000 a year for two years, and then dissolve the partnership, what should each receive?

Ans. A, $60000; B, $28000; C, $10000.

27. If of three partners A puts in $75000, B $60000, and C $40000, and A is to draw a salary of $2000 a year, B of $2500, and C of $3000, how ought a gain of $13500 at the end of one year to be divided?

Ans. A, $4571.43–; B, $4557.14+; C, $4371.43–.

28. January 1, A, B, C, and D formed a partnership, each putting in $20000. April 1, A put in $4000 more; June 1, B took out $5000; and October 1, C took out $6000. Each was to have for services $100 a month. A was sick and away from May 15 to July 15, and C took a vacation from June 20 to September 20, but B and D did not lose a day. If they gained during the year $13000, how ought it to be divided?

Ans. A, $3546.34+; C, $2948.14+;
B, $3091.31–; D, $3414.21–.

RATIO.

406. Oral Exercises.

1. If John is 16 years old and his father 48, how do their ages compare?

2. What part of 75 is 15?

3. I have $35 and James has $5; how many times as many dollars as James have I?

4. Compare 18 with 9; with 6; with 3; with 2; with 1; with 12; with 15.

407. Ratio *is the relation of one quantity to another of the same kind;* or, it is the quotient obtained by dividing one quantity by another of the same kind.

408. Ratio is usually indicated by two dots; thus, 8 : 4 expresses the ratio of 8 to 4.

The two quantities compared are the *terms* of the ratio; the first term is the *antecedent*, the second the *consequent*, and the two terms, collectively, a *couplet.*

409. The *antecedent* is a *dividend*, and the *consequent* a *divisor*.

Thus, $8 : 4 = 8 \div 4 = \frac{8}{4} = 2$,
and $3 : 12 = 3 \div 12 = \frac{3}{12} = \frac{1}{4}$.

410. An *inverse*, or *reciprocal*, ratio is a ratio inverted; that is, the antecedent becomes the consequent and the consequent the antecedent. Thus, the inverse ratio of 4 : 9 is 9 : 4.

411. The antecedent and consequent being a dividend and divisor, it follows that any *change in the antecedent causes a like change in the value of the ratio, and any change*

in the consequent causes an opposite change in the value of the ratio (Art. 81 and 118). Hence,

1. *Multiplying the antecedent multiplies the ratio; and dividing the antecedent divides the ratio* (Art. 79, a and b).

2. *Multiplying the consequent divides the ratio; and dividing the consequent multiplies the ratio* (Art. 79, c and d).

3. *Multiplying both antecedent and consequent by the same number, or dividing both by the same number, does not affect the ratio* (Art. 80, a and b).

412. The antecedent, consequent, and ratio are so related to each other, that if *two* of them are given the other can be found; thus: in $12 : 3 = 4$, we have

$$\text{Antecedent} \div \text{Consequent} = \text{Ratio},$$
$$\text{Antecedent} \div \text{Ratio} = \text{Consequent},$$
$$\text{Consequent} \times \text{Ratio} = \text{Antecedent}.$$

413. When there is but one antecedent and one consequent the ratio is said to be *simple;* thus, $15 : 5 = 3$, is a simple ratio.

414. When the corresponding terms of two or more simple ratios are multiplied together the resulting ratio is said to be *compound;* thus, by multiplying together the corresponding terms of the simple ratios,

$$6 : 3 = 2,$$
$$8 : 2 = 4,$$
$$10 : 2 = 5,$$

we have the compound ratio, $\overline{6 \times 8 \times 10 : 3 \times 2 \times 2 = 40}$.

5. **What is the ratio of 30 to 6?**

6. **What is the ratio of 4 to 11?**

7. **What is the *inverse* ratio of 20 to 4?** Ans. $\frac{4}{20} = \frac{1}{5}$

8. What is the inverse ratio of 3 to 7?

9. What is the ratio compounded of 9 to 4 and 6 to 5?

10. Which is the greater, the ratio of 8 to 7, or of 17 to 14?

11. Which is the greater, the ratio of 7 to 4, or of 18 to 13?

12. If 63 is the antecedent and 9 the consequent, what is the ratio?

13. If 72 is the antecedent and 8 the ratio, what is the consequent?

14. If 14 is the consequent and 4 the ratio, what is the antecedent?

PROPORTION.

415. Oral Exercises.

15. How does the ratio of 16 to 8 compare with the ratio of 24 to 12?

16. How does the ratio of 8 to 10 compare with 12 to 15?

416. Proportion is an equality of ratios. Thus, $8:6 = 4:3$ is a proportion.

The equality of two ratios is indicated by the sign of equality (=), or by four dots (: :). Thus,

$$12 : 3 = 8 : 2, \text{ or } 12 : 3 :: 8 : 2,$$

reads, 12 to 3 equals 8 to 2, or 12 is to 3 as 8 is to 2.

417. The first and last terms of a proportion are called *extremes;* the second and third, *means.*

418. If the means are the same number, this number is called a *mean* proportional. Thus, in $9:6=6:4$, 6 is a mean proportional between 9 and 4.

419. *In a proportion the product of the extremes is equal to the product of the means.*

For a proportion, as

$$3 : 4 = 6 : 8,$$

may be written

$$\frac{3}{4} = \frac{6}{8}.$$

Reducing these fractions to a common denominator, we have

$$\frac{3 \times 8}{32} = \frac{6 \times 4}{32}.$$

As these fractions are equal, and their denominators the same, their numerators must be equal, or $3 \times 8 = 6 \times 4$. But 3 and 8 are the extremes, and 6 and 4 the means.

420. It follows that *either extreme is equal to the product of the means divided by the other extreme; and either mean equal to the product of the extremes divided by the other mean.* That is, if any three terms of a proportion are given, the remaining term can be found. Hence, the name, *Rule of Three. A mean proportional between two numbers is equal to the square root* (Art. 438) *of their product.*

(17.) $8 : 2 = 12 : ?$
(18.) $6 : 9 = ? : 12$.
(19.) $5 : ? = 2 : 9$.
(20.) $? : 16 = 8 : 14$.
(21.) $\frac{1}{5} : \frac{1}{10} = \frac{1}{7} : ?$
(22.) $\frac{1}{8} : 10 = ? : 30$.
(23.) $7 : ? = 20 : \frac{1}{5}$.
(24.) $? : \frac{1}{3} = \frac{1}{4} : \frac{1}{12}$.

421. Written Exercises.

25. If 7 pounds of rice cost 63 cents, what will 11 pounds cost?

$7 : 11 = 63 :$ Ans.

Ans. 99 cents.

The cost evidently will increase as the quantity increases, that is, 11 pounds will cost as many times 63 cents as 11 pounds is times 7 pounds, or 7 pounds is to 11 pounds as 63 cents is to the required cost.

26. If 8 men can build a certain wall in 25 days, how long will it take 12 men to build the same wall?

$$12 : 8 = 25 : \text{Ans.}$$

Ans. $16\frac{2}{3}$ days.

In this case the number of days decreases in the same ratio as the number of men increases.

NOTE 1. When *more* requires *more*, as in Ex. 25, or *less* requires *less*, the proportion is called *direct*. When *more* requires *less*, as in Ex. 26, or *less* requires *more*, the proportion is called *inverse*. Thus, the more men employed the more work, and the less men the less work, in a given time; that is, the amount of work varies *directly* as the number of men. But the more men the less time, and the less men the more time, to do a given piece of work; that is, the time varies *inversely* as the number of men.

422. To solve examples by proportion,

Rule.

Make the number that is of the same kind as the required answer the third term.

If from the nature of the question the answer ought to be greater than this third term, of the two remaining terms make the greater the second, and the less the first; but if the answer ought to be less than the third term, make the less the second, and the greater the first.

Divide the product of the second and third terms by the first.

NOTE 2. If possible always cancel factors from the first with equal factors from either the second or third terms (Art. 80, b).

27. If a man earns \$48 in 4 months, how much will he earn in 9 months?

$$\cancel{4} : 9 = \$\cancel{48} : \text{Ans.}$$

$$\begin{array}{r} 12 \\ 9 \\ \hline \$108, \text{ Ans.} \end{array}$$

Since we are seeking for dollars, we make \$48 the third term; and then, as a man will earn more in 9 months than he will in 4 months, we make 9 the second term and 4 the first. Cancelling the equal factor 4 from the first and third terms, we have $\$12 \times 9 = \$108$, Ans.

28. If 20 bushels of wheat make 4 barrels of flour, how many bushels of wheat will be required to make 7 barrels of flour?

Ans. 35.

29. If a man can travel 120 miles in 3 days, how far can he travel in 10 days?

30. If 30 bushels of wheat make 6 barrels of flour, how many barrels of flour will 85 bushels of wheat make?

31. If a man travel 74 miles in 2 days, how long will it take him to travel 259 miles?

32. If 24 men perform a piece of work in 6 days, in how many days will 8 men perform the same work?

33. If a locomotive run 65000 miles in 8 weeks, how far, at that rate, would it run in 52 weeks?

34. If $30 pay for 20 yards of cloth, how many dollars will pay for 48 yards?

35. If 12 cords of wood cost $60, what will 28 cords cost?

36. If 5 cords of wood cost $25, how many cords may be bought for $95?

37. What will 8 tons of coal cost if 9 tons cost $54?

38. If 5 horses eat 16 bushels of oats in 6 weeks, how many bushels will 17 horses eat in the same time?

39. In how many days can 8 men build a house, if 14 men can build it in 82 days?

40. If the interest of $200 for 1 year is $12, what is the interest of $950 for the same time?

41. How many tons of coal can be bought for $84, when 6 tons cost $28?

42. How many tons of hay will 9 horses eat in 27 weeks, if 6 horses eat 15 tons in the same time?

43. If 8 horses eat a ton of hay in 22 days, how many horses will eat a ton in 32 days?

44. If $\frac{3}{7}$ of a ship cost $5679, what are $\frac{5}{7}$ of her worth?

45. If $\frac{3}{8}$ of a barrel of sugar cost $3.75, what will 9 barrels cost?

46. If a man walks 135 miles in 5 days of 9 hours each, in how many days of 10 hours each will he walk 135 miles?

47. How many yards of cloth $\frac{7}{8}$ of a yard wide are equal to 30 yards $1\frac{1}{4}$ yards wide?

48. Lent a friend \$600 for 8 months; afterwards he lent me \$500. How long may I keep it to balance the favor?

49. If, when flour is \$9 a barrel, a 4-cent loaf weighs 8 ounces, what ought it to weigh when flour is \$5 a barrel?

50. If 8 men can do a piece of work in 15 days, how many men must be added to the number to do it in 5 days?

51. A certain house was built by 40 workmen in 48 days, but, being burned, it is required to rebuild it in 30 days; how many men must be employed?

52. If 18 horses eat 85 bushels of oats in 4 weeks, how many bushels will 16 horses eat in the same time?

53. A garrison of 1200 men has provisions for 12 months; how long will the same provisions last if the garrison is reinforced by 150 men?

54. A ship's crew of 16 men has food for 56 days; how many men must be discharged that it may last 8 days longer?

55. If 0.95 of a ship cost \$5000, what will 0.675 of it cost?

56. At \$14 for a hundred pounds, what is the cost of $62\frac{1}{2}$ pounds?

57. If a steeple 150 feet high casts a shadow 210 feet, what is the length of the shadow cast, at the same time, by a staff 5 feet high?

NOTE 3. Since *each* of the three terms in the above example is in *feet*, the learner may be uncertain which number to place as the *third term;* but he has only to notice that he is required to find the length of a *shadow:* hence, the third term should be the number expressing the length of *shadow* in the given example, that is, 210 feet.

58. If a staff 5 feet long casts a shadow 7 feet, what is the height of a steeple which, at the same time, casts a shadow 210 feet?

59. If a staff 5 feet long casts a shadow 7 feet, how long is the shadow, cast at the same time, by a steeple which is 150 feet high?

60. If a steeple 150 feet high casts a shadow 210 feet, what is the height of a staff which, at the same time, casts a shadow 7 feet?

61. If 75 pounds of cheese are worth as much as 30 pounds of butter, how many pounds of cheese will pay for 24 pounds of butter?

62. If the interest of $600 for 6 months is $15, what principal will gain $64 in the same time?

63. If a man's salary amounts to $7200 in 4 years, what will it amount to in 15 years?

64. If a man's salary amounts to $18900 in 15 years, in how many years will it amount to $7560?

65. If $15\frac{1}{2}$ yards of silk that is $\frac{3}{4}$ of a yard wide will make a dress, how many yards of muslin that is $1\frac{1}{8}$ yards wide will be required to line it?

66. If 3 men can mow 10 a. 136 sq. rd. of grass in 4 days, by working 6 hours a day, how many days will it take them to mow the same area if they work only 4 hours a day?

67. If $\frac{3}{7}$ of an acre of land is worth $22.50, what is the value of $25\frac{3}{14}$ acres, at the same price?

NOTE 4. All the examples given above can be solved without the aid of Proportion, by what is sometimes called *Analysis* in distinction from *Proportion.*

COMPOUND PROPORTION.

423. Compound Proportion is an *equality* of two ratios, one of which is *compound* and the other *simple;* thus,

$$\left.\begin{array}{r} 3:12 \\ 16:\ \ 2 \end{array}\right\} = 18:9, \text{ is a compound proportion;}$$

$$\text{and } 48:24 \ = 18:9, \text{ is the same reduced to a simple form.}$$

NOTE. The *compound ratio* may consist of any number of couplets.

424. Written Exercises.

68. If 6 men in 8 hours thresh 30 bushels of wheat, in how many hours will 2 men thresh 5 bushels?

$$\left.\begin{array}{r} 2:6 \\ 30:5 \end{array}\right\} = 8 : \text{Ans.}$$

As the required answer is to be in hours, we make 8 hours the third term. To do the same work it will take 2 men longer than it will 6 men; hence, we make 2 the first and 6 the second term. To thresh 5 bushels will take less time than to thresh 30 bushels; hence, we make 30 the first and 5 the second term. The proportion then stands

$$2 \times 30 : 6 \times 5 = 8 : 4, \text{ Ans. } \quad \text{Hence,}$$

425. To solve questions in Compound Proportion,

Rule.

Write that given number which is of the same kind as the required answer for the third term. Take any two of the remaining terms of the same kind, and, considering the question as depending on these alone, arrange them as in simple proportion; arrange each pair of like terms by the same principles. Divide the product of the second and third terms by the product of the first terms.

NOTE 1. If possible always cancel factors in the second and third terms with equal factors in the first terms.

69. If 6 men in 15 days earn \$135, how many dollars will 9 men earn in 18 days?

$$\left.\begin{array}{r} \cancel{6} : 9 \\ \cancel{15} : \cancel{18} \\ \cancel{5} \quad \cancel{3} \end{array}\right\} = \$\underset{27}{\cancel{135}} : 9 \times \$27 = \$243, \text{ Ans.}$$

70. If 4 men, in 24 days of 9 hours each, build a wall 40 ft. long, 9 ft. high, and 4 ft. thick, in how many days of 6 hours each can 8 men build a wall 60 ft. long, 12 ft. high, and 5 ft. thick?

$$\left.\begin{array}{rcl}\cancel{8}\text{ men} & : & \cancel{4}\text{ men}\\ \cancel{2}\ \cancel{6}\text{ hours} & : & \cancel{9}\text{ hours}\\ \cancel{40}\text{ ft. long} & : & 3\ \cancel{60}\text{ ft. long}\\ \cancel{9}\text{ ft. high} & : & 3\ \cancel{12}\text{ ft. high}\\ \cancel{4}\text{ ft. thick} & : & 5\text{ ft. thick}\end{array}\right\} = \begin{array}{l}\cancel{24}\text{ days} : 45\text{ days, Ans.}\\ \cancel{3}\end{array}$$

71. If the freight for 15 boxes of sugar, each weighing 300 pounds, 40 miles, cost $6, what must be paid for carrying 50 boxes, weighing 360 pounds each, 95 miles?

72. If a family of 8 persons spend $900 in 6 months, how many dollars will be required for a family of 12 persons for 16 months?

73. If a family of 8 persons spend $900 in 6 months, how many months will $3600 sustain a family of 12 persons?

74. If a family of 8 persons spend $900 in 6 months, how large a family may be sustained 16 months for $3600?

75. If $100 gain $6 in 1 year, what will $600 gain in 9 months?

76. If $600 gain $27 in 9 months, what is the rate?

77. If $100 gain $6 in 1 year, in what time will $600 gain $27?

78. At 6 % what principal will gain $27 in 9 months?

79. If 12 horses eat 10 bushels of oats in 8 days, how many bushels will 30 horses eat in 40 days?

80. Of a chimney, which is to be 45 feet high, 9 feet was completed by 3 men in 2 days; how many men must be employed to build the remainder of it in 8 days?

81. If a 5-cent loaf weighs 9 oz. when wheat is $1 a bushel, how much bread ought to be bought for $0.50 when wheat is worth $0.75 a bushel?

82. If 4 barrels of flour serve a family of 6 persons 12 months, how many barrels will serve a family of 15 persons 18 months?

83. If 9 men dig a trench 84 ft. long, 5 ft. wide, and 3 ft. deep, in 4 days, how many men can dig a trenoh 420 ft. long, 3 ft. wide, and 2 ft. deep, in 6 days?

NOTE 2. All the dimensions of a surface, or a solid, always belong in the same term of a proportion; so that, having decided where one dimension belongs, the others can be written under this one at once. Thus, in Ex. 83, having decided that the 84 ft. (length) belongs in the first term, write the 5 ft. (width) and the 3 ft. (depth) under the 84; and under the 420 write the 3 and the 2.

84. If 4 men dig a trench 60 ft. long, 4 ft. wide, and 2 ft. deep in $6\frac{1}{2}$ days, how many days will it take 7 men to dig a trench 480 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. deep?

85. If 8 men, working 9 hours a day, dig a trench 45 rd. long, 4 ft. wide, and 3 ft. deep in 6 days, how many men, working 10 hours a day, can dig a trench 225 rd. long, 5 ft. wide, and 2 ft. deep in 3 days?

86. If 6 men, in 8 days of 10 hours each, build a wall 60 ft. long, 4 ft. wide, and $5\frac{1}{4}$ ft. high, how many days of $8\frac{1}{2}$ hours each will it take 15 men to build a similar wall 140 ft. long, $4\frac{1}{2}$ ft. wide, and $4\frac{1}{4}$ ft. high?

87. A garrison of 600 men have bread enough to allow 16 ounces a day to each man for 15 days; but, the garrison being reinforced by 200 men, how many ounces a day may each man have in order that the bread may last 20 days?

88. If a man travels 180 miles in 6 days, travelling 10 hours each day, how many miles will he go in 18 days, travelling, at the same rate, only 9 hours each day?

89. If 2 compositors, in 3 days of 10 hours each, set type for 40 pages, each page consisting of 34 lines of 48 letters each, how many compositors will set 576 pages of 40 lines of 51 letters each, in 12 days of 9 hours each?

90. If it takes 22 reams of paper to make 1000 copies of a book of 11 sheets, how many reams will be required to make 4500 copies of a book of 7 sheets?

91. If 18 horses or 15 oxen eat 3 tons of hay in 8 weeks, how much hay will 28 horses and 35 oxen eat in 9 weeks?

92. If a man, walking 11 hours a day for 9 days, travels 297 miles, in how many days of 10 hours each would he walk 540 miles, travelling at the same rate?

93. If 900 tiles, each 8 inches square, will pave a court, how many tiles that are 12 inches long and 6 inches wide will pave another court which is 3 times as long and half as wide?

94. If the freight for 8 hhd. of sugar, each weighing 700 lbs., 80 miles, cost $20, what must be paid for carrying 70 hhd., each weighing 350 lbs., 400 miles?

95. If 4 men, in 18 days of 10 hours each, build a wall 40 rd. long, 7 ft. high, and 3 ft. thick, how many men will be required to build a wall 49 rd. long, 6 ft. high, and 4 ft. thick, in 14 days of 9 hours each?

96. If $400 gain $15 in 9 months, what is the rate per cent?

97. If 8 shares in a bank yield their owner $27.50 in 3 months, how much will 15 shares yield in 3 years?

98. If 24 men can build a wall 75 rd. long, 9 ft. high, and 2 ft. thick in 16 days, working 10 hours a day, how many days will it take 80 men, working 8 hours a day, to build a similar wall, 400 rd. long, 8 ft. high, and $4\frac{1}{2}$ ft. thick?

99. If 5 compositors in 8 days of 10 hours each set type for 80 pages, each page containing 32 lines of 42 letters each, how many compositors will it take to set 480 pages of 28 lines of 40 letters in 5 days of 8 hours each?

100. At 5 % what principal will gain $359.20 in 4 years?

101. If a 4-cent loaf of wheat bread weighs 7 oz. when flour is $6.50 a barrel, what ought a 5-cent loaf to weigh if flour is $5 a barrel?

102. If it takes 25 reams of paper to make 1500 copies of a book of 8 sheets, how many copies of a book of 15 sheets can be made from 48 reams of paper?

INVOLUTION.

427. A **Power** of a number is the product obtained by taking the number *any number of times* as a *factor.*

428. **Involution** is the process of raising a number to a power.

The number involved is the *first power* of itself. It is also the *root* of the other powers.

429. The **Index** or **Exponent** of a power is a figure placed at the right and a little above the root *to show how many times it is used as a factor;* thus,

$4 \times 4 = 16 = 4^2$, or, the 2d power, or square of 4.
$4 \times 4 \times 4 = 64 = 4^3$, or, the 3d power, or cube, of 4.
$4 \times 4 \times 4 \times 4 = 256 = 4^4$, or, the 4th power of 4.
$4 \times 4 \times 4 \times 4 \times 4 = 1024 = 4^5$, or, the 5th power of 4.

Hence,

430. To involve a number to any required power,

Rule.

Take the number as a factor as many times as there are units in the index of the required power.

431. Oral Exercises.

1. What is the 3d power of 6? Ans. $6^3 = 6 \times 6 \times 6 = 216.$
2. What is the 5th power of 3? Ans. 243.
3. What is the 5th power of 4?
4. What is the 7th power of 3?
5. What is the 3d power of 15?
6. What is the 3d power of $\frac{4}{6}$?

$$\frac{4}{6} = \frac{2}{3};\ \left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2^3}{3^3} = \frac{8}{27},\ \text{Ans.}$$

7. What is the 2d power of $\frac{4}{14}$?

8. What is the 2d power of 0.002?

9. What is the square of $2\frac{1}{4}$?

$$2\frac{1}{4} = \frac{9}{4}; \ \left(\frac{9}{4}\right)^2 = \frac{81}{16} = 5\frac{1}{16}, \text{ Ans.}$$

NOTE 1. The number of decimal places in the power of a decimal is equal to the number of decimal places in the root multiplied by the index of the power (Art. 161).

NOTE 2. The powers of a number greater than unity are greater than the number itself, and the powers of a number less than unity are less than the number itself; thus, the cube of 2 is 8, which is greater than 2; and the square of $\frac{3}{2}$ is $\frac{9}{4}$, which is greater than $\frac{3}{2}$; but the square of $\frac{2}{3}$ is $\frac{4}{9}$, which is less than $\frac{2}{3}$.

EVOLUTION.

432. A **Root** of a number is one of the equal factors whose continued product is that number.

433. **Evolution** is the resolving of a quantity into as many equal factors as there are units in the index of the root.

434. There are two methods of indicating a root, one by means of the *radical sign,* $\sqrt{}$, and the other by means of a *fractional index.*

The figure placed over the radical sign is the *index* of the root, and is always the same as the denominator of the fractional index; thus, the *cube root of* 8 is $\sqrt[3]{8}$, or $8^{\frac{1}{3}}$.

The *square root* of the *cube* of 4, or the *cube* of the *square root* of 4, is $\sqrt{4^3}$, or $4^{\frac{3}{2}}$.

If no number is over the radical sign, 2 is understood.

435. **Evolution** is the reverse of **Involution.**

In Involution the root is given and the power required.

In Evolution the power is given and the root required.

436. A *perfect power* is a number whose root can be found. A *perfect square* is a number whose square root can

be found. A *perfect cube* is a number whose cube root can be found.

Note. Very few numbers are perfect powers. In most numbers we can only find an *approximate* root. Thus, 3 is the *approximate* square root of 10.

437. Every power and every root of 1 is 1.

Note. The roots of a number less than unity are greater than the number itself, and the roots of any number greater than unity are less than the number itself; thus, $\sqrt{\frac{4}{9}} = \frac{2}{3}$, which is greater than $\frac{4}{9}$; $\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$, which exceeds $\frac{27}{64}$; but $\sqrt{\frac{64}{36}} = \frac{8}{6}$, which is less than $\frac{64}{36}$; $\sqrt[3]{8} = 2$, which is less than 8.

SQUARE ROOT.

438. *To find the* **Square Root** *of a number is to resolve it into two equal factors, that is, to find a number which, multiplied into itself, will produce the given number.*

Numbers,	1,	10,	100,	1000.
Squares,	1,	100,	10000,	1000000.

439. Comparing the numbers above with their squares, we see that the square of any integral number less than 10 has either one or two figures; the square of any integral number less than 100 and over 9 has either three or four figures, and so on. That is, the square of a number consists of twice as many figures as the root, or of one less than twice as many.

440. Hence, to find the number of figures in the square root of a given number,

Begin at units and mark off the number into periods of two figures each, and there will be one figure in the root for each period of two figures in the square, and another figure in the root if a figure remains at the left of the full periods of the square.

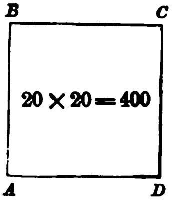
(For the Algebraic explanation of square root, see Appendix, page 362.)

441. Written Exercises.

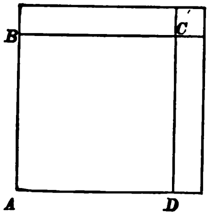
10. What is the length of one side of a square whose area is 576 square feet?

$$\begin{array}{r l}
 & 5\overset{\frown}{76}\ (24,\ \text{Ans.} \\
 & 4 \\
44) & \overline{176} \\
 & \underline{176} \\
 & 0
\end{array}$$

As 576 consists of three figures, its square root has two figures, and the square of the tens of the root must be in the 5 (hundreds). The greatest square in 5 (hundreds) is 4 (hundreds); therefore, the tens figure of the root must be 2.



Let *A C* represent the square whose side *A B* is 20 feet in length and area 400 square feet. Of the 576 square feet there remain 176 to be added to the square *A C*. As a square has two dimensions, to keep the figure a square both the length and the breadth must be increased equally, that is, equal additions must be made on *four* sides, or, more conveniently, on *two* adjacent sides. If we know the area of a rectangle and one dimension, we can find the length (Art. 201) by dividing the area by the given dimension. Now we know the length of the two sides, viz. $20 \times 2 = 40$, and this is sufficiently near the *whole* length of the additions to serve as a *trial divisor*. $176 \div 40 = 4 +$. If we place upon the two adjacent sides *B C*, *C D* additions 4 feet in width, we shall find that to complete the square there is lacking at *C* a small square whose length is exactly equal to the width of the two additions on *B C* and *C D*, that is, whose length is 4 feet. This small square increases the length of the two additions to 44 feet; that is, by adding this 4 feet to the trial divisor, 40 feet, we get the true divisor, 44 feet, and the area of the additions $44 \times 4 = 176$. Thus we get a square whose length is 20 ft. + 4 ft. = 24 ft., and whose area is 576 sq. ft.



NOTE. We do not divide 176 *square* feet by 40 *linear* feet. If the additions are 1 foot wide they will contain 40 *square* feet; they may, therefore, be as many feet wide as 40 *square* feet is contained times in 176 *square* feet.

442. Like reasoning applies, however many figures there are in the root. Hence, to find the square root of a number,

Rule.

1. Beginning at units, separate the given number into periods of two figures each.

2. Find the greatest square in the left-hand period, and write its root at the right of the given number.

3. Subtract the square of this root figure from the left-hand period, and to the remainder annex the next period for a dividend.

4. Double the root already found for a trial divisor, and omitting the right-hand figure of the dividend, divide and write the quotient as the next figure of the root, and also at the right of the trial divisor to form the true divisor.

5. Multiply the true divisor by this new root figure, and subtract the product from the dividend.

6. To the remainder annex the next period for a new dividend, double the part of the root already found for a trial divisor, and proceed as before until all the periods have been employed.

NOTE 1. The trial divisor being smaller than the true divisor, the quotient is frequently too large, *and a smaller number must be written in the root.*

11. What is the square root of 401956?

```
      40 19 56 (634, Ans.
      36
 123) 419
      369
1264) 5056
      5056
         0
```

12. What is the square root of 762129? Ans. 873.

13. What is the square root of 37636?

14. What is the square root of 67081?

```
      67081 (259, Ans.
      4
  45) 270
      225
 509) 4581
      4581
         0
```

In this example the trial divisor, 4, is contained in 27 six times; and the second remainder, 45, equals the divisor; still, the true root figure is 5.

15. What is the square root of 927369?

16. What is the square root of 9703225?

17. What is the square root of 21418384?

18. What is the square root of 31539456?

19. What is the square root of 89529444?

20. What is the square root of 5499025?

21. What is the square root of 29506624?

22. What is the square root of 12809241?

23. What is the square root of 16777216?

```
       16777216 (4096, Ans.
       16
  809) 7772
       7281
 8186) 49116
       49116
           0
```

When a root figure is 0, as in this example, we simply annex 0 to the trial divisor, and bring down the next period to complete the new dividend.

24. What is the square root of 16096144?

25. What is the square root of 36084049?

26. What is the square root of 36128505625?

NOTE 2. In finding the root of a *decimal*, the first period includes the *tenths* and the *hundredths*. If the last period is not full, annex 0.

27. What is the square root of 0.2116? Ans. 0.46.

28. What is the square root of 0.0729?

29. What is the square root of 28.3024?

30. What is the square root of 4444.8889?

31. What is the square root of 5.6?

```
     5.60 (2.36+,
     4
43)  160
     129
466) 3100
     2796
472)  30400
```

In this example we can only approximate to the root. By annexing successive periods of ciphers we can approximate nearer and nearer to the root. 2 is the square root to the *nearest unit;* 2.4, to the *nearest tenth;* and 2.37 (as the thousandths figure will be more than 5), to the *nearest hundredth.*

NOTE 3. Numbers that are perfect squares have the last figure of the last period 0, 1, 4, 5, 6, or 9.

32. What is the square root of 67 to the nearest thousandth? Ans. 8.185.

33. What is the square root of 473 to the nearest tenth?

34. What is the square root of 47.3 to the nearest hundredth?

443. To find the square root of a common fraction, or of a mixed number,

Rule.

Reduce the fraction or mixed number to its simplest form, and then take the square root of the numerator and denominator separately; or, if either term of the fraction, when reduced, is an imperfect square, reduce the fraction to a decimal and then proceed as in the previous examples.

35. What is the square root of $\frac{27}{48}$? $\sqrt{\frac{27}{48}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$, Ans.

36. What is the square root of $\frac{63}{448}$? Ans. $\frac{3}{8}$.

37. What is the square root of $\frac{28561}{117649}$?

38. What is the square root of $20\frac{1}{4}$?

$$\sqrt{20\tfrac{1}{4}} = \sqrt{\tfrac{81}{4}} = \tfrac{9}{2} = 4\tfrac{1}{2}, \text{ Ans.}$$

39. What is the square root of $18\frac{7}{9}$?

40. What is the square root of $40\frac{24}{25}$?

41. What is the square root of $\frac{4}{11}$? Ans. 0.603+.

42. What is the square root of $\dfrac{3\frac{1}{5}}{5\frac{2}{3}}$?

CUBE ROOT.

444. *To find the* **Cube Root** *of a number is to resolve it into three equal factors; that is, to find a number which, taken three times as a factor, will produce the given number.*

Numbers,	1,	10,	100,	1000.
Cubes,	1,	1000,	1000000,	1000000000.

445. Comparing the numbers above with their cubes, we see that the cube of any integral number less than 10 has less than four figures; the cube of any integral number less than 100 and more than 9 has less than seven and more than three figures; and so on. That is, the *cube* of a number consists of *three times* as many figures as the *root*, or of *one* or *two less* than three times as many.

446. Hence, to find the number of figures in the cube root of a given number,

Begin at the right, and mark off the number into periods of three figures each, and there will be one figure in the root for each period of three figures in the cube, and if there are one or two figures besides full periods in the cube, there will be a figure in the root for this part of a period.

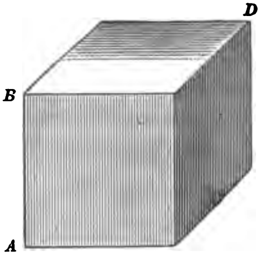
(For the Algebraic explanation of cube root, see Appendix, page 363.)

447. Written Exercises.

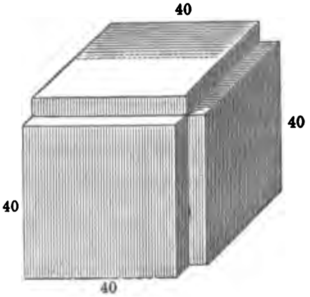
43. What is the length of one side of a cubical block that contains 74088 cubic inches?

		$74\widehat{088}$ (42, Ans.
Trial divisor,	4800	64
	240	10088
	4	10088
True divisor,	5044	0

As 74088 consists of five figures, its cube root has two figures, and the cube of the tens of the root must be in the 74 (thousands). The greatest cube in 74 (thousands) is 64 (thousands); therefore, the tens figure of the root must be 4.

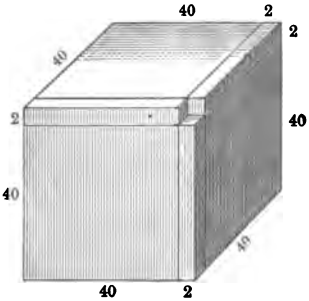


Let *A D* represent the cube whose linear dimension *A B* is 40 inches, and cubic contents 64000 cubic inches. Of the 74088 cubic inches there remain 10088 to be added to the cube *A D*. As a cube has three dimensions, to keep the figure a cube, the length, breadth, and thickness must be increased equally, that is, equal additions must be made on *six* sides, or, more conveniently, on three adjacent sides. If we know the volume of a rectangular prism and two of its dimensions, we can find the other dimension by dividing the volume by the product of the other two dimensions (Art. 211). Now we know the area of the face *B D*, viz. 40×40 sq. in. = 1600 sq. in. Therefore we know the area of the three adjacent faces, viz. 1600×3 sq. in. = 4800 sq. in. This is sufficiently near the area to be covered by the additions to serve as a trial divisor. $10088 \div 4800 = 2+$.



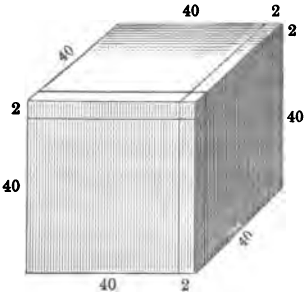
If now we place upon the three adjacent faces the three additions, each $40 \times 40 \times 2$, we shall find that to complete the cube there are lacking three corner-pieces, each 40 inches in length, 2 in breadth, and 2 in thickness, — that is, the length of each is the linear dimension of the original cube, and the other two dimensions are the same as the thickness of the three additions. There-

fore, the area to be covered by these three corner-pieces is $40 \times 2 \times 3$ sq. in. $= 240$ square inches. Placing these three corner-pieces in position, there still lacks, to complete the cube, a little cube whose linear dimension is 2 inches, the same as the thickness of the additions; and the area to be covered by this little cube is 2×2 square inches $= 4$ square inches. The total area to be covered by the additions necessary to keep the cubical form are, therefore,

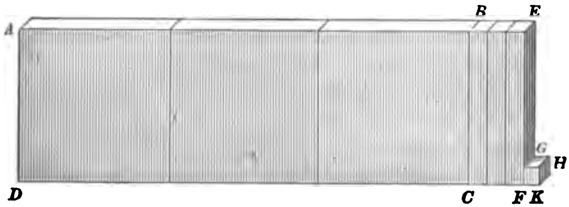


$$(40 \times 40 \times 3 + 40 \times 2 \times 3 + 2 \times 2) \text{ sq. in.} = 5044 \text{ sq. in.}$$

The other dimension of these seven pieces being 2 inches, their volume is 5044×2 cubic inches $= 10088$ cubic inches. Thus, the linear dimension, viz. 42 inches, of a cube whose volume is 74088 cubic inches has been found.



In this figure $A\,B\,C\,D$ represents the surface of the three additions that form the trial divisor, $B\,E\,F\,C$ the first addition to the trial



divisor, and $G\,H\,K\,F$ the second addition; and $A\,E\,G\,H\,K\,D$ represents the surface of all the additions that form the true divisor.

NOTE. We do not divide 10088 *cubic* inches by 4800 *square* inches. If the thickness of the additions is 1 inch, the cubic inches required to make the addi-

tions to the three faces will be 4800 *cubic* inches; the additions therefore will be as many inches thick as 4800 *cubic* inches is contained times in 10088 *cubic* inches.

448. If there are more than two figures in the root, the same reasoning applies. Hence, to find the cube root of a number,

Rule.

1. Beginning at units, separate the number into periods of three figures each.

2. Find by trial the greatest cube in the left-hand period write its root at the right of the given number, subtract the cube from the left-hand period, and to the remainder annex the next period for a dividend.

3. Square the root figure, annex two ciphers, and multiply this result by three for a trial divisor; divide the dividend by the trial divisor and write the quotient as the next figure of the root.

4. Multiply this root figure by the part of the root previously obtained; annex one cipher and multiply this result by three; add the last product and the square of the last root figure to the trial divisor, and the sum will be the true divisor.

5. Multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

6. Find a new trial divisor, and proceed as before, until all the periods have been employed.

NOTE. The notes in Art. 442, with slight modifications, are equally applicable in cube root.

44. What is the cube root of 21024576?

			21 $\overparen{024}$ $\overparen{576}$ (276, Ans.
			8
1st Trial Divisor	$20^2 \times 3 =$	1200	13024
	$20 \times 7 \times 3 =$	420	
	$7^2 =$	49	
1st True Divisor		1669	11683
2d Trial Divisor	$270^2 \times 3 =$	218700	1341576
	$270 \times 6 \times 3 =$	4860	
	$6^2 =$	36	
2d True Divisor		223596	1341576

The first trial divisor is contained 10 times in the dividend, yet the root figure is only 7. The true root figure can never exceed 9, *and must in all cases be found by trial.*

45. What is the cube root of 67917312?

	67 $\overparen{917}$ $\overparen{312}$ (408, Ans.
	64
480000	3917312
9600	
64	
489664	3917312

The 1st trial divisor, 4800, is larger than the 1st dividend, 3917; hence, we annex 0 to the root, 00 to the 1st trial divisor for the 2d trial divisor, and bring down the next period. The rule, followed literally, gives the same result.

NOTE 2. Prepare fractions and mixed numbers as in square root (Art. 443).

What is the value of each of the following expressions:

(46.) $\sqrt[3]{4330747}$? Ans. 163.
(47.) $\sqrt[3]{12326391}$? Ans. 231.
(48.) $\sqrt[3]{303464448}$? Ans. 672.
(49.) $\sqrt[3]{258474853}$?
(50.) $\sqrt[3]{82312875}$?
(51.) $\sqrt[3]{27081081027}$?
(52.) $\sqrt[3]{96702.579}$?
(53.) $\sqrt[3]{41.063625}$?
(54.) $\sqrt[3]{751.089429}$?
(55.) $\sqrt[3]{274\frac{5}{8}}$?
(56.) $\sqrt[3]{813\frac{1}{27}}$?
(57.) $\sqrt[3]{373\frac{31}{125}}$?

58. What is the cube root of 7854?

	$7\,\overset{\frown}{854}$ (19.8+,
	1
300	6854
270	
81	
651	5859
108300	995.000
4560	
64	
112924	903.392
	91.608

In this example we can only approximate to the root. By annexing successive periods of ciphers we can approximate nearer and nearer to the root. 20 is the cube root to the *nearest unit;* and (as the hundredths figure of the root will be more than 5) 19.9 is the cube root to the *nearest tenth.*

59. What is the cube root of 6 to the nearest hundredth?

60. What is the cube root of 450 to the nearest tenth?

61. What is the cube root of 0.07 to the nearest thousandth?

449. Miscellaneous Written Examples.

62. What is the 5th power of 15?

63. What is the 3d power of $5\frac{1}{5}$?

64. What is the square root of 85 to the nearest hundredth?

65. What is the square root of 8 to the nearest thousandth?

66. What is the square root of $\frac{75}{125}$?

67. What is the square root of $\dfrac{4\frac{1}{8}}{7\frac{1}{4}}$ to the nearest hundredth?

What is the value of each of the following expressions carried to three figures:

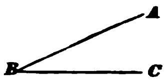
68. $\sqrt[3]{2}$?	73. $\sqrt[3]{846731.2}$?
69. $\sqrt[3]{84.3}$?	74. $\sqrt[3]{124876.5}$?
70. $\sqrt[3]{14.27}$?	75. $\sqrt[3]{278.3214}$?
71. $\sqrt[3]{4.6341}$?	76. $\sqrt[3]{1489123.1}$?
72. $\sqrt[3]{18.1245}$?	77. $\sqrt[3]{715432.27}$?

MENSURATION.

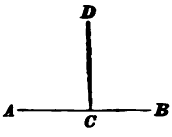
ANGLES.

450. An **Angle** is the difference in direction of two lines.

If the lines meet, the point of meeting, *B*, is called the *vertex;* and the lines *A B, B C*, the *sides* of the angle.

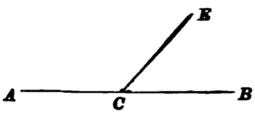


451. If a straight line meets another so as to make the adjacent angles equal, each of these angles is a *right angle;* and the two lines are perpendicular to each other. Thus, *A C D* and *D C B*, being equal, are right angles, and *A B* and *D C* are perpendicular to each other. A right angle contains 90°. (Art. 261.)



452. An **Acute Angle** is less than a right angle; as *E C B.*

453. An **Obtuse Angle** is greater than a right angle; as *A C E.*



Acute and obtuse angles are called oblique angles.

PLANE FIGURES.

454. A **Plane Figure** is a portion of a plane bounded by lines either straight or curved.

When the bounding lines are straight, the figure is a *polygon*, and the sum of the bounding lines is the *perimeter.*

455. Of Polygons, the simplest has three sides, and is called a *triangle;* one of four sides is called a *quadrilateral;* one of five, a *pentagon;* one of six, a *hexagon;* one of eight, an *octagon;* one of ten, a *decagon.*

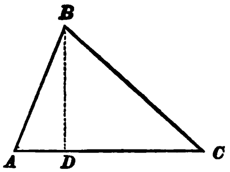
456. An **Equilateral Polygon** is one whose sides are equal each to each.

457. An **Equiangular Polygon** is one whose angles are equal each to each.

458. A **Regular Polygon** is one that is both equilateral and equiangular.

TRIANGLES.

459. A **Triangle** is a plane figure which is bounded by three lines; as *A B C.*



460. The *base* of a triangle (or of any other figure) is the side on which it is supposed to stand; as *A C.*

461. The *altitude* of a triangle is the perpendicular distance from the vertex of the angle opposite the base to the base, or to the base produced; as *B D.*

462. The area of a triangle is equal to *half the product of its base and altitude.*

NOTE. For demonstration of the principles of Mensuration, Geometry must be consulted.

1. The base of a triangle is 25 inches and the altitude 12 inches. What is the area? Ans. 150 sq. in.

2. The base is 18 ft. and the altitude 14 ft. What is the area?

3. The base is 10 yards and the altitude 15 yards. What is the area?

463. To find the area of a triangle when only the three sides are given,

From half the sum of the three sides subtract successively the three sides; find the square root of the product of these three remainders and the half sum.

4. Find the area of a triangle whose sides are respectively 5, 13, and 14 yards.

$$\frac{5+13+14}{2}=16.$$

$$\sqrt{(16-5)\times(16-13)\times(16-14)\times 16}=$$

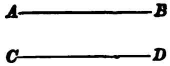
$$\sqrt{11\times 3\times 2\times 16}=\sqrt{1056}=32.5+ \quad \text{Ans. } 32.5 \text{ sq. yd.}$$

5. Find the area of a triangle whose sides are, respectively, 45, 55, and 60 feet.

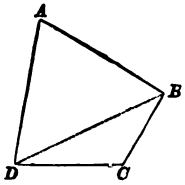
6. What is the area of a triangle whose sides are, respectively, 117, 221, and 250 rods?

QUADRILATERALS.

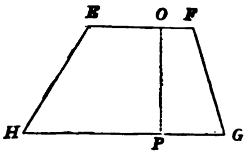
464. Parallel Lines are such as have the same direction; as AB and CD.



465. A **Trapezium** is a quadrilateral which has no two of its sides parallel; as $ABCD$.

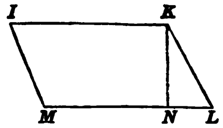


466. A **Diagonal** is a line joining the vertices of two angles of a polygon not adjacent; as DB.



467. A **Trapezoid** is a quadrilateral which has only two of its sides parallel; as *EFGH*.

468. The **Altitude** of a trapezoid is the perpendicular distance between its parallel sides; as *OP*.



469. A **Parallelogram** is a quadrilateral whose opposite sides are parallel; as *IKLM*.

470. The **Altitude** of a parallelogram is the perpendicular distance from the opposite side to the base; as *KN*.

471. The area of a parallelogram is equal to *the product of its base and altitude.*

NOTE. For the Rectangle see Arts. 197-201.

7. What is the area of a parallelogram whose base is 25 yards and altitude 8 yards?

472. The area of a trapezoid is equal to *half the product of its altitude and the sum of its parallel sides.*

8. The parallel sides of a trapezoid are 13 and 19 feet, and its altitude is 9 feet; what is its area? Ans. 144 sq. ft.

9. How many square feet can be covered with a board whose length is 12 feet, the wider end being 2 feet and the narrower 20 inches in width?

473. The area of a trapezium can be found *by finding the sum of the areas of the two triangles into which the trapezium is divided by a diagonal.*

[illegible]

A Board Foot [illegible]

[illegible]

13. How many feet, board measure, in a plank [illegible] feet long, [illegible] inches wide, and [illegible] inches thick?

[illegible] Ans.

14. Find the number of feet, board measure, in 25 boards, each 14 feet long, 9 inches wide, and $1\frac{1}{2}$ thick.

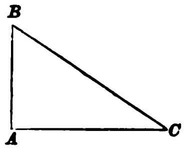
15. Find the number of feet, board measure, in 55 joists 3 by 4 (that is, the two smaller dimensions of the joists are 3 inches and 4 inches), each 16 feet long.

477. Boards less than an inch in thickness are reckoned the same as though they were an inch in thickness.

16. How many feet, board measure, in 75 boards, each 18 feet long, 8 inches wide, and $\frac{1}{2}$ inch thick? Ans. 900 bd. ft.

17. How many feet, board measure, in 45 boards, each 15 feet long, 10 inches wide, and $\frac{7}{8}$ inch thick?

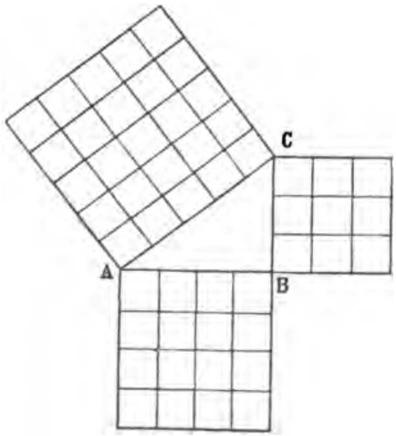
478. A *right-angled triangle* is one that has a *right angle*; as ABC.



The side BC opposite the right angle is called the *hypothenuse;* the other two sides, the *base* and *perpendicular.*

479. The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

Hence, the square of either of the two sides which form the right angle is equal to the square of the hypothenuse diminished by the square of the other side. This will be seen by counting the small squares in the square on the hypothenuse and those in the squares on the other two sides. Hence,



480. To find the hypothenuse when the base and perpendicular are given, *add the square of the base to the square of the perpendicular, and find the square root of the sum.*

481. To find either side about the right angle when the hypothenuse and the other side are given, *from the square of the hypothenuse subtract the square of the other given side, and find the square root of the remainder.*

18. The base of a right-angled triangle is 9 feet and the perpendicular is 12 feet; what is the hypothenuse?

$$\sqrt{9^2 + 12^2} = \sqrt{225} = 15. \qquad \text{Ans. } 15 \text{ ft.}$$

19. The hypothenuse of a right-angled triangle is 25 and the base is 15; what is the perpendicular?

$$\sqrt{25^2 - 15^2} = \sqrt{400} = 20, \text{ Ans.}$$

20. What is the distance on the floor from one corner to the opposite corner of a rectangular room 16 by 24 feet?

21. A wall 26 feet high has in front of it a ditch 14 feet wide. How long a ladder will it take to reach from the opposite side of the ditch to the top of the wall?

22. What is the length of the longest rod that, without bending, can be put into a box whose inner dimensions are 36, 24, and 12 inches?

23. If a pole 75 feet high is broken off 18 feet from the ground, how far from the foot of the pole will its top strike the ground?

24. How far from a tower 40 feet high must the foot of a ladder 50 feet long be placed that it may exactly reach the top of the tower?

25. If a rope 108 feet long, from a point in the street, will reach on one side to a window 75 feet high, and on the other to a window 45 feet high, how wide is the street?

26. The foot of a ladder 67 feet long stands 40 feet from a wall; how much nearer the wall must the foot be placed that the ladder may reach 10 feet higher?

CIRCLES.

482. The *circumference of a circle* (Art. 257) *is* 3.1416 *times its diameter;* or, *the diameter is equal to the circumference divided by* 3.1416.

$$\text{Circumference} = 3.1416 \times \text{Diameter}.$$
$$\text{Diameter} = \text{Circumference} \div 3.1416.$$

27. What is the circumference of a circle whose radius is 15 yards?

28. What is the diameter of a circle whose circumference is 57 rods?

29. What is the diameter of a tree whose circumference is 19 feet?

30. What is the circumference of a circular pond whose diameter is 56 yards?

483. The area of a circle is equal to *half the product of its circumference and its radius.* Or, it is equal to 0.7854 *times the square of its diameter.*

$$\text{Area} = \tfrac{1}{2}\ \text{Circumference} \times \text{Radius}.$$
$$\text{Area} = 0.7854 \times \text{Diameter}^2.$$

31. What is the area of a circle whose radius is 6 yards?

32. What is the area of a circle whose radius is 40 feet?

33. What is the area of a circle whose circumference is 18 inches?

34. What is the circumference of a circle whose area is 116 square feet?

35. The radii of two concentric circles are 40 and 54 feet; what is the area of the space bounded by their circumferences?

36. A has a circular lot of land whose diameter is 95 rods, and B a similar lot whose area is 750 square rods; compare these lots.

37. What is the difference between the perimeters of two lots of land each containing an acre, if one is a square and the other a circle?

SIMILAR SURFACES.

484. Similar Figures are figures that are of precisely the same form, without regard to their magnitude.

The *areas of similar figures are to each other as the squares of their corresponding lines;* and, conversely, *the corresponding lines of similar surfaces are to each other as the square roots of their areas.*

38. A has an acre of land one of whose sides is 20 rods in length; B has a piece of land of exactly similar form containing 169 acres. What is the length of the corresponding side of B's?

$1 : 169 = 20^2 : \text{Ans.}^2$, or, $1 : 13 = 20 : 260$. Ans. 260 rods.

39. Of two similar fields one contains 36 acres and the other 72. If a side of the first is 23 rods, what is the length of the corresponding side of the second?

40. The areas of two circles are to each other as 6 to 10, and the circumference of the smaller is 18 feet. What is the circumference of the greater?

41. The areas of two similar right triangles are respectively 361 and 225 square feet, and the hypothenuse of the first is 42 feet. What is the hypothenuse of the second triangle?

42. If water sufficient to fill a cistern in 50 minutes will pass through a pipe 2 inches in diameter, how long will it take to fill the cistern if the pipe is 5 inches in diameter, no allowance being made for friction?

43. If through a pipe 3 inches in diameter a vat can be filled in 4 hours and 16 minutes, what must be the diameter of a pipe through which it can be filled in 49 minutes, no allowance being made for friction?

44. If it costs $50 to paint a house 45 feet in length, what ought it to cost to paint a similar house 25 feet in length?

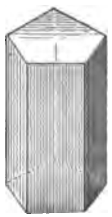
45. If a pipe 2 inches in diameter delivers 217 quarts of water a minute, how many quarts will a pipe 9 inches in diameter deliver a minute?

46. If a pipe 1.5 inches in diameter discharges 10 gallons a minute, how much will a pipe 3.5 inches in diameter discharge in the same time?

47. If we increase the length of the rope with which a horse is tied to a post in the centre of a field, from 12 to 15 feet, allowing 3 feet that the horse can reach beyond the end of the rope, how much have we increased the area over which he can feed?

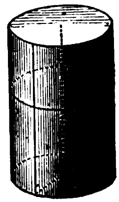
48. If it costs $ 45 to fence a field containing 10 acres, how much will it cost to enclose, with the same kind of fence, a field of similar shape containing 8 acres?

SOLIDS.



485. A **Prism** is a solid that has two equal, parallel faces, called *bases*, and all its other faces parallelograms.

NOTE. A prism is triangular, quadrangular, pentagonal, etc., according as its bases are triangles, quadrangles, pentagons, etc.



486. A round body whose diameter is the same throughout its entire length, and whose ends or bases are equal, parallel circles, is a **Cylinder.**

487. The *altitude* of a prism or cylinder is the perpendicular distance between its bases.

488. The surface of a prism or cylinder can be found by *multiplying the perimeter or circumference of the base by the length of the solid, and to the product adding the area of the two ends.*

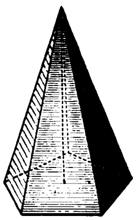
49. What is the surface of a prism whose length is 20 inches and base 6 inches square? Ans. 552 sq. in.

50. What is the surface of a cylinder whose length is 30 yards and diameter 2 yards?

489. The solid contents of a prism or cylinder is equal to *the product of the area of its base and its altitude.*

51. What are the contents of a cylinder whose length is 25 feet and diameter 15 feet?

52. What are the contents of a quadrangular prism whose length is 15 feet and base a rectangle 3 by 4 feet?



490. A **Pyramid** is a solid, having a polygon for its *base,* and for its other faces triangles which meet at a common point, called the *vertex* of the pyramid.

491. The *slant height* of a pyramid is the perpendicular distance from the vertex to one side of the base.

NOTE. A pyramid is triangular, quadrangular, etc., according as its base is a triangle, quadrangle, etc.

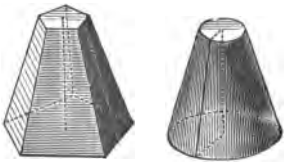
492. A solid that is like a pyramid, except that its *base* is a *circle,* is a **Cone.**

493. The *altitude* of a pyramid or cone is the perpendicular from the vertex to the base.

494. The contents of a pyramid or of a cone can be found by *multiplying the area of its base by one third of its altitude.*

53. What are the contents of a cone whose base is 15 inches in diameter and whose altitude is 42 inches?

54. What are the contents of a pyramid whose altitude is 18 inches, and whose base is a triangle having sides 4, 6, and 8 inches, respectively?



495. A **Frustum** of a pyramid or cone is the part remaining after a portion next the vertex has been cut off by a plane parallel to the base. The two ends are called the *upper and lower bases.*

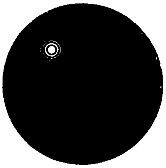
496. The contents of a frustum of a pyramid or cone can be found by *multiplying the sum of the areas of the two bases, added to the mean proportional between the two bases, by one third of the altitude of the frustum.*

55. What are the contents of the frustum of a quadrangular pyramid, whose altitude is 12 feet and whose bases are 6 feet and 4 feet square? Ans. 304 cu. ft.

56. What are the contents of the frustum of a cone whose altitude is 42 inches and whose bases are 24 inches and 18 inches in diameter?

57. If the inner diameter of the bottom of a tub is 21 inches, and of the top 24 inches, and the height is 12 inches, how many gallons will it hold?

497. A **Sphere** is a solid bounded by a curved surface, of which every point is equally distant from a point within, called the *centre*.



The *diameter* of a sphere is a line passing through the centre, and limited in both directions by the surface.

498. The surface of a sphere is equal to *the product of its circumference and its diameter.*

Surface of a Sphere = Circumference × Diameter.

58. What is the surface of a sphere whose diameter is 100 inches? Ans. 31416 sq. in.

59. What is the surface of a cannon ball whose diameter is 6 inches?

60. What is the surface of a globe whose diameter is 2 feet?

499. The contents of a sphere is equal to *the product of its surface and one third of its radius.* Or it is equal to 0.5236 ($\frac{1}{6}$ of 3.1416) *times the cube of its diameter.*

$$\text{Volume of a Sphere} = \text{Surface} \times \tfrac{1}{3}\ \text{Radius}.$$
$$= 0.5236 \times \text{Diameter}^3.$$

61. What are the contents of a sphere whose diameter is 100 inches? Ans. 523600 cu. in.

62. What is the volume of a cannon ball whose diameter is 8 inches?

63. What is the volume of a globe whose diameter is 3 feet?

64. What is the volume of a ball whose radius is 2 inches?

SIMILAR SOLIDS.

500. Similar Solids are solids which are of precisely the same form.

The *solid contents of similar solids are to each other as the cubes of their corresponding lines;* and, conversely, *the corresponding lines of similar solids are to each other as the cube roots of their contents.*

65. If a cannon ball 8 inches in diameter weighs 64 pounds, what is the weight of a cannon ball 4 inches in diameter?

$$8^3 : 4^3 = 64 \text{ lb.} : \text{Ans., or } 2^3 : 1^3 = 64 \text{ lb.} : \text{Ans.};$$
$$\text{that is, } 8 : 1 = 64 \text{ lb.} : 8 \text{ lb., Ans.}$$

66. If a sphere of silver $2\frac{1}{2}$ inches in diameter is worth \$64, what is the diameter of a sphere of silver that is worth \$1728?

$$64 : 1728 = \overline{2\tfrac{1}{2}\text{ in.}}^3 : \overline{\text{Ans.}}^3, \text{ or } \cancel{4} : \overset{3}{\cancel{12}} = 2\tfrac{1}{2}\text{ in.} : \text{Ans.}$$

Ans. $7\frac{1}{2}$ in.

67. Of two eggs of exactly the same shape, the longest axis of one is $2\frac{3}{4}$ inches, and of the other 2 inches. Compare their volumes.

68. If two men own a conical stack of hay 15 feet in height, what must be the altitude of the frustum left after one has taken half of the hay from the top?

69. If a pail whose height is 22 inches holds 10 quarts, what must be the height of a similar pail to hold 12 quarts?

70. If a horse whose girth is 6 feet weighs 950 pounds, what will a horse whose girth is 7 feet weigh?

71. If the altitude of a box holding a bushel is 20 inches, what is the altitude of a box of similar shape holding 19.2 quarts?

501. Miscellaneous Written Examples.

72. What is the area of a trapezoid whose altitude is 18 feet and parallel sides 25 and 35 feet respectively?

73. Find the area of a trapezium whose sides, taken in order, are 28, 23, 24, 15 feet, and the diagonal from the end of the first to the end of the second side is 30 feet.

74. How many feet, board measure, in 8 boards 15 feet long, 18 inches wide, and $\frac{3}{4}$ inch thick?

75. How many feet, board measure, in 35 joists 20 feet long, 3 by 4 inches in the other two dimensions?

76. How many feet, board measure, in a tapering stick of timber 40 feet long, 12 by 15 inches at one end, and 10 by 12 inches at the other?

77. What ought to be paid for a board 25 feet long, 20 inches wide at one end and 14 inches at the other, and $1\frac{1}{2}$ inches thick, at $18 a thousand feet?

78. What will be the cost of 50 joists, each 18 feet long, 4 by 5 inches in the other dimensions, at $12.50 a thousand feet?

79. If a ladder 80 feet long will reach from a point in the street on one side to a window 45 feet from the ground, and on the other to a window 55 feet from the ground, how wide is the street?

80. If a rope $\frac{1}{2}$ inch in diameter will support 500 pounds, how many pounds will a cable of the same material $3\frac{3}{4}$ inches in diameter support?

81. How many bushels will it take to fill a box 4 ft. long, 2 ft. wide, and $2\frac{1}{2}$ ft. high?

82. How many bushels of grain will a bin that is 10 ft. long, 6 ft. wide, and 5 ft. high hold?

83. A bin 8 ft. long and 4 ft. wide contains 100 bushels of grain. To what height is it filled?

502. Miscellaneous Oral Exercises.

1. If 8 barrels of flour cost $48, what will 11 barrels cost?

2. If a man sells a watch for $\frac{5}{8}$ of its cost, receiving $40, what did it cost?

3. If a boy buys 3 balls at 5 cents apiece, 2 sticks of candy for 2 cents, a pound of nuts for 10 cents, and pays for them with a fifty-cent piece, how much change ought he to receive?

4. If by selling land at $80 an acre I lose 20 %, how must I sell it to gain 40 %?

5. A piece of sheeting shrank $\frac{1}{10}$ of itself in washing, and then measured 18 yards. How long was the piece before washing?

6. Bought a chest of tea weighing 50 pounds for $25. What must I sell it for a pound to make 15 %? If I sell it for 40 cents a pound do I gain or lose, and what per cent?

7. Bought a pair of oxen and a horse for $250. The oxen cost $\frac{2}{3}$ as much as the horse; what was the cost of the horse?

8. What is the interest of $8 for a year at 5 %? at 8 %? at 9 %?

9. If 4 men build $\frac{3}{4}$ of a rod of wall in an hour, how many rods will 3 men build in 8 hours?

10. What date is 13 days after February 18, 1896?

11. How many miles in 1280 rods?

12. A boy bought some raisins for 28 cents, some figs for 7 cents, a comb for 6 cents, a brush for 5 cents, and gave the shopkeeper 50 cents. What ought the boy to receive in change?

13. If my agent sells $400 worth of goods at $2\frac{1}{2}$ % commission, what is the amount of his commission?

14. What will 3 quarts of molasses cost at 50 cents a gallon?

15. If the price of land rises from $0.60 a square foot to $0.75, what is the per cent of rise?

16. A boy bought 2 hens at $0.46 apiece and paid for them with eggs at 4 for 7 cents, making up any fraction of an egg with money. How many eggs and how much money did the boy give for the hens?

17. How many cubic inches in a cistern whose dimensions are 2 feet long, 8 inches deep, and 5 inches wide?

18. How many hours from 9.30 A. M., to-day, to 4.45 P. M., to-morrow?

19. What part of anything is $12\frac{1}{2}$ % of it?

20. How much will 16 eggs cost at 20 cents a dozen?

21. How many days from April 3d to July 7th?

22. If a boy misses 20 out of 50 words, what per cent does he misspell? What ought his rank mark to be on a scale of 10? on a scale of 8?

23. Reduce 18 rods to feet; to inches.

24. What is the square of $3\frac{1}{2}$?

25. Which is the greater, the ratio of 5 to 2, or of 7 to 3?

26. What will 18 ounces of butter cost at 32 cents a pound?

27. What is the inverse ratio of 2 to 6?

28. What per cent of anything is $\frac{3}{8}$ of it? $\frac{3}{4}$ of it? $\frac{2}{3}$ of it?

29. What will 20 eggs cost at eighteen cents a dozen?

30. How many feet in $8\frac{1}{2}$ yards?

31. At 64 cents a pound what is $\frac{3}{4}$ of a pound of tea worth?

32. In 160 quarts how many bushels? how many pecks?

33. If a boy has $0.50 and spends $0.10, what per cent of his money does he spend?

34. If 51 cents are divided equally among 3 boys, how many cents will each have?

35. 25 is 5 % of what number? 10 %? $12\frac{1}{2}$ %?

36. At $0.125 a quart what must I pay for 1000 quarts?

37. What is $33\frac{1}{3}$ % of $3? $16\frac{2}{3}$ %? $12\frac{1}{2}$ %? 25 %?

38. How many hours in 540 minutes?

39. A man being asked how many sheep he had, replied that if he had as many more, $\frac{1}{2}$ as many more, and 10 sheep, he should have 100; how many had he?

40. If I pay $185 a ton for rice, what do I pay a pound?

41. What date is it 19 days after July 17th?

42. What is 8 % of $5? of $50? of $500? of $5000?

43. What is the ratio of 5 to 9?

44. In 6 tons how many pounds?

45. If in a school of 300 scholars 95% are present on Monday, what is the number present?

46. Travelling from Albany, with the correct Albany time, at the end of the second day I found that the sun set 20 minutes later than at Albany. Was I east or west of Albany? How many degrees?

47. What per cent of $40 is $5? $8? $10? $16? $25? $30?

48. In 512 gills how many pints? quarts? pecks? bushels?

49. If land falls from $0.75 a square foot to $0.60, what per cent does it fall?

50. If a quantity of provision will serve 5 men 8 days, how long will it serve 4 men?

51. If I buy at $8 a thousand pounds, what do I pay a pound?

52. If I sell for 75 cents a book which cost me 80 cents, what per cent do I lose?

53. If I sell a horse for which I paid $ 150 so as to gain 25 %, what do I receive for the horse?

54. If I buy calico at 10 cents a yard and sell at $12\frac{1}{2}$, what do I gain per cent?

55. How many eggs at 20 cents a dozen will pay for $4\frac{1}{2}$ pounds of raisins at 10 cents a pound?

56. If a collector takes $ 8 for collecting $ 400, what per cent does he take?

57. Sold 5000 pounds of pork at 9 cents a pound, and lost 10 %. What per cent should I gain by selling it at 12 cents a pound?

58. A man paid $ 20 for a harness, and sold it for $\frac{4}{5}$ of what he paid. What did he get for it?

59. When 4 % is lost on cheese sold at 12 cents a pound, what was the cost?

60. A man and his wife together used a barrel of flour in 4 months, but one lasted the woman alone 9 months. How long would a barrel of flour last the man alone?

61. A merchant loses 10 % of his capital by bad debts, and one third of the remainder by a fire, and is then worth $ 12000. What was his capital previous to these losses?

62. A boy spent $ 0.25, which was 10 % of all he had; how much money did he have?

63. What is the interest of $ 5 at 6 % for 2 years? for 3 years 6 months?

64. If I buy knives for $ 16 a dozen, and sell them for $ 2 each, how much do I gain per cent?

65. A merchant, owning $\frac{4}{9}$ of a ship, sold $\frac{3}{4}$ of his share for $ 4000; what was the value of the ship?

66. What is the amount of $ 10 for 2 years 6 months at 5 %? at 8 %?

503. Miscellaneous Written Examples.

67. The product of three numbers is 228; one is $9\frac{1}{2}$ and another is $7\frac{1}{3}$. What is the third number?

68. $\frac{1}{3}$ of $\frac{2}{7}$ of $28\dfrac{3\frac{6}{7}}{4\frac{2}{7}} + \dfrac{39\frac{1}{2}}{105}$ equals what?

69. Add $\frac{3}{8} + 0.16 + \frac{2}{5} + 0.008 +$ one and one tenth $+ 4\frac{1}{2} +$ 10.05.

70. The sum of two numbers is $143\frac{1}{2}$; their difference is $17\frac{1}{4}$. Give the numbers.

71. Find the sum, difference, product, and quotient of $\frac{7}{9}$ and $\frac{2}{7}$.

72. Find the greatest common divisor and the least common multiple of 18, 48, 72, 66.

73. Change $\frac{3}{4}$ to a decimal fraction, multiply by 0.0008, and divide the product by 0.02.

74. Reduce $\frac{3}{4}$ of $\frac{58}{63}$ of $\dfrac{2\frac{1}{4}}{6}$ to a decimal.

75. If $30\frac{3}{4}$ pounds of tea cost \$ $35\frac{1}{7}$, what cost $361\frac{9}{10}$ pounds?

76. Add $\frac{2}{7}$, $\frac{3}{11}$, and $\frac{4}{13}$, and multiply their sum by 1.43.

77. What number multiplied by $\frac{1}{2}$ of itself will produce $12\frac{1}{2}$?

78. How many bricks, each 8 inches long and 4 inches wide, will be required to pave a sidewalk 10 rods long and 9 feet wide, allowing 4 inches in width for the edgestone?

79. How many days, hours, etc., in 87 % of a year?

80. On a certain railroad a freight train runs at an average rate of 17.6 feet a second, including stops; how many miles will it run in 24 hours?

81. In how many days will a ship, sailing $13\frac{9}{10}$ miles an hour, traverse a distance equal to the earth's circumference, 25000 miles?

82. One million bricks, each 8 in. long, 4 in. wide, and 2 in. thick, are piled in the form of a cube. How long is a side of the pile?

83. What sum must I invest at 6 %, to have an annual income of \$ 1000?

84. Find the mean proportional between 0.00125 and 800.

85. A collector of the taxes of a town retains for collecting $2\frac{1}{2}$ %, which is \$ 739. How much does the town receive?

86. What per cent of an acre is a square yard?

87. A field in the form of a right-angled triangle, with the two short sides of equal length, contains two acres. What is the length of the longest side?

88. The fore wheel of a carriage is 10 feet, and the hind wheel 12 feet in circumference; how many times will each turn round in running from Boston to Worcester, 44 miles?

89. An Oswego coal-dealer bought (at wholesale), at a mine in Pennsylvania, 1540 (long) tons anthracite coal at \$ 3.50 a ton. The freight to Oswego was \$ 2040, and the loss in transportation was 30 (long) tons. The coal was retailed for \$ 5.50 a (short) ton. What was the gain?

90. Express 37817.8535 hours in years (of 365 days), days, hours, minutes, and seconds.

91. When gold was quoted at $127\frac{1}{2}$, how many gold dollars and how much fractional currency ought to be received for a ten-dollar bill?

92. The sun crosses the meridian at the Worcester High School 2 minutes 59 seconds later than at the Cambridge Observatory. What is the difference in longitude?

93. If you travel till you find that the sun sets 55 minutes earlier than at the place from which you started, how many degrees east, or west, have you gone, and in which direction?

94. C. Rollins bought 25 casks of oil, 125 gallons each, at 76 cents a gallon, and paid \$ 15.96 for drayage. If there should be a waste of 3 % in measuring, and 5 % of the sales prove bad debts, and it should cost 2 % of the remainder to collect it, for how much a gallon must I sell it to make a net profit of 30 % on the whole cost, no allowance being made for interest?

95. An importer sold cloth to a wholesale dealer at a profit of 10 %. The wholesale dealer sold it to a retailer at a profit of 10 % on the cost to himself. The retailer sold it at an advance of 20 % on what it cost him. Now if the retailer sold it for \$726, what did it cost the importer?

96. H invested a certain sum in flour and G invested twice as much. In selling, H lost 10 % and G gained 10 %. G received \$260 more than H. How much did each invest?

97. Find three numbers each greater than 50, each not prime, that are mutually prime?

98. The sum of two numbers is $21\frac{1}{7}$, and their difference is $7\frac{1}{3}$. What are the numbers?

99. Reduce $\frac{4}{5}$, $\frac{1}{3}$, 0.13, 1.7 to fractions having the common denominator 33.

100. One rod is 11 % of what distance?

101. What number multiplied by $\frac{3}{4}$ of itself will produce 27?

102. Find the square root of 49098049.

103. What number must be multiplied by $\dfrac{2\frac{3}{5}}{4\frac{1}{2}}$ that the product may equal 1?

104. A and B together can build a wall in 4 days; A and C in 5 days; and B and C in 6 days. If the three build the wall together and receive \$18.50 for the work, how ought the money to be divided?

105. The first newspaper despatch ever sent by a human voice over a wire was by Professor A. Graham Bell's Telephone, from Salem to Boston, Feb. 12, 1877. How long ago was that? State the answer in years, months, and days.

106. Find the balance due Sept. 30, 1896, on a note for \$1475, dated June 3, 1895, on which was paid, Sept. 17, 1895, \$200; Jan. 3, 1896, \$300; Aug. 2, 1896, \$400; interest at 6 %.

107. Note dated Jan. 12, 1892, for $1500, interest at 6%.

INDORSEMENTS: Oct. 24, 1892, $48; Jan. 10, 1893, $31; July 12, 1893, $150; Oct. 17, 1894, $75. What was due June 18, 1895?

108. A note for $50, dated Jan. 1, 1895, was on interest at 6% till April 15, 1895, when $25.87 was paid upon it. What was due Jan. 1, 1896?

109. Memorandum: Nov. 29, 1892, gave a note for $1500, with 6% interest. June 13, 1894, paid $125; Oct. 25, 1894, paid $350; March 4, 1895, took up the note. How much was the last payment?

110. On the 1st of July James Merchant bought 750 barrels of flour for $6.50 a barrel, cash; on the 12th he paid 15 cents a barrel for freight. Aug. 1 he sold the flour for $7.20 a barrel, payable, one half in cash, the other half by a 6 months' note. Aug. 18 he had the note discounted at a bank at 6%. How much did he make, allowing money to be worth 6%?

111. N. W. Belcher bought 100 barrels of flour at $6.50 a barrel, cash, and paid $0.50 a barrel for freight and other expenses; after keeping it 4 months he sold it for $7.50 a barrel, cash. Allowing money to be worth 6%, how much did he gain, and what per cent?

112. H. M. Valentine gets a note for $800, payable in 90 days, discounted at a bank; when due he is unable to pay it, and gets it renewed for 90 days more. How much does the bank receive as discount in both these transactions, money being worth 6%?

113. The capital stock of a certain bank is $200000, and the directors declare a dividend of 5%. If the sum set aside to pay the dividend is subject to a tax of 3%, what sum must be set aside that the directors may pay each stockholder 5% on his stock?

114. My agent sells for me 400 barrels of apples at $1.25 a barrel. If he discounts to the purchaser 5% for cash, and

charges me 2 % commission on his cash receipts, how much does he pay over to me?

115. A merchant takes account of stock annually, and at the end of three years finds that he has lost 10 % each year, and now is worth $ 15000. What was his capital at first?

116. If a broker sells goods that cost $ 10000 at a profit of 6 %, and charges 5 % on the amount received, as commission, how much does the owner of the goods receive as profit?

117. A merchant sells through a broker goods to the amount of $ 1343.78, and pays the broker a commission of 4 % on that amount, after which he still has 4 % profit left. What did the goods cost?

118. A and B can perform a piece of work in 5 days, B and C in 8 days, and A and C in 6 days; in what time can each of them do the work alone, and how long would it take them to perform the work together?

119. If 5 tons of coal are equivalent to 9 cords of wood as fuel, and a family burns 12 cords of wood in a year, how much will they save in a year by changing from wood to coal when wood sells at $ 4 a cord and coal at $ 6.50 a ton?

120. A merchant bought eggs at the rate of 4 for 5 cents and sold them at the rate of 4 for 7 cents, clearing $ 8. How many eggs did he buy?

121. A grocer bought a box of soap weighing 72 pounds at 7 cents a pound. Before he sold it, it lost $\frac{1}{8}$ of its weight. He sold it at $8\frac{1}{2}$ cents a pound. Did he make or lose? How much?

122. A man paid $ 500 for a horse, a carriage, and a harness. The horse cost $\frac{1}{5}$ more than the carriage, and 4 times as much as the harness. What did each cost?

123. A gentleman bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of the remainder to his eldest son, and $\frac{1}{5}$ of what then remained to his daughter, who received $ $684.66\frac{2}{3}$. What was the value of the estate?

124. The denominator of a fraction is $\frac{\frac{3}{8} \times \frac{1}{3} + \frac{4}{7} \text{ of } 8\frac{1}{6}}{18\frac{7}{8} \text{ of } \frac{5}{6} + \frac{2}{9}}$, and the numerator is $\frac{6}{7}$ of the denominator. What is the value of the fraction?

125. If a grocer's scales give only $15\frac{1}{2}$ ounces for a pound, out of how much money does the grocer cheat a customer who buys tea to the amount of $ 16?

126. If $\frac{2}{3}$ of a bushel of corn is worth $\frac{3}{7}$ of a bushel of wheat, and corn is worth $ 0.63 a bushel, how much wheat can be bought for $ 24.50?

127. A man paints two sides of a board-fence, 6 feet high, in $5\frac{1}{4}$ days, working 10 hours a day. If he can paint 8 square yards an hour, how long is the fence?

128. For what principal must I write a note on 4 months, at $7\frac{3}{10}$ %, to enable me to receive at a bank, on its date, $ 300?

129. Find the cube root of 4436184.

130. How many cubic yards of earth must be thrown out to dig a ditch 2 feet deep and 3 feet wide, just within the boundary of a garden 16 rods long and 12 rods wide?

131. A and B engage in trade. A puts in $ 10000 and at the end of 5 months takes out a certain sum; B puts in $ 6000 and after 3 months puts in $ 4000 more; at the end of the year A's gain is $ 1800 and B's $ 2250. What sum did A withdraw at the end of 5 months?

132. What number multiplied by $\frac{2}{3}$ of itself will give 54?

133. If 24 men, in 9 days of 12 hours each, build a wall 200 feet long, 6 feet high, and 2 feet thick, how many men, in 72 days of 10 hours each, can build a wall 950 feet long, 8 feet high, and 5 feet thick?

134. If on February 20 I buy $10000 United States 6 % bonds at $110\frac{1}{4}$, and $10000 City of Chicago 7's at 103, and accrued interest from January 1 on the latter, what will they cost me?

135. A certain savings bank pays 5 % a year, adding the accrued interest to the principal at the end of each 6 months, and in it Mr. J. deposits every six months $20 on the day when interest begins. What sum will stand to his credit in the bank on the day after he makes his sixth deposit?

136. A room 15 by 20 feet is to be carpeted with tapestry carpet, $\frac{3}{4}$ of a yard wide. In order to make the figure match a breadth cannot be split, and if the breadths are laid to run lengthwise of the room, each breadth must be 10 inches longer than the room, and if the other way, 8 inches. What is the least number of yards of this carpet that must be bought to carpet the room, and what will it cost at $0.95 a yard?

137. How many more yards of carpet must be bought to carpet the room named above than to carpet a room whose floor contains the same number of square feet, but is so shaped that none of the carpet needs to be turned under?

138. Sold to Luther Farwell goods as follows:

Jan. 17, 1894,	on 6 m.,	275 yd. cloth,	@	$2
Mar. 13, "	" 3 m.,	900 gal. oil,	"	0.20
June 16, "	" 4 m.,	75 bbl. sugar,	"	8

Also bought of him:

Feb. 19, 1894,	on 4 m.,	80 tons coal,	@	$6
May 24, "	" 6 m.,	10 tons hay,	"	15
July 7, "	" 5 m.,	3 carriages,	"	150
" 25, "	" 4 m.,	1 horse,	"	200

When ought the balance of this account to begin to draw interest?

139. If I deposit, on the day when interest begins, $200 in a savings bank that pays 6 % a year, adding the accrued interest at the end of each 6 months, what sum will stand to my credit at the bank the day after I make my eighth deposit?

140. How large a square floor can be laid with 2704 square feet of boards?

141. Find the square root of the square root of 16 times 81.

142. What is the square root of 9 times the square of 12?

143. A rectangular piece of land containing 25 acres has its length to its breadth as 4 : 3. Find the length and the breadth.

144. What is the side of a square equivalent to a rectangular field that is 121 rods long and 64 rods wide?

145. How many feet of fence would be required to enclose an acre of land in the form of a square?

146. What is the length of one side of a square field containing 15 acres and 1 square rod?

147. If it costs $312 to enclose a field 216 rods long and 24 rods wide, what will it cost to enclose a square field of equal area with the same kind of fence?

148. I hold three notes bearing interest from date: the first, dated July 3, 1894, is $530; the second, dated Oct. 9, 1894, is $740; the third, dated Feb. 6, 1895, is $630. If a single note of $1900 is substituted for these three, when ought it to be dated?

149. A person owes a debt of $1680, due in 8 months, of which he pays $\frac{1}{3}$ in 3 months, $\frac{1}{4}$ in 5 months, $\frac{1}{8}$ in 6 months, and $\frac{1}{6}$ in 7 months. When is the remainder due?

150. If I buy $1200 worth of goods, half on 2 months' and half on 4 months' credit, what can I afford to pay at the time of purchase, if money is worth 6%?

151. A note of $1250, dated July 5, 1892, was paid June 6, 1894, with interest at 8%. What was the amount paid?

152. If 15 men, working 10 hours a day, can do a certain piece of work in 18 days, how many days will it take 13 men to do the same work by working 8 hours a day?

153. In what time will $800 at 8% gain $150?

154. What is the present worth of $1609.30, due in 10 months 24 days, when money is worth 5%?

PRACTICAL EXAMPLES.

504. The following examples are arranged according to subjects, and are to be selected from at discretion.

PROPERTIES OF NUMBERS.

1. What are the prime factors of 1440?

2. Resolve 25740 into its prime factors.

3. Find the prime factors of 10917.

4. Resolve 29925 into its prime factors.

5. Prove that 859 is prime.

6. Is 991 prime?

7. Is 2501 prime?

8. What is the greatest common divisor of 12, 16, 84, and 144?

9. What is the greatest common divisor of 1181 and 2741?

10. Find the greatest common divisor of 340200, 583200, and 2268000.

11. What is the least common multiple of 25, 75, 100, 10, 5, 200, and 150?

12. Find the least common multiple of 8, 12, 72, 92, and 364.

13. Find the least common multiple of 5, 10, 12, 20, and 36.

14. Multiply the greatest common divisor of 18, 45, 63, and 99 by their least common multiple.

15. What is the smallest sum of money that can be made up of 2-cent, or 3-cent, or 5-cent, or 10-cent, or 25-cent pieces?

16. A can walk round an island in 12 days, B in 16 days, and C in 20 days. If they start at the same time from the same point and walk in the same direction, in how many days will they all come together again, and how many times will each have walked round the island?

17. A can hoe a row of corn in a certain field in 30 minutes, B can hoe a row in 20 minutes, and C can hoe a row in 35 minutes. What is the least number of rows that each can hoe in order that all may finish together?

COMMON FRACTIONS.

18. Reduce $\frac{23820}{39700}$ to its lowest terms.

19. Reduce $\frac{16200}{24840}$ to its lowest terms.

20. Find the greatest common divisor of both terms, and reduce the fraction $\frac{945}{4620}$ to its lowest terms.

21. Reduce $\frac{3}{12}$, $\frac{16}{128}$, $\frac{7}{16}$, and $\frac{9}{27}$ to fractions having the least common denominator.

22. Reduce $\frac{3}{8}$, $\frac{7}{16}$, $\frac{11}{24}$, and $\frac{5}{48}$ to fractions having a common denominator.

23. Reduce $\frac{5}{7}$, $\frac{7}{8}$, $\frac{5}{21}$, $\frac{3}{16}$ to fractions having a common denominator.

24. Find the sum of $4\frac{3}{4}$, $5\frac{7}{8}$, $17\frac{9}{11}$, and $14\frac{23}{44}$.

25. Add $\dfrac{3\frac{1}{4}}{5\frac{2}{3}}$, $\frac{5}{6}$, and $\frac{8}{9}$ of $\frac{3}{16}$.

26. Add $\frac{5}{6}$, $\frac{4}{9}$, $2\frac{2}{15}$, and $7\frac{3}{20}$.

27. Find the sum of $\dfrac{\frac{3}{5} \text{ of } \frac{5}{6}}{\frac{1}{2}}$ and $\dfrac{\frac{4}{7} \text{ of } \frac{7}{12}}{\frac{4}{9} \times \frac{9}{4}}$.

28. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{12}{39}$ of $\frac{16}{21}$ of $8\frac{2}{3}$ to its simplest form.

29. Divide $\frac{7}{8}$ of $\frac{16}{19}$ by $\frac{4}{11}$ of $\frac{35}{38}$.

30. Find the product of $3\frac{1}{3} \times 2\frac{1}{5} \times 4\frac{3}{10} \times 5\frac{1}{4}$.

31. Reduce $\dfrac{\frac{1}{2} \text{ of } \frac{5}{7} \text{ of } 7\frac{3}{8}}{12\frac{9}{14}}$ to its simplest form.

32. What number multiplied by $33\frac{1}{3}$ gives 1000?

33. What number divided by $37\frac{1}{2}$ gives 64?

34. What number increased by $\frac{5}{16}$ of itself gives 252?

35. The sum of two numbers is 954 and one of the numbers is $3\frac{1}{2}$ times the other; what are the numbers?

36. What part of $\frac{5}{8}$ is $\frac{1}{3}$?

37. What part of $\dfrac{10\frac{1}{2}}{\frac{7}{3}}$ is $\dfrac{\frac{3}{4} \times \frac{2}{5}}{\frac{1}{3} \text{ of } 4}$?

38. Find the simplest expression for $\dfrac{\frac{1}{5} \times 1\frac{1}{4} \times 4\frac{1}{2}}{\frac{5}{56} \times 1\frac{1}{3} \times 3\frac{1}{2}} - \dfrac{2\frac{1}{4} + 4\frac{1}{3}}{5\frac{1}{2} + 1\frac{1}{12}}$.

39. What number is that from which if you take $\frac{1}{7}$ of itself, $3\frac{2}{5}$ times the remainder minus 1 will be 50?

40. If 5 be added to both terms of the fraction $\frac{3}{7}$, will its value be increased or decreased, and how much?

41. Divide $6\frac{2}{5} \times \dfrac{3\frac{1}{2} \text{ of } 7}{4}$ by $\frac{1}{2}$ of $\frac{3}{5}$.

42. If $\frac{5}{8}$ of an acre cost \$17.50, what will be the cost of $15\frac{3}{4}$ acres?

43. If $\frac{5}{8}$ of a dollar will buy $\frac{9}{10}$ of a basket of peaches, what will 48 baskets cost?

44. What part of $\dfrac{12\frac{1}{2}}{4}$ is $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$?

45. Multiply the quotient of $14\frac{2}{5}$ divided by $6\frac{6}{7}$ by the quotient of $5\frac{5}{8}$ divided by $7\frac{7}{11}$.

46. If a horse travels $6\frac{1}{4}$ miles an hour, how many hours will it take him to travel as far as a rail-car will run in 6 hours, the car running $22\frac{1}{2}$ miles an hour?

47. A and B, 325 miles apart, travel towards each other. When A has travelled $\frac{7}{13}$ of the whole distance and B $\frac{4}{5}$ of the remainder, how far apart are they?

48. If A can do a piece of work in 10 days, B in 12, and C in 15, how long will it take the three together to do it?

49. A gentleman left his son an estate, $\frac{1}{4}$ of which he spent in a year and $\frac{5}{12}$ of the remainder in 6 months more, when he had only \$ 1400 remaining; what was the value of the estate?

50. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of \$ 7 for 5 yards, and sold them out at the rate of \$ 11 for 7 yards, and gained \$ 200 by the bargain; how many bales were there?

51. If $\frac{12}{33}$ of a ton of hay cost \$ 9.50, what will a ton cost?

52. If $\frac{18}{25}$ of a cask of oil is worth \$ 86, what is the value of 8 casks?

53. A can do a piece of work in 8 days of 10 hours each, and B can do the same work in 12 days of 8 hours each. If they agree to complete this piece of work together in 5 days, how many hours a day must they work?

54. If A can do a piece of work in 12 days, A and B in 7 days, and B and C in 5 days, how long will it take A and C together to do it?

55. A can do $\frac{1}{6}$ of a piece of work in 4 days; B $\frac{1}{5}$ in 5 days; C $\frac{1}{4}$ in 6 days; D $\frac{1}{3}$ in 7 days. How long will it take them all to do it together?

DECIMALS.

56. Add $0.0\frac{1}{2}$, $0.083\frac{1}{3}$, $0.004\frac{1}{6}$, $0.00011\frac{1}{4}$, and $5\frac{1}{2}$.

57. Reduce to common fractions 0.375, 0.016, $0.037\frac{1}{2}$, 0.0025, $0.043\frac{1}{3}$.

58. $1.44 \div 12 + 144 \div 12 + 0.0144 \div 1.2 + 14400 \div 0.012 + 14.4 \div 0.0012 = ?$

59. Divide 74.86 by 0.001. 60. Divide 0.001 by 74.86.

61. Divide 57.3 by 191000. 62. Divide 191000 by 57.3.

63. Reduce to a simple decimal $0.05\frac{1}{250}$.

64. Divide 0.0001 by 359.

65. Multiply 580.045 by 0.0025 and divide the product by 15000.

66. Reduce $\dfrac{2\frac{1}{2} \times \frac{3}{8}}{\frac{5}{8} \times 3\frac{1}{3}}$ to a decimal fraction.

67. Multiply 124 by 10; that product by 0.01; divide that by $\frac{1}{1000}$ of 10; add these results and divide the sum by $0.003\frac{1}{3}$.

68. Divide 32.22 by 10, and this quotient by 100; multiply this quotient by 10, this product by 1000; this product by $\frac{1}{10}$; and give the sum of the five results.

69. The product of three numbers is $\dfrac{2.5}{4}$; two of the numbers are 8.03 and 0.06. What is the third?

70. Find the sum of 449, and 3 tenths, and 3 millionths; from that sum subtract 246, and 3 thousandths; multiply the remainder by 5 thousandths; and divide the product by 4 ten-thousandths.

71. Divide fifty thousand by twenty-five thousandths. Also, divide seven millionths by thirty-five hundred.

72. Reduce $\frac{7}{80}$ to a decimal, and 0.005 to a common fraction.

73. Divide four thousand ten-thousandths by two thousandths. Also, divide three hundred, and sixty-three millionths by three hundred.

74. Reduce $\frac{3}{80}$ to a decimal fraction. Also, reduce 0.00585 to a common fraction in its lowest terms.

75. Divide seventeen hundred, and eighty-five thousandths by five thousand. Also, divide six hundred and fifty-four by six hundred-thousandths.

UNITED STATES MONEY.

76. A farmer sold $330\frac{1}{2}$ pounds of maple sugar at 9 cents a pound and took his pay in muslin at $16\frac{3}{4}$ cents a yard. How many yards did he receive?

77. What will be the cost of 12350 feet of boards at $20 a thousand?

78. A bricklayer furnished 17650 bricks at \$8 a thousand and 15 days' work at \$1.75 a day. Make out his bill.

79. Bought a cask of molasses, containing 64 gallons, for \$16; but 11 gallons having leaked out, at what price a gallon must I sell the remainder to gain \$2.55?

80. How many pounds of tea at 65 cents a pound will pay for 2 barrels of rice, each weighing 325 pounds, at 9 cents a pound?

81. What will it cost to fence both sides of a road 165.5 rods long at \$3.20 for each rod of fence?

82. A man bought a horse for \$126 and 8 sheep at \$3.50 a head. How much wood at \$5.50 a cord must he give to pay the bill?

83. If 1.25 acres of land are worth \$87.50, what is the worth of 16.5 acres of the same land?

84. What will 87.5 yards of cloth cost if $1\frac{3}{4}$ yards cost \$1.26?

85. Bought wheat at 94 cents a bushel, to the amount of \$59.22, and sold it for \$70.56; what was the selling price a bushel?

86. In 1860 I bought cotton at $8\frac{1}{3}$ cents a pound, which I sold in 1862 at $90\frac{7}{8}$ cents a pound. What did I gain on 1000 pounds?

87. How many cheeses, each weighing 45 pounds, at 9 cents a pound, will it take to pay for 5 barrels of beans at \$8.10 a barrel?

88. Find the amount of the following bill:

GREENFIELD, MASS., April 3, 1895.

J. A. WINCHESTER,

To H. P. NEWTON, Dr.

For	2550	feet	boards	@	\$16.50	a thousand
"	8726	"	"	"	18.00	"
"	4760	"	"	"	15.00	"

89. Find the amount due on the following bill:

BOSTON, June 14, 1895.

W. H. STICKNEY,

Bought of A. P. BARNES.

18	pounds	tea	@	$62\frac{1}{2}$	cents
25	"	rice	"	9	"
37	"	cheese	"	$8\frac{3}{4}$	"
2	gallons	molasses	"	45	"
		Cr.			
	Cash,				$ 10.00

90. Find the amount of the following bill:

NEW YORK, May 16, 1895.

A. H. SMYTH, Dr.

To B. H. KERN & Co.

For	40	yards	ingrain	carpet	@	$ $0.87\frac{1}{2}$
"	52	"	tapestry	"	"	0.95
"	18	"	oilcloth	"	"	0.45
"	63	"	straw	"	"	0.50
"	6	mats			"	1.50

91. W. J. Hyde sold to the city of Boston 150 tons of coal at $ 4.30 a ton, 175 tons at $4.17 a ton, 250 tons at $ 3.95 a ton, and 215 tons at $ 3.84 a ton. Make out a receipted bill, dated Jan. 1, 1895.

92. Mrs. N. F. Cutting bought of Hale, Everett & Co. of Albany, New York, May 7, 1895, 25 yds. calico @ 9 c.; 43 yds. sheeting @ $12\frac{1}{2}$ c.; 12 yds. silk @ $ 1.50; 14 yds. flannel @ 42 c.; 6 handkerchiefs @ 28 c.; 2 pairs kid gloves @ $1.62. Make out Mrs. C.'s bill.

93. L. Miller, of Woonsocket, sold A. F. Wood the following items: April 4, 1895, 18 yds. of black silk at $ 1.50 a yard; April 10, 4 pieces of muslin, 27 yds. in each, at 11 cents a yard; April 25, 5 dress patterns at $ 4.75 each; May 8, $18\frac{1}{2}$ yds. of linen at $ 0.75 a yard. Mr. Wood paid $ 75 on account. Prepare this bill for settlement.

94. C. P. Thayer bought of J. H. Cushing of Buffalo, N. Y., May 6, 1895, 2 horses at \$95 each, 3 cows at \$27 each, 1 carriage at \$127, 1 light buggy at \$67. Make out Mr. Thayer's account.

95. Philip Davis is debtor to William Richmond of Brockton, as follows: For $16\frac{1}{4}$ yards linen at 22 cents a yard; $7\frac{1}{2}$ yards flannel at $62\frac{1}{2}$ cents a yard; $\frac{1}{2}$ dozen handkerchiefs at $37\frac{1}{2}$ cents each; and $2\frac{3}{4}$ yards drilling at $15\frac{1}{2}$ cents a yard. This bill was paid November 23, 1895. Make out a receipted bill in proper form.

96. Suppose you buy of D. Appleton & Co. of New York, 8 reams of note paper at \$2.25 a ream; 3500 envelopes at \$2.75 a thousand; 25 boxes of steel pens at \$$1.12\frac{1}{2}$ a box; 4 French Dictionaries at \$1.50 each; and 5 photograph albums at \$3.75 each. Make out a bill for D. A. & Co. against yourself, in regular form.

DECIMAL WEIGHTS AND MEASURES.

97. If a man can travel 256 meters in 0.05 of an hour, how long will it take him to travel a kilometer?

98. How many liters are there in a rectangular cistern 3.8^{m} long, 1.5^{m} wide, and 8^{dm} deep?

99. What must be paid for a pile of wood whose dimensions are 3 by 2 by 55.5 meters, at \$1.75 a ster?

100. How many meters of carpeting, 8 decimeters wide, will it take to cover a room a dekameter long and 7.6 meters wide?

101. If it is necessary to carpet the room whose dimensions are given in Ex. 100, without splitting a breadth, if 1 centimeter is allowed to turn under at each end of the breadths, what is the least length of carpet that will carpet the room?

102. How many cubic meters in a marble block 20.5^{m} long, 1.5^{dm} wide, and 402^{cm} high?

103. What will be the cost of a block of marble 2.5^{m} long, 0.75^{m} wide, and 0.2^{m} thick, at $0.24 a cubic decimeter?

104. How many hektars are there in a lot 255^{m} long by 150^{m} wide?

105. Brass contains 52 parts of copper to 39 of zinc. How much of each metal is there in 844.97 grams of brass?

106. If a cistern is 5^{m} long, 2.5^{dm} wide, and 92^{cm} deep, how many cubic meters of water will it hold? How many liters? How many kilos (Art. 223) of cast iron?

107. How many hektoliters of wheat will a bin contain that is 3^{m} long, 2^{m} wide, and 1^{m} deep?

108. How long is a pile of wood 4.6^{m} wide and 3^{m} high that contains 189 sters?

109. What is the capacity in liters of a cubical vat that measures 2.5 meters in every direction?

110. If a cistern is 3.5^{m} long, 18^{dm} wide, and 85^{cm} deep, what is its capacity in dekaliters? How many metric tons of water will it hold?

111. How many ars in a lot of land 175^{m} square?

112. How many sters of wood in a pile 4^{m} long, 2.5^{m} high, and 1.5^{m} wide?

113. How many liters of water will a cistern contain whose dimensions are 3^{m} long, 25^{dm} wide, and 154^{cm} deep? How many kilos?

COMPOUND NUMBERS.

114. Reduce $\frac{1}{24}$ of a bushel to quarts and pints.

115. Reduce 5 yd. 2 ft. 6 in. to the decimal of a rod.

116. Reduce $\frac{53}{71}$ of a gallon to quarts, pints, etc.

117. What part of a mile is 25 ft. 6 in.?

118. Reduce 0.795 of a mile to lower denominations.

119. What part of 16 acres 72 sq. rods is 3 acres 16 sq. rods?

120. How many bags, each holding 2 bush. 3 pk. 7 qt., will it take to hold 124 bush. 2 pk. 3 qt.?

121. What will be the cost of 5 tons 1520 pounds of coal at $6 a ton?

122. What will be the cost of a pile of wood 60 ft. long, 4 ft. wide, and 5 ft. 9 in. high, at $4.50 a cord?

123. How many bushels will a bin contain that is 4 ft. square and 6 ft. deep?

124. How many square feet in the entire surface of a rectangular block of marble 8 ft. long, 4 ft. wide, and 3 ft. thick?

125. A farm containing 360 a. 42 sq. rods is divided into 23 equal parts; how much land does each contain?

126. How many yards of cloth that is 27 in. wide will it take to cover 12 tables, each 6 feet by 3 feet?

127. Find the cost of 84 feet of boards at $20 a thousand.

128. How many bricks 8 in. long and 4 in. wide will cover a floor 24 ft. long and 16 ft. wide?

129. There are two bins: the inside dimensions of one are $10 \times 8 \times 6$ ft.; the capacity of the other equals an 8-feet cube. Which holds the more and how many bushels more?

130. A lot of land is 25 rods square. Two acres are reserved and the rest sold at $2 a square rod. What is received for the part sold?

131. What is the amount due for 700 feet of boards at $22.50 a thousand feet, and 912 pounds of hay at $14.50 a ton?

132. What cost 3 a. 100 sq. rd. of land at $43 an acre?

133. What will it cost to dig a cellar 40 ft. long, 32 ft. wide, and 5 ft. deep, at $0.25 a cubic yard?

134. How many square yards in the walls of a room 36 ft. long, 24 ft. wide, and 12 ft. high? Find the cost of plastering

the walls of the room at $0.25 a sq. yd., allowing for a mop-board 12 inches high, but making no allowance for doors or windows.

135. What are the cubic contents of a prism of marble 8 ft. 9 in. long, 3 ft. 4 in. wide, and 1 ft. 8 in. thick?

136. How deep is a cellar 35 ft. long and 27 ft. wide, if 175 cubic yards of earth are removed in digging it?

137. If a cubic inch of gold is beaten out so as to cover a space 95 ft. 3 in. long and 19 ft. 4 in. wide, how thick will it be?

138. If in a shower the water falls to the depth of $1\frac{1}{2}$ inches, how many gallons will fall upon a township that is 5 miles square?

139. Find the area of a rectangular lot that is 45 rd. 6 ft. by 18 rd. 10 ft.

140. At $75 an acre, what will 8 acres 33 square rods and 175 square feet of land cost?

141. A room is 16 ft. long, 14 ft. wide, and 12 ft. high; what will it cost to plaster it at 10 cents a square yard, allowing 10 inches for the width of the mop-board?

142. A room is 27 ft. long, 18 ft. wide, and 10 ft. high. How many pieces of paper $\frac{1}{2}$ yd. wide (9 yd. in a piece) will the side walls require, no allowance being made for doors, windows, etc.?

143. On a railroad 57 m. 133 rd. $11\frac{1}{2}$ ft. long there are 9 stations, including those at the two ends of the road. What is the average distance between the stations?

144. How many bushels of grain will a bin hold that is 8 ft. 5 in. long, by 2 ft. 6 in. wide, by 6 ft. deep?

145. A street 650 ft. long and 72 ft. wide averages 4.5 feet below grade. Find the cost of filling it in at $0.45 a cubic yard.

146. If a bushel of coal weighs 56 pounds, how many tons may be transported on a train of 18 cars, the box on each car being 20 ft. long, 7 ft. wide, 15 in. high?

147. A owns $\frac{5}{8}$ of a field and B the remainder; $\frac{2}{5}$ of the difference between their shares is 4 acres 136 square rods. How many acres in each one's share?

148. Mr. Willey has a room 23 ft. long and 16 ft. wide to be carpeted with carpet $\frac{3}{4}$ yd. wide and worth \$0.87$\frac{1}{2}$ a yard. The breadths of carpet are to run lengthwise of the room. To make the figure match there is a loss on each breadth of carpet of 15 inches and a breadth cannot be split. What will it cost Mr. Willey for his carpet?

149. If Mr. Willey puts the breadths of carpet across the room, and there is a loss of 9 inches on each breadth, what will it cost him to carpet his room?

150. Find the absolute difference in time due to a difference of $37° 40' 20''$ in longitude.

151. The absolute difference in the time of two places is 2 h. 27 m. 8 sec.; what is the difference in longitude?

152. Find the absolute difference of time between Washington, $77° 2' 48''$ W., and New Orleans, $90° 7'$ W.

153. Find the absolute difference in time between Boston, $71° 4' 9''$ W., and Chicago, $87° 35'$ W.

154. Find the absolute difference of time between Constantinople, $28° 59'$ E., and Boston.

155. Find the absolute difference in time between New York, $74° 0' 3''$ W., and Mexico, $103° 45'$ W.

156. Find the absolute difference in time between St. Helena, $5° 24'$ W., and Washington.

157. Annapolis, Md., and Cincinnati, O., are both in latitude $39°$ N. (nearly). On this parallel a degree of longitude is 53.47 miles; and the distance between the places is 422.433 miles. What is the absolute difference in time?

158. Find the absolute difference in time between the Cape of Good Hope, $18° 24'$ E., and Cape Horn, $67° 21'$ W.

159. The absolute difference of time between Portland and Chicago (west of Portland) is 1 h. 9 m. 32 sec. Portland is 70° 15′ W. What is the longitude of Chicago?

160. Find the absolute difference in time between Berlin, 13° 23′ 53″ E., and San Francisco, 122° 26′ 15″ W.

161. What is the longitude of a place where the sun rises 3 h. 10 m. 50 sec. earlier than it does at Philadelphia, 75° 10′ W.?

PERCENTAGE.

162. What is 5 % of 20? What is 20 % of 60 % of 120?

163. 25 is 4 % of what number? 125 is 100 % of what number?

164. What per cent of 125 is 150? What per cent of 150 is 125?

165. A person worth $40000 gave 30 % of it to his son, and the amount given was 75 % of what the son had before. How much had the son after receiving the father's gift?

166. A man owes me a debt, and pays me 40 % of it; then 20 % of the remainder; then 50 % of the second remainder. How much then remains to be paid?

167. A drover bought a lot of cattle and afterward bought 500 more, and found that the number last purchased was just 40 % of all he then had. How many had he at first?

168. What is $\frac{3}{8}$ % of 125 % of 160 bushels?

169. One third is $\frac{1}{3}$ % of what?

170. How many gold dollars could have been bought for $100 in currency, and how much currency left, when gold was 147?

171. The highest point reached by gold during the war was 284. What at that time was $1 in currency worth in gold?

172. What was gold quoted at when a dollar in currency was worth $0.75 in gold?

173. A collector of the taxes of a town receives $2\frac{1}{2}$ % for collecting, and is paid \$ 739. How much does the town receive?

174. A merchant sold 86.55 tons of coal at \$ 5.24 a ton, making a profit of \$100.63. How much did the coal cost him a ton, and what per cent did he make by the transaction?

175. If $\frac{3}{4}$ of an article is sold for $\frac{1}{2}$ of its cost, what is the per cent of loss?

176. I have a house which I am willing to sell for \$ 5000, but by so doing I shall lose 15 %. What was the cost of the house?

177. Sold cloth for \$ 96, and thereby lost 6 % of the cost; what was the cost?

178. I sell goods for \$ 384 more than they cost me and gain $12\frac{1}{2}$ %. What did the goods cost?

179. A merchant sold two boxes of goods for \$ 60 each; on one he gained 20 % and on the other he lost 20 %. Did he gain or lose by the transaction, and how much?

180. Paid \$ 2 for 3 pounds of tea, and sold 2 pounds for \$ 3. What is the per cent of profit?

181. Sold $\frac{1}{6}$ of a lot of apples for $\frac{1}{3}$ of the cost of the whole. What is the per cent of gain on the part sold?

182. If the price of flour is raised from \$ 6.25 to \$ $6.62\frac{1}{2}$, what per cent is the rise?

183. A person sold two farms for \$ 1890 each; for one he received 25 % more than he paid, for the other 25 % less. How much did he gain, or lose, by the transaction?

184. What per cent is made in buying coal by the *long* ton and selling it at the same nominal rate by the ton of 2000 pounds?

185. A grocer purchased a lot of teas, on which he lost 16 % by selling them for \$ 4200. What did he pay?

186. Bought land for $10 an acre; what must I ask for it if I pay for brokerage and other expenses of sale 4 % of the price obtained and make 20 % on the cost?

187. A merchant, finding a case of linen for which he paid $0.87½ a yard, damaged in transportation, sold it for 25 % less than the ordinary retail price, and yet sold it at cost. What was the ordinary retail price?

188. A man bought 2000 bushels of wheat for $1250. He finds 15 % of it worthless. For how much must he sell the remainder a bushel so as to gain 20 % on the whole cost?

189. A sells a horse to B, losing 10 % of the cost; B sells the horse to C, losing also 10 %; C sells him to D, gaining 10 %, and D sells him to E, gaining 10 %, and receiving $294.03. What did the horse cost A?

190. Bought a horse, buggy, and harness for $500. The buggy cost 37½ % less than the horse, and the harness cost 70 % less than the buggy. What was the cost of each?

191. A lot of gloves that cost me $0.80 a pair, getting wet, I sold at a reduction of 16⅔ % from the previous selling price, and yet gained 12½ % on the cost. What was the original selling price?

192. Sold a lot of goods for $200 and thereby gained 15 %; what per cent should I have gained had I sold them for $220?

193. A merchant sold a quantity of goods for $874. He deducts for cash 5 % of the bill, and finds that he has made 15 % on his investment. What did he pay for the goods?

194. A merchant shipped a cargo of flour worth $3597 from New York to Liverpool. For what must he insure it at 3¼ %, to cover the value of the flour and premium?

195. A school building is insured for ⅝ of its value, at a premium of 1¼ %, amounting to $125.25. What is the value of the building?

196. A house worth $12000 is insured for ⅔ of its value. What is the premium at ⅜ %?

197. For what sum must I insure a ship worth \$ 43500 so as to cover the property and the premium of 2 % ?

198. What will it cost to insure a store and stock worth \$ 25000 for $\frac{3}{4}$ of its value at $\frac{5}{8}$ %, including \$ 1.25 for the policy ?

199. What would be the cost of insuring my house for \$ 12750 at $\frac{3}{4}$ %, and \$ 1 for the policy ?

200. A house is worth \$ 3240. It is insured for $\frac{2}{3}$ of its value at $\frac{3}{4}$ %. What is the amount of the premium ?

201. What will it cost to insure a house worth \$ 6000, at $\frac{1}{2}$ % on $\frac{3}{4}$ of its value, and \$ 1800 worth of furniture on $\frac{4}{5}$ of its value at $\frac{3}{5}$ %, allowing \$ 1 to pay for the policy ?

202. Sold 1500 cases of boots @ \$ 52. Find the commission at 2 % and the proceeds.

203. Sent an agent \$ 4680 to invest in flour at \$ 7.50 a barrel, after deducting his commission of 3 %. How many barrels did he buy ?

204. A's commission of $\frac{1}{4}$ % for collecting the taxes of a town amounted to \$ 1150 ; what amount of taxes did he collect ?

205. A broker bought for me 150 shares of stock at 115 ; what did the stock cost me, including brokerage of $\frac{1}{4}$ % ?

206. My agent sells for me 5750 bushels of potatoes at 70 cents a bushel, for which he has a commission of $1\frac{1}{2}$ % ; he invests the net proceeds in iron at \$ 42 a ton, after deducting his commission of 1 % for buying. How many tons does he buy ?

207. A commission merchant sold 500 pieces of muslin, each piece containing 21 yards, at \$ 0.23 a yard. What is his commission at $2\frac{1}{2}$ % ?

208. A commission merchant sold consignments of cotton for \$ 5640, and after deducting his commission and \$ 76.50 for expenses, he remitted to the consignor \$ 5422.50 as the net proceeds. What per cent was his commission ?

209. Sent goods to my agent in Liverpool which he sold for $ 75000; what sum can he invest for me if his commission for selling is 2 % and for investing 1 %?

210. If a collector who charges 4 % commission on what he collects pays me $ 259.20 on a bill of $ 375, what per cent of the bill does he collect?

211. I send to my agent in New York $ 1700, with instructions to purchase woollen goods, after deducting his commission of 3½ %. What was his commission? What was the sum invested in goods?

212. Sent to my agent in Lyons $ 5640, which he is to invest in silks, after deducting his commission of 4 %. What can he invest and what is his commission?

213. Sent to my agent $ 5000, to invest in flour. He purchased 700 barrels at $ 6.75 a barrel, and charged me 3 % commission. For how much ought he to give me credit?

214. Sent my agent 10000 bushels of wheat which he sold for $ 0.56 a bushel, having a commission of ½ % for selling. I directed him to invest the proceeds in sugar at $ 0.05 a pound, after deducting his commission of 1 %. How many pounds did he buy?

215. My agent sells 500 cases of boots, of 12 pairs each, at $ 3.50 a pair, commission ½ %; he invests the net proceeds in cotton at 15 cents a pound, after deducting his commission of 2 %. How many pounds does he buy?

216. A merchant sold a quantity of goods for $ 18775. He deducts 4 % for cash, and then finds that he has made 10 %. What did he pay for the goods?

217. I sold goods on commission, kept 5 % for my services, and remitted the balance, $ 237.50. For how much did I sell the goods?

218. An agent collects 75 % of a bill of $ 160 and charges 5 % for collecting. What sum does the debtor pay? What sum does the agent retain? What does the creditor receive?

219. A creditor pays an agent for collecting 5 % and receives \$ 256.50 on a bill due from a debtor who pays only 45 %. How much does the agent retain? How much did the debtor pay? How much was the bill?

220. An agent who charges 5 %, receives \$ 15.75 for collecting a bill of a debtor who pays 65 %. What does the debtor pay? What does the creditor receive? What was the amount of the bill?

221. In Ex. 220, what per cent of the bill does the creditor receive? What per cent of this sum does the agent receive? What per cent of the whole bill does the agent retain?

(222.)

July 2, 1895.

ASA PACKER,

Bought of ROBERT JAMESON.

197 yds. carpet,	@	\$ 2.25 a yd.
32 " oil-cloth,	"	1.45 "
2 doz. handkerchiefs,	"	0.67 each
2 pieces of linen, each containing 26 yds.	"	0.75 a yd.

Make out Mr. P.'s bill, deducting 5 % from its face.

223. Mr. Lamson bought 100 shares of bank stock at 105, and sold them at a profit of \$ 350. What premium did he get?

224. A broker bought for me, at 12 % discount, 100 shares of mining stock of the par value of \$ 25 a share. A few days later he sold the same stock at $3\frac{1}{2}$ % discount. Allowing the broker a commission of $\frac{1}{8}$ % on each transaction, how much did I gain?

225. Bought 50 shares of railroad stock at $82\frac{3}{8}$ and sold it at $7\frac{5}{8}$ below par. How much did I gain?

226. The taxes of a certain village are \$ 1280; the property is valued at \$ 106000. What is the rate of taxation?

227. It is proposed to grade and pave a street one mile long, at a total expense of \$ 63360. How much are A, B, and

C assessed, who own respectively 24, 39, and 51 feet frontages on this street, provided the owners of property are assessed $\frac{1}{3}$ of the expense?

228. The real estate of a certain town is valued at $ 2400000, the personal property at $ 1800000, and there are 800 persons who pay a poll-tax of $ 1.50 each. If the town raises $ 64200, what is A's tax, whose property is inventoried at $ 12000, and who pays one poll-tax?

229. In a certain town a tax of $ 5000 is to be assessed. There are 500 polls, each assessed 75 cents, and the valuation of the taxable property is $ 370000. What will be the tax of a man whose property is valued at $ 7500, and who pays for two polls?

230. A owns real estate appraised at $ 5500, and personal property to the amount of $ 3700. What is his annual tax, the poll-tax being $ 2, and the rate of taxation being 16 mills on a dollar?

INTEREST.

Find the interest with the

	Principal.	Time.	Rate.
(231.)	$ 480,	11 m. 29 d.,	7.3 %.
(232.)	$ 756.84,	8 m. 17 d.,	6 %.
(233.)	$ 1250,	2 y. 3 m. 12 d.,	6 %.
(234.)	$ 45,	12 y. 27 d.,	$6\frac{3}{4}$ %.
(235.)	$ 342.50,	3 m. 29 d.,	7 %.
(236.)	$ 981.75,	6 m. 11 d.,	$7\frac{1}{2}$ %.
(237.)	$ 516.80,	4 m. 17 d.,	6 %.
(238.)	$ 107.93,	1 y. 3 m. 19 d.,	8 %.
(239.)	$ 1786,	3 y. 3 m. 3 d.,	6 %.
(240.)	$ 755,	4 y. 11 m. 27 d.,	8 %.

Find the amount with the

	Principal.	Time.	Rate.
(241.)	$ 250,	4 m. 25 d.,	6 %.
(242.)	$ 785.75,	6 m. 18 d.,	8 %.
(243.)	$ 247,	3 m. 25 d.,	5½ %.
(244.)	$ 375.18,	1 y. 4 m. 8 d.,	7 %.
(245.)	$ 415.25,	1 y. 8 m. 17 d.,	6½ %.
(246.)	$ 1201.09,	1 y. 5 m. 29 d.,	5 %.
(247.)	$ 2753,	1 y. 9 m. 16 d.,	6 %.
(248.)	$ 879.84,	2 y. 2 m. 2 d.,	6 %.
(249.)	$ 743.15,	2 y. 4 m. 8 d.,	7½ %.
(250.)	$ 1508.33,	3 y. 3 m. 3 d.,	7 %.

Find the interest with the

	Principal.	Time.	Rate.
(251.)	$ 238.50,	Sept. 22 to Jan. 3,	6 %.
(252.)	$ 5216.50,	Apr. 1 to Sept. 24,	7½ %.
(253.)	$ 720,	Mar. 10 to Oct. 3,	7.3 %.
(254.)	$ 5937.50,	Apr. 3 to Jan. 28,	6 %.
(255.)	$ 2486.75,	Mar. 13 to July 9,	7½ %.
(256.)	$ 3945,	June 9 to Oct. 12,	6 %.
(257.)	$ 1827.43,	April 10 to Nov. 7,	5 %.
(258.)	$ 2473.18,	Jan. 7 to May 19,	5½ %.
(259.)	$ 3786.32,	June 8 to Dec. 31,	6 %.
(260.)	$ 5863.45,	May 15 to Nov. 5,	7 %.

Find the amount with the

	Principal.	Time.	Rate.
(261.)	$ 32.25,	Nov. 15 to July 25,	7½ %.
(262.)	$ 75.37,	Jan. 8 to Aug. 30,	6 %.
(263.)	$ 87.45,	Feb. 17 to Oct. 8,	6 %.
(264.)	$ 175.67,	Sept. 5 to Dec. 18,	7 %.

	Principal.	Time.	Rate.
(265.)	$193.43,	July 8 to Oct. 14,	5 %.
(266.)	$217.65,	May 17 to Nov. 11,	7.3 %.
(267.)	$375.18,	Apr. 24 to Sept. 5,	6 %.
(268.)	$219.75,	June 7 to Dec. 1,	6½ %.
(269.)	$845.85,	Mar. 8 to Sept. 3,	5½ %.
(270.)	$2845.15,	Aug. 27 to Nov. 20,	6 %.

Find the interest with the

	Principal.	Time.	Rate.
(271.)	$72.50,	Aug. 8, 1894, to July 22, 1895,	6 %.
(272.)	$85.75,	Apr. 18, 1892, to Jan. 25, 1895,	6 %.
(273.)	$1843.15,	Oct. 15, 1894, to July 12, 1895,	7.3 %.
(274.)	$500,	Jan. 6, 1890, to July 2, 1895,	7 %.
(275.)	$2500,	Sept. 20, 1886, to Mar. 27, 1895,	4½ %.
(276.)	$876.26,	Oct. 12, 1889, to July 21, 1893,	7.3 %.
(277.)	$714.35,	May 1, 1894, to June 5, 1895,	5 %.
(278.)	$2875.18,	June 7, 1894, to Mar. 8, 1895,	6 %.
(279.)	$3635.76,	Sept. 5, 1894, to Aug. 21, 1895,	6 %.
(280.)	$5843.25,	Nov. 19, 1894, to Sept. 9, 1895,	7 %.

Find the amount with the

	Principal.	Time.	Rate.
(281.)	$87.45,	Nov. 8, 1894, to May 9, 1895,	6 %.
(282.)	$95.87,	Jan. 8, 1892, to Apr. 7, 1894,	6 %.
(283.)	$74.35,	May 29, 1894, to Sept. 12, 1895,	7 %.
(284.)	$115.75,	June 18, 1894, to July 5, 1895,	5 %.
(285.)	$227.25,	Sept. 6, 1893, to Jan. 1, 1895,	5½ %.
(286.)	$375.48,	Dec. 18, 1894, to Aug. 5, 1895,	6 %.
(287.)	$583.50,	Sept. 7, 1894, to July 8, 1895,	6½ %.
(288.)	$785.25,	July 9, 1894, to May 3, 1895,	7 %.
(289.)	$1275.55,	Oct. 27, 1893, to Jan. 3, 1895,	7.3 %.
(290.)	$3548.12,	Mar. 18, 1893, to July 7, 1894,	8 %.

Find the exact interest with the

	Principal.	Time.	Rate.
(291.)	$65.25,	Aug. 12, 1895, to Jan. 3, 1896,	6 %.
(292.)	$89.75,	Dec. 3, 1894, to July 9, 1895,	6 %.
(293.)	$175.50,	Feb. 12, 1894, to Mar. 8, 1895,	7 %.
(294.)	$250.15,	May 17, 1893, to Feb. 10, 1894,	6 %.
(295.)	$348.62,	Apr. 30, 1894, to Jan. 23, 1895,	5 %.
(296.)	$576.95,	Aug. 7, 1894, to Feb. 28, 1895,	$5\frac{1}{2}$ %.

297. What is the interest of $560, from June 5 to Sept. 4, at $\frac{3}{4}$ % a month?

298. What is the interest of $85.35, from Mar. 19 to July 8, at 1 % a month?

299. What is the interest of $117.85, from Jan. 17, 1895, to July 5, 1895, at 2 % a month?

Find the rate with the

	Principal.	Interest.	Time.
(300.)	$ 300,	$ 18,	9 m.
(301.)	$ 1200,	$ 48,	8 m.
(302.)	$ 45,	$ 8.10,	4 y. 6 m.
(303.)	$ 595,	$ 33.32,	8 m. 12 d.
(304.)	$ 800,	$ 40,	10 m.

Find the rate with the

	Amount.	Interest.	Time.
(305.)	$ 790.02,	$ 142.02,	3 y. 7 m. 25d.
(306.)	$ 2612.50,	$ 112.50,	6 m.
(307.)	$ 1746.25,	$ 96.25,	1 y. 5 m. 15 d.
(308.)	$ 54.99,	$ 9.99,	1 y. 10 m. 6 d.

Find the rate with the

	Amount.	Principal.	Time.
(309.)	$ 705,	$ 600,	2 y. 6 m.
(310.)	$ 226.20,	$ 180,	3 y. 6 m.
(311.)	$ 56.25,	$ 50,	2 y. 6 m.
(312.)	$ 163.30,	$ 142,	3 y.
(313.)	$ 600,	$ 500,	3 y.

Find the principal with the

	Amount.	Time.	Rate.
(314.)	$ 86.85,	2 y. 7 m. 18 d.,	6 %.
(315.)	$ 600,	2 y. 3 m. 24 d.,	7 %.
(316.)	$ 1524.10,	3 y. 10 m. 9 d.,	7 %.
(317.)	$ 485.40,	1 y. 3 m.,	5 %.
(318.)	$ 129332.04,	2 y. 18 d.,	8 %.

319. What is the present worth of $ 325, due in 1 y. 6 m. 20 d., at 8 %?

320. What is the present worth of a note at 30 days without grace, for $ 2072.25, when interest is 5 %?

321. A note for $ 520 was given Oct. 1, 1894, payable in one year without grace and without interest. What was its value Feb. 1, 1895, money being worth 6 %?

322. What is the present worth of $ 500, due in 1 y. 6 m., at 6 %?

323. What is the present worth of $ 756, due in 1 y. 4 m.?

324. Find the present worth of $ 890, due in 1 y. 6 m., money being worth 8 %.

325. If I pay a debt of $ 1410, 2 y. 6 m. before it is due, what discount should be made, money being worth 7 %?

Find the time with the

	Principal.	Interest.	Rate.
(326.)	$450,	$6.75,	6 %.
(327.)	$725,	$10.15,	7 %.
(328.)	$560,	$9.10,	6½ %.
(329.)	$175,	$39.37,	7 %.
(330.)	$450,	$18.125,	5 %.

Find the time with the

	Amount.	Interest.	Rate.
(331.)	$150,	$29.63,	6 %.
(332.)	$909.45,	$63.45,	6 %.
(333.)	$908,	$158,	8 %.
(334.)	$7588,	$588,	7 %.

Find the time with the

	Amount.	Principal.	Rate.
(335.)	$179.85,	$165,	6 %.
(336.)	$3775,	$3000,	10 %.
(337.)	$298.45,	$254,	5 %.

338. In what time will the interest of $120, at 8 %, equal the principal?

Find the principal with the

	Interest.	Time.	Rate.
(339.)	$13,	8 m.,	6 %.
(340.)	$800,	2 y. 6 m.,	10 %.
(341.)	$171.70,	3 y. 4 m. 12 d.,	6 %.
(342.)	$384,	4 y. 3 m. 6 d.,	6 %.
(343.)	$627,	1 y. 10 m. 24 d.,	7 %.

344. What sum must I invest at 6 % to have an annual income of $1000?

345. At what rate will a sum of money at simple interest triple itself in 37 y. 6 m.?

346. How long does it take a sum of money to quadruple itself at 8 %?

347. Find the date at which $500, put at 6 % interest July 8, 1895, will amount to $876.25.

348. At what date will a sum of money which is put at 8 % interest Jan. 4, 1895, be tripled?

349. The interest of a certain sum for 4 years was half of the sum. What was the rate?

350. Find the date at which a sum of $234, which was put at simple interest at 9 % Oct. 25, 1890, amounted to $351.

PARTIAL PAYMENTS.

(351.)

$3400. BOSTON, MASS., July 7, 1892.

On demand, I promise to pay James Tucker, or order, three thousand four hundred dollars, with interest. Value received.

JOHN FLETCHER.

INDORSEMENTS: Sept. 13, 1893, $1250; Mar. 4, 1895, $150. What was due May 1, 1895?

(352.)

$325. LOWELL, MASS., Sept. 2, 1892.

On demand, I promise to pay J. R. Wait, or order, with interest at 8 %, three hundred twenty-five dollars. Value received.

D. F. JOHNSON.

INDORSEMENTS: July 17, 1893, $55.50; Aug. 7, 1894, $15; Dec. 17, 1895, $84. What was due Sept. 2, 1896?

(353.)

$17440. NEWPORT, R. I., March 7, 1891.

Three months after date, I promise to pay J. F. Smith, or order, seventeen thousand four hundred forty dollars, with interest. Value received. F. W. SARGENT.

INDORSEMENTS: June 23, 1891, $5000; Oct. 20, 1892, $1000; Jan. 7, 1893, $1225; July 6, 1893, $500.

What was due June 19, 1894?

(354.)

$445. PORTLAND, ME., May 19, 1891.

On demand, I promise to pay James Clarke, or order, four hundred forty-five dollars, with interest. Value received.

J. P. PETERS.

INDORSEMENTS: Jan. 12, 1893, $25; Feb. 19, 1894, $208.94; May 8, 1895, $144.17. What was due Jan. 16, 1896?

(355.)

$785. PROVIDENCE, R. I., June 7, 1887.

Eight months after date, I promise to pay E. D. Fox, or order, with interest, seven hundred eighty-five dollars. Value received.

P. J. LOAMES.

INDORSEMENTS: July 10, 1889, $50; Aug. 13, 1891, $75; Dec. 26, 1893, $300; July 3, 1894, $35.

What was due March 7, 1895?

(356.)

$700. ALBANY, N. Y., Dec. 7, 1887.

On demand, I promise to pay P. White, or order, seven hundred dollars, with interest at 7%. Value received.

W. F. WHITMAN.

INDORSEMENTS: Jan. 9, 1889, $44.50; Sept. 7, 1891, $407.75, July 3, 1894, $25. What was due July 17, 1896?

(357.)

$1200. BOSTON, May 10, 1894.

On demand, I promise to pay John Andrew, or order, one thousand two hundred dollars, with interest. Value received.

JAMES SMITH.

INDORSEMENTS: Aug. 6, 1894, $250; Dec. 20, 1894, $15; April 5, 1895, $18.50; July 8, 1895, $375; Dec. 10, 1895, $100. What was due April 10, 1896?

358. A note of $2584, dated Feb. 17, 1891, had the following

INDORSEMENTS: Aug. 17, 1891, $500; May 25, 1892, $100; Oct. 19, 1893, $900. What was due Jan. 1, 1895?

359. A note of $750, dated March 5, 1890, had the following

INDORSEMENTS: Jan. 10, 1891, $200; Oct. 11, 1894, $300. What was due March 3, 1896?

360. A note of $757.14, dated May 17, 1893, had the following

INDORSEMENTS: Jan. 8, 1894, $40; July 9, 1894, $20; July 30, 1894, $500; May 17, 1896, $20.14.

What was due July 30, 1896?

361. May 12, 1893, I gave a note for $680 on demand, with interest at 6 %. Aug. 22, 1894, paid $50; April 18, 1895, paid $95. What was due July 7, 1896?

362. On a note for $425, at 8 %, dated March 25, 1895, were the following

INDORSEMENTS: June 1, 1895, $75; Dec. 30, 1895, $120. What was due Sept. 1, 1896?

363. A man buys a lot of land for $5000, and at the end of each year pays $500 towards principal and interest (6 %). How much will be due after the fifth payment?

(364.)

$317.35. BALTIMORE, MD., July 5, 1894.

On demand, I promise to pay J. A. Lenter, or order, three hundred seventeen dollars $\frac{35}{100}$, with interest. Value received.

JAMES W. MAYNARD.

INDORSEMENTS: Sept. 3, 1894, $65; Nov. 13, 1894, $75; Jan. 7, 1895, $150. What was due May 1, 1895?

(365.)

$75. NEW YORK, Jan. 2, 1895.

On demand, I promise to pay James Osgood, or order, seventy-five dollars, with interest at 6 %. Value received.

EMMONS WISE.

INDORSEMENTS: May 6, 1895, $15; June 18, 1895, $25. What was due July 31, 1895?

Find the amount at compound interest with the

	Principal.	Time.	Rate.
(366.)	$ 530,	3 y. 2 m. 19 d.,	6 %.
(367.)	$ 325,	2 y. 8 m. 12 d.,	6 %.
(368.)	$ 25,	3 y. 5 m.,	6 %.
(369.)	$ 571.45,	4 y. 4 m. 4 d.,	6 %.
(370.)	$ 200,	30 y.,	7 %.
(371.)	$ 300,	20 y.,	6 %.

Find the compound interest with the

	Principal.	Time.	Rate.
(372.)	$ 850,	3 y. 6 m. 10 d.,	6 %.
(373.)	$ 252,	2 y. 6 m. 24 d.,	6 %.
(374.)	$ 1100,	3 y.,	7 %.
(375.)	$ 635,	2 y. 3 m. 6 d.,	8 %.

376. What is the amount of $ 500, from Jan. 1 to Oct. 25, at 8 %, compounded quarterly?

377. What is the amount of $ 1200 for 2 years at 6 %, compound interest, payable quarterly?

378. If $ 2500 at compound interest amounts to $ 4320 in 3 years, what is the rate? What is the rate if it is at simple interest?

379. What is the present worth of $ 800, due in 3 years, money being worth 6 %, compound interest?

380. What sum of money at 10 %, compound interest, will amount to \$ 7320.50 in 4 years?

381. What is the compound interest of \$ 1 for 71 years, allowing it to double every 11 years 10 months?

382. Find the compound interest, at 6 %, of \$ 2500 for 1 y. 8 m., interest compounded quarterly.

BANK DISCOUNT.

Find the discount and the proceeds of a note with the

	Face.	Date.	Time.	Discounted.	Rate.
(383.)	\$ 640,	May 1,	4 m.,	July 3,	8 %.
(384.)	\$ 287.50,	Mar. 18,	3 m.,	May 1,	7 %.
(385.)	\$ 186.75,	Oct. 12,	6 m.,	Dec. 9,	$7\frac{1}{2}$ %
(386.)	\$ 625,	Feb. 22,	75 d.,	Mar. 19,	6 %.
(387.)	\$ 600,	Sept. 15,	90 d.,	Nov. 20,	6 %.
(388.)	\$ 500,	Apr. 19,	60 d.,	May 19,	7 %.
(389.)	\$ 846,	June 20,	45 d.,	July 5,	6 %.
(390.)	\$ 755.64,	May 7,	90 d.,	May 7,	6 %.
(391.)	\$ 417.13,	Sept. 6,	4 m.,	Sept. 6,	7 %.
(392.)	\$ 639.50,	Mar. 3,	90 d.,	Mar. 15,	6 %.
(393.)	\$ 2500,	May 7,	4 m.,	July 7,	6 %.
(394.)	\$ 200,	Dec. 26,	30 d.,	Dec. 26,	7 %.

Find the avails of a note with the

	Face.	Date.	Time.	Discounted.	Rate.
(395.)	\$ 470.66,	Feb. 7,	60 d.,	Feb. 7,	6 %.
(396.)	\$ 26.21,	Apr. 19,	4 m.,	June 25,	$7\frac{1}{2}$ %.
(397.)	\$ 418.90,	Nov. 11,	6 m.,	Jan. 3,	7 %.
(398.)	\$ 568.40,	Oct. 17,	3 m.,	Nov. 20,	6 %.
(399.)	\$ 981.50,	Sept. 25,	5 m.,	Nov. 30,	7 %.
(400.)	\$ 784,	July 10,	100 d.,	Aug. 23,	7 %.

	Face.	Date.	Time.	Discounted.	Rate.
(401.)	\$ 625,	Feb. 9,	90 d.,	Mar. 1,	$7\frac{1}{2}$ %.
(402.)	\$ 524.10,	Apr. 19,	75 d.,	June 3,	$7\frac{1}{2}$ %.
(403.)	\$ 3000,	July 17,	3 m.,	July 17,	7 %.
(404.)	\$ 1975,	May 1,	60 d.,	May 1,	7.3 %.
(405.)	\$ 700,	Nov. 1,	69 d.,	Nov. 1,	6 %.
(406.)	\$ 5768.45,	Jan. 7,	4 m.,	Jan. 7,	6 %.
(407.)	\$ 7543,	June 15,	4 m.,	July 15,	7.3 %.
(408.)	\$ 600,	June 21,	60 d.,	July 21,	8 %.
(409.)	\$ 307.65,	Feb. 13,	60 d.,	Feb. 13,	7 %.
(410.)	\$ 513.50,	May 17,	90 d.,	July 20,	$7\frac{1}{2}$ %.
(411.)	\$ 700,	Apr. 8,	4 m.,	Apr. 8,	6 %.
(412.)	\$ 400,	Dec. 7,	$3\frac{1}{2}$ m.,	Dec. 7,	7 %.

413. What is the difference between the bank discount and the true discount (Art. 353) of a 60 days' note for \$ 450, discounted at 6 %?

414. What is the difference between the true and bank discount of a 3 months' note for \$ 7867.45, at 7 %?

415. To obtain \$ 17865, from a bank on a note due in 3 months, discounting at 6 %, what must be the face of the note?

416. A merchant wishes to borrow \$ 1000 at a bank. For what amount must he make a 60 days' note, if he gets it discounted at 6 %?

417. How large a note, having 4 months to run, must I offer for discount, at a bank discounting at 6 %, in order to realize \$ 500 net?

418. If I wish to get \$ 75755.14 from a bank discounting at 7 %, on a 60 days' note, what sum must be named in the note?

419. What must be the face of a note at 60 days, the proceeds of which, when discounted at a bank at 6 %, are \$ 100?

420. For what principal must I write a note on 4 months, at $7\frac{3}{10}$ % per annum, to enable me to receive at a bank, on its date, $ 300?

421. For what sum must a 90 days' note be written that the proceeds may be $ 360, discount being 6 %?

422. Wishing to borrow $ 500 at a bank, for what sum must my note be given at 90 days, discount being 7 %?

423. For what sum must a note be written, payable in 4 months, that I may receive $ 640 on it when discounted at a bank, at 6 %?

424. For what sum must a 60 days' note be written in order to receive from a bank, discounting at 7 %, $ 540?

425. I desire to use to-day $ 680, which I can obtain by getting my note at 30 days discounted at a bank at 5 %. For what sum must I write the note?

426. A owns $\frac{3}{4}$ of a ship which is worth $ 68000. If B buys $\frac{2}{3}$ of A's share at this rate, for what sum must B draw a 60 days' note that A may get his money from the proceeds obtained from a bank, discounting at 7 %?

427. For what sum must I give my note, payable in 4 m. 12 d., at a bank, discounting at 7 %, to obtain $ 973.75?

428. What must be the face of a 4 months' note to obtain, on its date, $ 1550, at 6 %?

429. For what sum must a note on 5 months, at 8 %, be written, to obtain $ 450 from a bank?

430. What must be the face of a 90 days' note to obtain $ 100 at a bank discounting at 6 %?

EQUATION OF PAYMENTS.

431. May 7, 1895, a merchant has due him $ 300 to be paid in 60 days, $ 500 to be paid in 120 days, and $ 750 to be paid in 180 days. What is the equated time for the payment of the whole?

432. Dec. 12, 1895, a man buys \$500 worth of goods on 3 months' credit; \$600 on 4 months'; \$300 on 6 months'; and \$1000 on 2 months'. What is the equated time of payment?

433. John Nay holds three notes, the first of \$350, due Mar. 13, 1895, the second of \$500, due July 5, 1895, the third of \$275, Sept. 30, 1895. What is the equated time for the payment of the three?

434. March 7, 1895, A owes B \$470 due April 21, \$310 due May 30, and \$850 due June 9. Find the equated time of payment.

435. There is due to a merchant \$800, $\frac{1}{6}$ of which is to be paid in 2 months, $\frac{1}{3}$ in 3 months, and the remainder in 6 months; but the debtor agrees to pay $\frac{1}{2}$ down. How long may the debtor retain the other half so that neither party shall sustain loss?

436. Aug. 7, 1895, a merchant in Brooklyn owes John Jones \$2000, of which \$500 is to be paid down, \$275 in 5 months, \$400 in 8 months, and the remainder in 10 months. What is the equated time of payment?

437. William Durgin buys goods on 6 months' credit, and the date of the delivery and the amounts are as follows: \$575, March 18, 1895; \$415, April 7; \$833, May 8; and \$326, May 19. Find the equated date of payment.

438. Bought the following bills on 45 days: Jan. 16, 1895, \$1145; Jan. 27, \$513; Feb. 1, \$215; Feb. 7, \$672. What is the equated time of payment?

439. Find the equated time for paying the balance of the following account.

Dr.	JAY, FARWELL & CO.				Cr.
1895.			*1895.*		
Jan. 29	Mdse., 2 m.	\$519	Jan. 2	Mdse., 3 m.	\$357
Feb. 5	Mdse., 3 m.	423	" 25	Mdse., 2 m.	738
" 19	Mdse., 2 m.	967	Feb. 4	Mdse., 3 m.	421

440. The following account was settled by a 5 months' note. When should the note be dated to be due (with grace) at the equated time of payment?

Dr. FRANCIS STOCKTON. **Cr.**

1894.			1894.		
Feb. 13	Mdse., 4 m.	$985	Feb. 3	Mdse., 2 m.	$769
" 19	Mdse., 2 m.	744	" 15	Mdse., 3 m.	321
" 27	Mdse., 3 m.	659	" 26	Mdse., 2 m.	565
Mar. 8	Mdse., 2 m.	475	Mar. 5	Mdse., 3 m.	913

441. The following items were all bought on 30 days. What is the equated time for payment of the balance of the account?

Dr. SWAN, HICKS & CO. **Cr.**

1895.			1895.		
May 3	Mdse.,	$85	Apr. 29	Mdse.,	$117
" 8	Mdse.,	55	May 5	Mdse.,	87
" 12	Mdse.,	317	" 14	Mdse.,	113
" 23	Mdse.,	97	" 17	Mdse.,	210
" 31	Mdse.,	212	" 21	Mdse.,	128

BONDS.

442. What will be my annual income from $7350, invested in United States 4's at 98?

443. My income from an investment in $4\frac{1}{2}$ % bonds is $900. The bonds cost me a premium of 3 %. How much did I invest?

444. If I invest my capital in United States 5's at $104\frac{1}{4}$, what per cent on my investment do I receive?

445. When 5 % government bonds are at 104, what amount must be invested in them to yield an annual income of $800?

446. What is the least sum of money that must be invested in $100 bonds paying 6 % annually at $3\frac{1}{2}$ % premium, to bring from them an annual income of not less than $1000?

447. From which shall I get the greater per cent of income, an investment in United States 5's at 104¼, or in United States 4's at 99½?

448. If I buy bonds at 9½ above par and receive a semi-annual interest of 3½ %, what per cent do I receive annually on my investment, allowing 7 % interest on the interests received?

449. What do I receive for 32 State Bonds, par value $ 100, which a broker sells for me at 15¼ % discount (brokerage ¼ %)?

450. Which pays the greater per cent on the investment, and how much greater, 6 % bonds at 90, or 8 % bonds at 130?

451. If both are redeemed at the end of two years from their purchase, which is the better investment, Boston 6's at 102, or Boston and Maine 7's at 108?

452. In order to realize 6 % annually for my investment, what must I give for bonds that pay a semi-annual interest of 3 % if I immediately reinvest the semi-annual interest at 6 %?

453. What sum of money must be invested in $ 100 bonds that sell for 95½, that pay 6 % annually, to produce an income of $ 1500?

454. If 5 % bonds are bought at 90, what is the rate of income on the investment?

455. A person having 49 hundred-dollar bonds of the denomination 4½'s sold them at 104 and invested the proceeds in 4's at 98. Did he increase or diminish his income, and how much?

PARTNERSHIP.

456. Divide $ 620 among three persons so that their shares shall be in the proportion of 3, 7, and 10.

457. Three persons engage in trade with a joint capital of $ 37680. A puts in $ 6 as often as B puts in $ 10, and as often as C puts in $ 14. Their annual gain is equal to C's stock. What is each partner's gain?

458. Three men enter into partnership: A furnished \$ 4000 and B \$ 6000; they gained \$ 1680, of which C's share was \$ 840. Required C's stock, and A's and B's gain.

459. Four companies have 60, 90, 150, 225 men respectively. The general wants 70 men. How many must each company furnish?

460. H, P, and S have a capital of \$ 1440. They gain \$ 1080, of which H has \$ 3 as often as P \$ 5 and S \$ 7. What is the sum of the capital and gain of each?

461. A and B agree to divide their year's profits, \$ 7980, so that A shall have more than B by 10 % of the whole. What is the share of each?

462. What is the share of each if A takes of the \$ 7980 more than B by 10 % of B's?

463. What is the share of each if B takes of the \$ 7980 less than A by 10 % of A's?

464. Divide \$ 299 into three parts which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{8}$.

465. Three persons purchased a horse together. A gave \$ 20, B gave 40 % more than A, and C gave $15\frac{1}{2}$ % less than both the others. What fractional part of the horse does each own?

466. A and B were partners: A put in \$ 3626.25, and B \$ 6527.25. They gained \$ 2626.68. What was the gain of each?

467. Divide \$ 8600 into shares that shall be to each other as 8, 7, 6, and $\frac{1}{2}$.

468. A, B, and C enter into partnership; A put in \$ 500 for 18 months; B, \$ 380 for 13 months; C, \$ 270 for 9 months. They lost \$ 818.50; what was each man's loss?

469. A, B, and C formed a copartnership. A furnished $\frac{2}{5}$ of the capital for 6 months; B $\frac{1}{3}$ of the capital for 10 months; C the balance for 12 months. The whole gain was \$ 1560. What was the gain of each?

470. A, B, and C entered into partnership Jan. 1, 1895. A put in $ 5000, B $ 7000, and C $ 10000. Four months later they took in D, who put in $ 4000. Jan. 1, 1896, on settling they found that they had made $ 8000. What was each man's share of the gain?

471. A, B, and C began business Jan. 1, with $ 650, furnished by A; April 1, B put in $ 500; July 1, C put in $ 450. The profit for the year was $ 375; divide it.

472. A and B enter into partnership. A puts in $ 6000 Jan. 1, B puts in nothing until April 1. How much must he then put in, in order that the partners may share the gain equally at the end of the year?

473. Jan. 1, 1895, three persons began business with $ 1300 furnished by A; March 1, B put in $ 1000; Aug. 1, C put in $ 900. The profits at the end of the year were $ 750. Find the gain of each partner.

474. Ames and Howe entered into partnership the 1st of January, and each put in $ 2000. The 1st of May Ames put in $ 1000 more. At the end of the year the profits proved to be $ 2800. What should each receive?

475. Three men agree to do a piece of work for $ 7832. The first man furnishes 25 men, 24 days, 9 hours a day; the second 20 men, 25 days, 10 hours a day; the third 30 men, 20 days, 12 hours a day. How much ought each of the contractors to receive?

476. A, B, and C hire 10 acres of pasture for $ 140, in which A keeps 60 head of cattle for 20 days, B 20 head for 40 days, and C 30 head for 30 days. What proportion of the rent should each pay?

477. Jan. 1, 1895, A, B, and C form a partnership for 1 year, and each furnishes $ 2000. May 1, A furnishes $ 1000 more; June 1, B furnishes 1500, and C withdraws $ 500; Oct. 1, A withdraws $ 500, and B and C furnish $ 1000 each. Having gained $ 3000, at the close of the year the partnership is dissolved. What is each partner's share of the gain?

478. A, B, and C hire a pasture for \$ 106. A pastures 10 horses for 9 weeks, B 60 cows for 12 weeks, and C 15 oxen for 8 weeks. If 5 horses are counted as equal to 12 cows, and 4 cows to 3 oxen, how much ought each man to pay?

479. Jan. 1, A and B form a partnership, A putting in \$ 4000, and B \$ 6000. April 1, A put in \$ 2000 more, and June 1, B put in \$ 1000 more. Oct. 1, each took out \$ 1300; at the end of the year they had gained \$ 7000. What share of the gain belonged to each?

480. A begins business with \$ 5000; at the end of 2 months B puts in \$ 3000; at the end of 3 months C puts in \$ 6000. At the end of 5 months the profits amount to \$ 10560. What is each man's share of the stock and of the gain?

481. A's capital is in 8 months, B's 9 months, and C's 10 months. A's gain is \$ 720, B's \$ 540, C's \$ 980. What part of the whole capital, \$ 8610, did each own?

PROPORTION, OR ANALYSIS.

(482.) $8 : 15 = 12 : ?$.	(486.) $4 : ? = ? : 16$.
(483.) $9 : 16 = ? : 40$.	(487.) $\frac{4}{5} : 5\frac{1}{2} = ? : \frac{11}{24}$.
(484.) $18 : ? = 3 : 50$.	(488.) $? : 2\frac{1}{3} = \frac{1}{4} : \frac{3}{7}$.
(485.) $? : 40 = 30 : 50$.	(489.) $\frac{5}{8} : ? = \frac{1}{3} : \frac{2}{5}$.

490. What is the ratio of $\frac{4}{9}$ of 6 to 4?

491. The consequent is $3\frac{1}{4}$, the antecedent $\frac{13}{28}$; what is the ratio?

492. The ratio is $2\frac{2}{5}$, the antecedent $\frac{1}{2}$ of $\frac{5}{8}$; what is the consequent?

493. What is the mean proportional between 16 and 64?

494. Find a mean proportional between 12 and 0.75.

495. What is the third proportional to 3 and 30?

496. If \$ 250 will buy 18 yards of cloth, how many yards can be bought for \$ 147?

497. If a church steeple 150 feet high casts a shadow 275 feet, what will be the length of a shadow cast by a man 6 feet tall?

498. A garrison of 800 men have provisions enough to last them 60 days, allowing each man 2 pounds a day. After 20 days a detachment of 200 men leave; how long will the remaining provisions supply the men that remain?

499. For what must hay be sold a ton to gain 12 %, if by selling it at $ 16 there is a gain of 28 %?

500. If a staff 4 feet long cast a shadow 6 feet long, what is the height of a steeple whose shadow at the same time is 173 feet?

501. If a 5-cent loaf weighs 8 ounces when flour is $ 6 a barrel, what ought it to weigh when flour is $ 7.50 a barrel?

502. A takes 6 steps while B takes 7; but 4 of A's steps equal 5 of B's. Which is the faster walker?

503. If 6 men can build 73 feet of wall 4 feet high in 5 days, how many feet can they build in 33 days?

504. How much will it cost to make a walk 8 feet wide and 225 feet long, when it costs $ 30 to make one 10 feet wide and 90 feet long?

505. If 4 men can dig a ditch 24 rods long in 20 days, how many rods can 6 men build in 8 days?

506. If 9 horses eat 42 bushels of grain in 12 days, how many bushels would be required for 63 horses 7 days?

507. A block of granite is $8 \times 5\frac{1}{2} \times 2\frac{1}{4}$ feet, and weighs 16335 pounds; what is the weight of a granite column 20 feet high and $2\frac{1}{2}$ feet square?

508. If I employ 25 men 24 days of 10 hours each to do a piece of work, how many men must I employ to do 6 times as much in 30 days of 8 hours each?

509. If $ 400 in 9 months at 7 % produce $ 21, what will be the interest on $ 360 for 8 months at 6 %?

510. If 6 men can build a wall 80 rods long, 10 feet high, 9 feet thick, in 100 days of 9 hours, how many days of 10 hours will be required by 15 men to build a wall 200 rods long, 9 feet high, and 5 feet thick?

511. If 6 men in 15 days earn $ 135, how many dollars will 9 men earn in 18 days?

512. If a bin 8 feet long, 4½ feet wide, and 2½ feet deep, hold 67½ bushels, how deep must another bin be made, that is 18 feet long and 3⅝ feet wide, to hold 450 bushels?

513. If 6 men, in 16 days of 9 hours each, build a wall 20 rods long, 6 feet high, and 4 feet thick, in how many days of 12 hours each, will 28 men build a wall 400 rods long, 4 feet high, and 8 feet thick?

514. If 14 men, working 8 hours each day, can do a piece of work in 12 days, how many men will do the same amount of work in 16 days, if they work 7 hours each day?

515. If 36 men, in 20 days of 10 hours each, can make a wall 100 rods long, 4 feet high, and 3 feet thick, in how many days of 12 hours each, can 10 men make a wall 75 rods long, 5 feet high, and 4 feet thick?

516. If an army of 5000 men have provisions for 15 days, at the rate of 20 ounces a day to each man, and they are reinforced with 3000 men, upon what daily allowance must each man be put that the provisions may last 20 days?

517. A vessel has provisions for 50 men for 4½ months, allowing each man 1½ pounds a day. How long would the same provisions furnish 75 men, allowing 14 ounces a day to each man?

518. If 5 men can dig a ditch 25 rods long, 4 feet deep, and 3 feet wide, in 7 days of 8 hours each, how many men will it take to dig a ditch 750 rods long, 4½ feet deep, and 5 feet wide, in 14 days of 9 hours each?

519. If 7 men can mow a field 17 rods wide and 50 rods long in 2 days, mowing 5 hours a day, how many days will it

take 2 men to mow a field 40 rods wide and 51 rods long, if they mow 4 hours a day?

520. If 8 men can build 30 rods of a stone-wall in 18 days, how many men will it take to build 480 rods of the same wall in 16 days?

521. If a family of 13 persons can live on \$468 for 4 months, how many dollars would support the family for 9 months if 3 persons more are added?

522. If 6 men can build a wall 80 rods long, 6 feet high, and 3 feet thick in 84 days, working 10 hours a day, how many days will it take 30 men to build a similar wall 500 rods long, $3\frac{3}{4}$ feet thick, and 8 feet high, if they work only 7 hours a day?

523. If 4 men dig a ditch 98 rods long, 2 feet wide, and 1 foot deep in 5 days of 8 hours each, how many men can dig a ditch 588 rods long, $1\frac{1}{2}$ feet wide, and 3 feet deep in 20 days of 9 hours each?

524. If 30 men build a wall 90 rods long, 8 feet high, and 4 feet thick in 16 days of 10 hours each, how many days of 8 hours each will 60 men require to build a wall 120 rods long, 10 feet high, and 3 feet thick?

525. If 15 pipes, each delivering 15 gallons a minute, fill a cistern in 4 hours 16 minutes, how many pipes, each delivering 18 gallons a minute, will it take to fill a cistern 4 times as large in 8 hours 32 minutes?

526. If 8 iron bars, each 5 feet long, 4 inches wide, and 3 inches thick, weigh 960 pounds, what is the weight of 28 bars, each $6\frac{1}{2}$ feet long, $3\frac{1}{2}$ inches wide, and $2\frac{1}{2}$ inches thick?

527. If 25 men can do a piece of work in 15 days, how many men will it take to do 4 times as much in $\frac{1}{3}$ of the time?

528. How deep must a bin 6 feet long and 5 feet wide be to hold 180 bushels?

529. If a bin 7 feet long, $2\frac{1}{2}$ feet wide, and 2 feet deep holds 28 bushels, how deep must a bin 20 feet long, 1 foot 6 inches wide, be to contain 116 bushels?

530. If a workman charges \$24 for 8 days' work of 8 hours each, what should he charge for 6 days' work of 10 hours each?

531. If a bar of silver 1 foot 6 inches long, 4 inches wide, and 2 inches thick is worth \$1240, what is the value of a bar of gold 1 foot 3 inches long, 8 inches wide, and 1 inch thick, the weight of a cubic inch of silver being to the weight of a cubic inch of gold as 10 to 19, and the value of an ounce of silver to an ounce of gold as 2 to 33?

532. If 18 men can build a wall 140 rods, 6 feet high, 3 feet thick, in 9 days, working 10 hours a day, how many hours a day must 25 men work to build a wall 220 rods long, 5 feet high, and 4 feet thick, in 12 days?

533. If 6 men dig a trench 15 yards long, 4 yards broad, and 5 feet deep in 3 days of 12 hours, in how many days of 8 hours will 8 men dig a trench 20 yards long, 8 yards broad, and 8 feet deep?

534. What is the cost of $36\frac{1}{2}$ yards of cloth, $1\frac{1}{2}$ yards wide, if $2\frac{1}{2}$ yards, $1\frac{3}{8}$ yards wide, cost \$$3.37\frac{1}{2}$?

535. If 7 men build 280.5 rods of wall in 121 days, in how many days will 11 men build 170 rods?

536. If a horse at the rate of $6\frac{1}{2}$ miles an hour can travel $\frac{3}{8}$ of a certain journey in $4\frac{3}{8}$ days, how many days will it take him to travel $\frac{4}{7}$ of the same journey at the rate of $8\frac{3}{8}$ miles an hour?

537. If 6 men working 9 hours a day can build a wall in 15 days, in how many days will a party of men build this wall, working 10 hours a day, if the number of men is equal to the number of days?

538. If a man spends every 5 months what he earns in 2 months, and earns \$530 every $2\frac{1}{2}$ months, how much does he save in a year?

POWERS AND ROOTS.

539. What is the cube of $\frac{7}{3}$? of 0.007?

540. What is the fourth power of 5? of 0.5? of 0.005?

541. What is the third power of 30? of 3? of 0.3? of 0.03?

542. Compare 10 times the third power of 6 with 10 times 6 multiplied by 3.

543. Find the square root of 0.4; of 0.04; of $1\frac{1}{4}$.

544. Find the square root of 452.634.

545. What is the square root of 134.8575?

546. What is the square root of 0.9? of 0.09?

547. What is the square root of 747.4756?

548. Compare the square root of 35 with 10 times the square root of 3.5.

549. Multiply the $\sqrt{\frac{98}{18}}$ by $\sqrt{1014049}$.

550. Find the square root of 0.035; of 2.5; of $2\frac{1}{4}$; of $\frac{2}{3}$; of $\frac{1008}{1183}$.

Find the value of

(551.) $\sqrt{1296}$.

(552.) $\sqrt{106929}$.

(553.) $\sqrt{196.8409}$.

(554.) $\sqrt{78.6769}$.

(555.) $\sqrt{49.2804}$.

(556.) $\sqrt{104.8576}$.

(557.) $\sqrt{747.4756}$.

(558.) $\sqrt{4.190209}$.

(559.) $\sqrt{442.6816}$.

(560.) $\sqrt{733.3264}$.

Find the value of

(561.) $\sqrt{0.5}$.

(562.) $\sqrt{79}$.

(563.) $\sqrt{417}$.

(564.) $\sqrt{287}$.

(565.) $\sqrt{3.1}$.

(566.) $\sqrt{25.39}$.

(567.) $\sqrt{531.5}$.

(568.) $\sqrt{195}$.

(569.) $\sqrt{8175}$.

(570.) $\sqrt{45.18}$.

571. How many more feet of fence will it take for a rectangular lot 81 rods long and 25 rods wide, than for a square lot of the same area?

572. What is the least integral factor by which 2016 can be multiplied so that the product shall be a perfect square?

573. What is the least integral factor by which 2016 can be divided so that the quotient shall be a perfect square?

574. What is the least integral factor by which 792 can be multiplied so that the product shall be a perfect cube?

Find the value of

(575.) $\sqrt[3]{2197}$.

(576.) $\sqrt[3]{21952}$.

(577.) $\sqrt[3]{3176523}$.

(578.) $\sqrt[3]{13144256}$.

(579.) $\sqrt[3]{731432.701}$.

(580.) $\sqrt[3]{1.157625}$.

(581.) $\sqrt[3]{341532099}$.

(582.) $\sqrt[3]{748.613312}$.

(583.) $\sqrt[3]{39512.447416}$.

(584.) $\sqrt[3]{751.089429}$.

Find the value of

(585.) $\sqrt[3]{49\frac{8}{27}}$.

(586.) $\sqrt[3]{34568.63}$.

(587.) $\sqrt[3]{9358}$.

(588.) $\sqrt[3]{51}$.

(589.) $\sqrt[3]{77869}$.

(590.) $\sqrt[3]{0.0001488777}$.

(591.) $\sqrt[3]{6100}$.

(592.) $\sqrt[3]{751}$.

(593.) $\sqrt[3]{15}$.

(594.) $\sqrt[3]{0.0093}$.

595. Find the cube root of 8; of $\frac{125}{729}$; of 1.25; of $6\frac{1}{4}$; of 0.008.

596. The cube of 3.5 is the square root of what number?

597. Find the cube root of 0.27; of 0.027.

598. What is the cube root of 8.144865728?

599. What is the cube root of 205692449327.

600. Find the cube root of $1\frac{1}{8}$.

601. Find the cube root of 41063.625.

602. Find the cube root of the square of 64.

603. Find the square of the cube root of 64.

604. What is the length of one side of a cubical pile that contains 256 cords of wood?

605. The contents of a cubical block of granite are 4913 solid feet. What are the superficial contents?

606. Required the length of one side of a cubic cistern that will contain as much water as a moat 64 feet long, 18 feet wide, and 12 feet deep.

607. What is the depth of a cubic box that will exactly hold a bushel?

608. How many square feet in the surface of a cube containing 42875 cubic inches?

609. How many square feet in the surface of a cube containing 8998912 cubic feet?

MENSURATION.

610. The base of a triangular lot is 244 feet and the altitude 108 feet; what is the area?

611. The three sides of a triangle are 208, 315, and 125 feet; what is the area?

612. What is the area of a parallelogram whose base is 128 feet and altitude 72 feet?

613. How many square feet are there in a plank 20 feet long, and 14 inches wide at one end and 11 inches at the other?

614. What is the area of a lot of land whose four sides are, successively, 25, 38, 18, and 34 rods, and the diagonal from the beginning of the first side to the end of the second 42 rods?

615. In a piece of ribbon $2\frac{1}{4}$ inches wide and 167 feet long, how many square yards?

616. Of a street 50 feet wide how many feet in length make one acre?

617. How many acres in a street 2 miles long and 50 feet wide?

618. A built a square house, 40 feet long on each side; B built a house containing the same area, 80 feet long; which has the greater extent of wall, and how much greater?

619. How many bricks 8 inches by 4 will be required to lay a sidewalk 4 rods long and 6 feet wide?

620. In a square field of one acre, if a man mow a space one rod in width around it within the line, how many square rods does he mow?

621. What part of 7 feet square are 7 square feet?

622. How many rods of fence will be required to enclose 640 acres of land in a square form?

623. A farmer sets out an orchard of 600 trees so that the number of rows is to the number of trees in a row as 2 to 3. The trees are 25 feet apart, and no tree is within $12\frac{1}{2}$ feet of the fence; how many square feet of land in the field?

624. What is the least number of yards of carpet that is $\frac{3}{4}$ of a yard wide that must be bought to carpet, without splitting a breadth, a room 17 feet long by 16 feet 6 inches wide? Which way must the carpet be put down? How many more yards does it take than it would to carpet a room whose floor contained the same number of square feet, but so shaped that none of the carpet need to be turned under?

625. A room 17 feet 6 inches wide and 24 feet long is to be carpeted with a piece of tapestry carpet $\frac{3}{4}$ of a yard wide, that costs $ 1.10 a yard. In order to make the figure match if the carpet runs lengthwise of the room, half a yard must be allowed on each cutting, while if the carpet runs across, four inches must be allowed on each cutting; and a breadth cannot be split. How much less will it cost to lay the carpet one way rather than the other?

626. How many feet, board measure, in a plank 12 feet 4 inches long, 2 feet 3 inches wide, and 4 inches thick?

627. How many feet, board measure, in a 4-inch plank that is 15 feet 6 inches long, 2 feet 3 inches wide at one end and 1 foot 11 inches wide at the other end?

628. How many feet, board measure, in 8 half-inch boards each 8 inches wide and 13 feet 6 inches long?

629. How many feet, board measure, in 6 joists, 3 inches by 4 inches, and 11 feet long?

630. How many feet, board measure, in a plank 16 feet long 23 inches wide at one end, 15 at the other, and $3\frac{1}{2}$ inches thick?

631. How many feet, board measure, in 12 sticks of square timber each 32 feet 6 inches long, and the other dimensions 8 inches by 10 inches?

632. How many feet, board measure, in a stick of timber 24 feet long, 10 inches wide, and 8 inches thick?

633. What will it cost at $ 1.25 a rod to fence a lot containing 1 acre 32 square rods, if the lot is 3 times as long as it is wide?

634. What is the value of a lot of land $5\frac{1}{2}$ rods long, 60 feet wide in front, 48 feet wide in the rear, at $ 0.68 a square foot?

635. The area of a rectangular piece of land is 50 acres, and the length of the piece is to its breadth as 5 to 1; what are the length and breadth?

636. A tree 150 feet high, standing upon the bank of a stream, was broken off 25 feet from the ground, and, falling across the stream, the top just reached the opposite shore. What was the width of the stream?

637. If a line 160 feet long will reach from the top of a steeple 130 feet high to the opposite side of the street, what is the width of the street?

638. Two vessels whose courses are exactly at right angles meet in mid-ocean. One sails 9 miles an hour, the other 12. How far apart will they be in 24 hours?

639. What is the length of the diagonal of a square lot which contains 16 square rods?

640. A and B start from the same point, each walking 12 hours a day. A walks due north, at the rate of a mile in 15

minutes, while B walks due east, at the rate of a mile in 12 minutes. How far apart will they be at the end of the fourth day?

641. A field in the form of a right-angled triangle, with the two short sides of equal length, contains six acres. What is the length of the longest side?

642. The diagonal of a square field measures 200 feet in length. Required the contents of the field.

643. A ladder resting on the ground 21 feet from a house just reaches a window which is 28 feet high; how long is the ladder?

644. A tree 64 feet high is broken off 24 feet high, the part broken off turning upon the stump as upon a hinge; at what distance from the bottom of the tree does the top strike the ground?

645. A ladder 54 feet long will reach a window 25 feet high on one side of a street, and, without moving the foot, it will reach a window 16 feet high on the other side; how wide is the street?

646. The length of a ladder which will reach from the middle of a street 80 feet wide to the eaves of a house is 50 feet; what is the height of the house?

647. The distance from the top of a building 72 feet high to the base of the building on the opposite side of the street is 120 feet; what is the width of the street?

648. What is the length of a diagonal of a room whose dimensions are $30 \times 20 \times 10$ feet?

649. What is the longest straight rod that can be put into a box 5 feet long, 3 feet wide, and 2 feet deep?

650. What is the distance from an upper corner to the opposite lower corner of a room 16 feet long, $14\frac{1}{2}$ feet wide, and 10 feet high?

651. Two ships sail from the same port, one due north 128 miles, the other due east 72 miles. How far are the ships from each other?

652. There are two columns on the same plane, one 70 feet and the other 50 feet high. There is between these a column 5 feet high, the top of which is 100 feet from the summit of the higher, and 80 feet from the summit of the lower. Required the distance between the tops of the two columns.

653. A horse is tied with a rope 38 feet long. If he can reach 2 feet more, how many square rods of land can he graze over?

654. A pond is 25 rods in diameter. How many like this will it take to equal in area one 140 rods in diameter?

655. Allowing that a horse tied with a halter can reach 3 feet more than the length of the halter, how long a rope must be used to allow a horse to feed over just an acre of ground?

656. What is the difference in the expense of fencing a circular 10-acre lot and one of the same area in a square form, if the fence costs 75 cents a rod?

657. The diameter of a circle is 10 inches; how many inches in the side of the inscribed square?

658. What is the side of the greatest square stick of timber that can be hewn from a log 18 inches in diameter?

659. A circular field is 600 rods in circumference. What is the side of a square field that contains the same area?

660. What is the diameter of a circular pond that contains 625 times as much area as one 48 rods in diameter?

661. The area of a triangle is 24 square feet, and one side of it is 8 feet. What is the corresponding side of a similar triangle containing 96 square feet?

662. What is the side of a square that contains 361 times as much area as one whose side is 75 feet?

663. What is the side of a square equal in area to a circle 100 feet in diameter?

664. A circular field contains 10 acres. What is the length of its diameter?

665. What is the area of a circular pond which shall contain 45 times as much area as one 35 rods in diameter?

666. If a pipe 5 inches in diameter will discharge a certain quantity of water in 16 hours, in what time will a pipe 3 inches in diameter discharge the same quantity?

667. A bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep; what are the dimensions of a similar measure that holds half a bushel?

668. If a certain cistern can be emptied by a given pipe in $5\frac{3}{4}$ hours, how long will it take a pipe whose diameter is $2\frac{1}{2}$ times the diameter of the given pipe to empty it?

669. If a man dig a square cellar that measures 9 feet each way in 3 days, how long will it take him to dig one of the same depth measuring 12 feet each way?

670. What is the side of an equilateral triangle that shall contain 35 times as much area as one whose side is 6 feet?

671. If a pipe 3 inches in diameter will empty a cistern in 8 minutes, what is the diameter of a pipe that will empty it in 18 minutes?

672. How many bricks, 8 inches long, 4 inches wide, and 2 inches thick, will be required to build a wall 30 feet long, 8 feet high, and 2 feet thick?

673. How many bricks, whose dimensions are 8 by 4 by 2 inches, will it take to build the walls of a house 50 feet long, 32 feet wide, and 24 feet high, the walls to be 1 foot 6 inches thick, and no allowance made for doors and windows?

674. How many bricks, 8 by 4 by 2 inches, will be required to build a house 36 feet long, 27 feet wide, and 23 feet high, the walls being 1 foot 4 inches thick, the house having 5 doors, each 4 feet wide and 8 feet high, and 30 windows, each 3 feet wide and 6 feet high, no allowance being made for the space occupied by the mortar?

675. A garden whose breadth is 6 rods, and length twice its breadth, has a wall 2 feet thick and 4 feet high around it, outside of the line; what was the cost of this wall at 4 cents a cubic foot?

676. What will be the cost of digging a ditch around the garden above mentioned, within and adjacent to the wall, 3 feet wide, and $2\frac{1}{2}$ feet deep, at $\frac{5}{8}$ of a cent a cubic foot?

677. What would be the cost of walling the garden above mentioned, the central line of the wall to be on the bounding line, the wall to be 2 feet thick and $4\frac{1}{2}$ feet high, at $6\frac{1}{4}$ cents a cubic foot?

678. How many bricks, each 8 inches long, 4 inches wide, and 2 inches thick, will make a cubical pile 13 feet each way?

679. How high is a pile of wood which contains 10 cord feet if it is 9 feet long and 4 feet wide?

680. If a sod $1\frac{1}{2}$ inches thick is taken from the entire surface of a field of one acre, how many cubic yards are taken?

681. Find the solid contents of a cone whose base is 3 feet in diameter and whose altitude is 8 feet.

682. How many cubic feet in a globe 5 feet in diameter?

683. How many gallons will a circular cistern hold that is 6 feet in diameter and 10 feet deep?

684. How many quarts of milk will a pail contain whose diameter at the top is 11 inches, at the bottom 9, and whose depth is 8 inches?

685. How many pints does a coffee-pot hold that is 7 in. across the bottom, $4\frac{1}{2}$ in. across the top, and 8 in. high?

686. If the diameter of the earth is 7912 miles, how many square miles in its surface?

687. What is the diameter of a globe whose volume is 728 cubic yards?

688. How long is the edge of the greatest cube that can be cut from a 6-inch globe?

689. The weights of 3 stacks of hay of similar form are respectively 3, 5, and 7 tons, and the slant height of the least is 12 feet. Find the slant height of the other two.

690. What are the dimensions of a box similar to, and that will hold 3 times as much as, a box 8 by 6 by 4 inches?

691. If an iron ball 4 inches in diameter weighs 8 pounds, what is the weight of a ball 50 inches in diameter?

692. If a stack of hay 26 feet high weighs 29 tons, what is the height of a similar stack which weighs 9 tons?

693. How many bullets $\frac{1}{2}$ of an inch in diameter will be required to make a ball 4 inches in diameter?

694. If a loaf of sugar 10 inches high weighs 8 lb., what is the height of a similar loaf weighing 1 lb.?

695. Suppose the diameter of the earth is 7912 miles, and that it takes 1404928 bodies like the earth to make one as large as the sun, what is the diameter of the sun?

696. A bin is 15 feet long, 5 feet wide, and 3 feet deep; what is the edge of a cubical box that will hold the same quantity of grain?

697. If a ball 8 inches in diameter weighs 50 pounds, what is the diameter of a ball that weighs 164 pounds?

698. If a bell 6 inches high, 4 inches in diameter, and $\frac{1}{2}$ of an inch thick weighs 4 lb., what are the dimensions of a similar bell that weighs 740 lb.?

699. If a globe of gold 1 inch in diameter is worth $100, what is the diameter of a globe worth $3600?

700. A, B, and C own equal shares of a conical stack of hay 20 feet in height. If A takes his from the top, B next, and C last, what is the altitude of the part that each takes?

APPENDIX.

NOTATION AND NUMERATION.

505. In addition to the table on page 4 the periods above billions are trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, etc.

506. And also, in addition to the table on page 102, the names of the decimal periods below billions are formed by changing the *s* of the names of the integral periods to *ths*.

507. The method of numeration, given on pages 4 and 102, is called the French method, and is the one generally used in this country.

508. In the *English Method* a period is six figures.

ENGLISH NUMERATION TABLE.

Trillions,	Hundred Thousand Billions,	Ten Thousand Billtons,	Thousand Billions,	Hundred Billions,	Ten Billions,	Billions,	Hundred Thousand Millions,	Ten Thousand Millions,	Thousand Millions,	Hundred Millions,	Ten Millions,	Millions,	Hundred Thousands,	Ten Thousands,	Thousands,	Hundreds,	Tens,	Units.
6,	**5**	**8**	**7**	**3**	**5**	**1,**	**4**	**2**	**9**	**8**	**6**	**7,**	**3**	**1**	**8**	**7**	**3**	**5.**
	3d period, Billions.						2d period, Millions.						1st period, Units.					

509. In the French and English methods the written expressions are the same as far as, and including, hundred millions. But an English billion is a thousand French billions, an English trillion a million French trillions, etc.

510. The **Roman Notation,** or that used by the Romans, employs *seven capital letters* to express numbers, namely:

I,	V,	X,	L,	C,	D,	M.
1,	5,	10,	50,	100,	500,	1000.

All other numbers are expressed by combining and repeating these letters.

511. The Roman Notation is based on the following principles:

1. When two or more numbers of equal value are united, or when a letter of less value follows one of greater, the sum of their values is indicated; thus, XXX stands for 30, LXV for 65, CC for 200, MDCLXVII for 1667.

2. When a letter of less value is placed before one of greater, the difference of their values is indicated; as, IX stands for 9, XL for 40, XC for 90.

3. When a letter of less value stands between two of greater value, the less is to be taken from the sum of the other two; as, XIV stands for 14, XIX for 19, CXL for 140.

4. A letter with a line over it represents a number one thousand times as great as the same letter without the line; thus, X stands for ten, but $\overline{\text{X}}$ stands for one thousand times ten, that is, ten thousand; M stands for one thousand, but $\overline{\text{M}}$ for one thousand times one thousand.

TABLE OF ROMAN NUMERALS.

I	1	XIV	14	C	100
II	2	XV	15	CCCC, or DC	400
III	3	XVI	16	D	500
IV	4	XVII	17	DCCCC, or CM	900
V	5	XVIII	18	M	1000
VI	6	XIX	19	MD	1500
VII	7	XX	20	MDCLXV	1665
VIII	8	XXI	21	MDCCXLIX	1749
IX	9	XXX	30	MDCCCLXXIX	1879
X	10	XL	40	$\overline{\text{V}}$	5000
XI	11	L	50	$\overline{\text{L}}$	50000
XII	12	LX	60	$\overline{\text{C}}$	100000
XIII	13	XC	90	$\overline{\text{M}}$	1000000

512. The Roman Numerals are seldom used, except for numbering the pages of a preface, the divisions of a discourse, and the sections, chapters, and other divisions of a book.

CONTRACTIONS IN MULTIPLICATION.

513. To multiply by 9, 99, or any number whose figures are 9's,

Annex as many 0's *to the multiplicand as there are* 9's *in the multiplier, and from the number so formed subtract the multiplicand; the remainder will be the product sought.*

1. Multiply 786 by 999.

$$\begin{aligned} 786 \times 1000 &= 786000 \\ 786 \times \quad\;\; 1 &= \quad\;\; 786 \\ 786 \times \;\; 999 &= 785214, \textbf{ Ans.} \end{aligned}$$

2. Multiply 437 by 99.
3. Multiply 8764 by 999.
4. Multiply 18432 by 9999.

514. To multiply by aliquot parts of 100,

Move the decimal point two places to the right, and take such fraction of the result as the multiplier is of 100.

Thus, $160 \times 12\frac{1}{2} = 16000 \times \frac{1}{8} = 2000$.

515. To multiply by aliquot parts of 1000,

Move the decimal point three places to the right, and take such fraction of the result as the multiplier is of 1000.

Thus, $273 \times 333\frac{1}{3} = 273000 \times \frac{1}{3} = 91000$.

NOTE. The same principle may be used when the multiplier is an aliquot part of 1 with any number of ciphers annexed. (See Art. 172.)

CONTRACTIONS IN DIVISION.

516. To divide by a composite number,

Divide the dividend by one factor of the divisor, and the quotient so obtained by another factor, and so on till all the factors of the divisor have been used. The last quotient will be the required quotient.

NOTE. If the divisor, greater than 12, is composed of factors not greater than 12, the work of division is lessened.

5. Divide 5432 by 56.

$$\begin{array}{r} 7)\,\underline{5432} \\ 8)\,\underline{776} \\ 97, \text{ Ans.} \end{array}$$

6. Divide 8712 by 72.
7. Divide 17658 by 81.

517. To divide by 9, 99, or any number whose figures are 9's,

Cut off from the right of the dividend as many places as there are 9's in the divisor; the number at the left of the point will be the quotient; and the remainder will be the same as the quotient plus the figures at the right of the point. If the remainder exceeds the divisor, divide this remainder again as before; and add the second quotient to the first for the true quotient.

Thus, 9 in 10, once and one remainder; 9 in 50, 5 times and 5 remainder; 9 in 71, 7 times and 7 + 1 remainder; 9 in 67, 6 times and 6 + 7 = 13 remainder, and 9 in 13, once and 4 remainder, therefore, 9 in 67, (6 + 1) times and 4 remainder.

99 in 100, once and one remainder; 99 in 700, 7 times and 7 remainder; 99 in 817, 8 times and 8 + 17 = 25 remainder.

999 in 1000, once and one remainder; 999 in 314526, 314 times and 314 + 526 = 840 remainder; 999 in 7869843, 7869 times and 7869 + 843 = 8712 remainder; and 999 in 8712, 8 times and 8 + 712 = 720 remainder; therefore, 999 in 7869843, (7869 + 8 =) 7877 times, and 720 remainder.

8. Divide 1648 by 99.

9. Divide 187432 by 999.

10. Divide 2784567 by 9999.

518. To divide by aliquot parts of 100,

Move the decimal point two places to the left, and multiply the result by 100 *divided by the given divisor.*

$$\text{Thus, } 84800 \div 33\tfrac{1}{3} = 848 \times 3 = 2544.$$

519. To divide by aliquot parts of 1000,

Move the decimal point three places to the left, and multiply the result by 1000 *divided by the given divisor.*

$$\text{Thus, } 647000 \div 250 = 647 \times 4 = 2588.$$

DIVISIBILITY OF NUMBERS.

520. Explanation of principles in Art. 94:

(a.) As 2, 4, 6, 8, and 10 are divisible by 2, any number of tens, or any number of tens plus 2, 4, 6, or 8, is divisible by 2; that is, any number whose unit figure is 0 or an even number is divisible by 2.

(c.) As 4 will divide 100, it will divide any number of times 100 plus any number divisible by 4; that is, any number is divisible by 4 when 4 will divide the number expressed by the two right-hand figures.

(d.) As 5 will divide 10, it will divide any number of times 10, or any number of times 10 plus 5; that is, any number whose unit figure is 0 or 5 is divisible by 5.

(g.) As 8 will divide 1000, it will divide any number of times 1000 plus any number divisible by 8; that is, any number is divisible by 8 when 8 will divide the number expressed by the three right-hand figures.

(h.) As 9 is one less than 10, there are as many 9's in any number as there are 10's, and a remainder equal to the number of 10's plus the units figure: thus 10 is 1 nine and 1 remainder; 80 is 8 nines and 8 remainder; 16 is 1 nine and 1 + 6, or 7, remainder; 42 is 4 nines and 4 + 2, or 6, remainder. (See Art. 517.)

	7000	contains	777	nines,	and a	remainder	of	7
	600	"	66	"	"	"		6
	20	"	2	"	"	"		2
	1	"	0	"	"	"		1
Hence,	7621	"	845	"	"	"		7 + 6 + 2 + 1.

7 + 6 + 2 + 1, or 16, contains 1 nine and a remainder of 1 + 6, or 7 ; that is, 7621 contains 845 + 1, or 846, nines, and a remainder equal to the excess over 9 of the sum of its figures. In like manner, if any number is divided by 9, the remainder is equal to the excess over 9 of the sum of its figures. Hence, any number is divisible by 9 when the sum of its figures is divisible by 9.

NOTE 1. This explains the method of proof in addition, subtraction, etc., called "casting out the 9's."

(b.) As 9 is divisible by 3, any number of times 9 plus 3, or 6, will be divisible by 3 ; that is, any number is divisible by 3 when the sum of its figures is divisible by 3.

(e.) As 3 is not an even number, and every even number is divisible by 2, it is evident that any even number divisible by 3 is divisible by 6.

NOTE 2. Every *prime* number, but 2 and 5, has 1, 3, 7, or 9 for its unit figure.

521. For further aid in determining the factors of numbers, we have the following

TABLE OF PRIME NUMBERS FROM 1 TO 997.

1	41	101	167	239	313	397	467	569	643	733	823	911
2	43	103	173	241	317	401	479	571	647	739	827	919
3	47	107	179	251	331	409	487	577	653	743	829	929
5	53	109	181	257	337	419	491	587	659	751	839	937
7	59	113	191	263	347	421	499	593	661	757	853	941
11	61	127	193	269	349	431	503	599	673	761	857	947
13	67	131	197	271	353	433	509	601	677	769	859	953
17	71	137	199	277	359	439	521	607	683	773	863	967
19	73	139	211	281	367	443	523	613	691	787	877	971
23	79	149	223	283	373	449	541	617	701	797	881	977
29	83	151	227	293	379	457	547	619	709	809	883	983
31	89	157	229	307	383	461	557	631	719	811	887	991
37	97	163	233	311	389	463	563	641	727	821	907	997

GREATEST COMMON DIVISOR.

522. To explain the method of finding the greatest common divisor of numbers given in Art. 100, the following principles are given:—

1. *A common divisor of two numbers is also a common divisor of the sum or the difference of any multiples of each.*

Thus, 16 is divisible by 4; it is evident that any other number less, or greater, than 16 by any integral number of 4's, is also divisible by 4. It therefore follows that 4 will divide any multiple of 16 plus or minus any number of 4's.

The same method of reasoning applies to all numbers.

2. *The greatest common divisor of two numbers is also the greatest common divisor of the less and the remainder after dividing the greater by the less.*

Suppose it is required to find the greatest common divisor of 27 and 21.

$$\begin{array}{r} 21)\ 27\ (1 \\ \underline{21} \\ 6 \end{array}$$

According to the first principle stated above, as $27 = 21 + 6$, any divisor of 21 and 6 must be a divisor of 27; and as $6 = 27 - 21$, any divisor of 27 and 21 must be a divisor of 6; that is, the divisors of 27 and 21 and of 21 and 6 are identical, and therefore the greatest common divisor of 27 and 21 must also be the greatest common divisor of 21 and 6.

In like manner, the greatest common divisor of 21 and 6 is the greatest common divisor of 6 and the remainder after dividing 21 by 6.

The same method of reasoning applies to all numbers. Hence we derive the rule given in Art. 100, page 65.

523. To add or subtract two fractions whose numerators are each unity,

Rule.

To add take the sum of the denominators, to subtract take the difference of the denominators, for a numerator, and in either case take the product of the denominators for a denominator.

11. Add $\frac{1}{7}$ and $\frac{1}{8}$. $\frac{1}{7}+\frac{1}{8}=\frac{8+7}{56}=\frac{15}{56}$, Ans.

12. Take $\frac{1}{5}$ from $\frac{1}{4}$. $\frac{1}{4}-\frac{1}{5}=\frac{5-4}{20}=\frac{1}{20}$, Ans.

13. Add $\frac{1}{6}$ and $\frac{1}{5}$.

14. From $\frac{1}{4}$ take $\frac{1}{9}$.

524. GREATEST COMMON DIVISOR OF FRACTIONS.

15. Find the greatest common divisor of $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{8}{15}$.

Solution. The greatest common divisor of the numerators, 2, 4, and 8, is 2; and the greatest common divisor of thirds, ninths, and fifteenths is forty-fifths; that is, is a fraction whose numerator is 1, and denominator the least common multiple of the denominators. Therefore the greatest common divisor of $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{8}{15}$ is $\frac{2}{45}$. Hence,

525. To find the greatest common divisor of two or more fractions,

Rule.

Divide the greatest common divisor of the numerators by the least common multiple of the denominators.

Find the greatest common divisor of

(16.) $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{7}{8}$.

(17.) $1\frac{1}{5}$, $2\frac{2}{3}$, and $\frac{6}{7}$.

(18.) $7\frac{1}{4}$, $2\frac{3}{8}$, and 8.

(19.) $\frac{12}{13}$, $\frac{8}{9}$, and $\frac{4}{3}$.

(20.) $2\frac{1}{4}$, $5\frac{1}{2}$, $6\frac{1}{3}$, and $4\frac{2}{5}$.

526. LEAST COMMON MULTIPLE OF FRACTIONS.

21. Find the least common multinle of $\frac{2}{9}$, $\frac{4}{15}$, and $\frac{8}{21}$.

Solution. The least common multiple of the numerators, 2, 4, and 8, is 8; and the least common multiple of ninths, fifteenths, and twenty-firsts, is thirds; that is, is a fraction whose numerator is 1 and denominator the greatest common divisor of the denominators. Therefore the least common multiple of $\frac{2}{9}$, $\frac{4}{15}$, and $\frac{8}{21}$ is $\frac{8}{3}$, or $2\frac{2}{3}$. Hence,

527. To find the least common multiple of two or more fractions,

Rule.

Divide the least common multiple of the numerators by the greatest common divisor of the denominators.

Find the least common multiple of

(22.) $\frac{2}{5}$, $\frac{7}{10}$, and $\frac{14}{15}$.

(23.) $2\frac{2}{3}$, $4\frac{1}{6}$, and $6\frac{1}{9}$.

(24.) $1\frac{1}{2}$, 7, $\frac{3}{10}$.

(25.) $\frac{1}{12}$, $3\frac{1}{2}$, 4, and 6.

(26.) 2, $5\frac{1}{2}$, 8, and $9\frac{1}{4}$.

27. If A and B start from the same point, at the same time, to walk round an island, and A can walk round it in $\frac{2}{3}$ of a day, and B in $\frac{3}{4}$ of a day, how many days will it be before they will be together again? How many times round the island must each go?

CIRCULATING DECIMALS.

528. Circulating Decimals are decimals in which the same figure, or set of figures, is repeated without limit. Thus, if we attempt to reduce $\frac{1}{3}$ to a decimal, we have 0.33333, etc., the 3's repeating without limit. $\frac{4}{11} = 0.363636$, etc., the 36 repeating without limit. Instead of repeating the fig-

ures, and approximating nearer and nearer to the real value of the fraction, the repeating figures (sometimes called *repetends*), are written but once, and a dot placed over the first and the last figure. Thus, $\frac{2}{3} = 0.\dot{6}$; $\frac{9}{37} = 0.\dot{2}4\dot{3}$.

529. If the repeating figures begin from the decimal point, the decimal is called a *pure circulating decimal.* For example, $0.\dot{3}$, or $0.\dot{0}6\dot{1}$.

530. If the repeating figures do not begin from the decimal point, the decimal is called a *mixed circulating decimal.* For example, $0.24\dot{8}5\dot{7}$.

531. Circulating decimals arise from reducing to decimals common fractions which, in their lowest terms, have in their denominators other factors than 2 and 5.

For a common fraction cannot be expressed as an exact decimal unless its denominator can be multiplied so as to become an exact power of 10; and no number can be multiplied so as to become a power of 10 unless it is composed of the factors 2 or 5 or 2 and 5.

532. If a common fraction whose denominator is 9, 99, or any number whose figures are 9's, is reduced to a decimal, the decimal will be a circulating decimal whose figures are the numerator with ciphers prefixed, if necessary, to make as many repeating figures as there are figures in the denominator. Thus, $\frac{3}{9} = 0.\dot{3}$; $\frac{4}{99} = 0.\dot{0}\dot{4}$; $\frac{37}{999} = 0.\dot{0}3\dot{7}$. (The principle is the same as in Art. 517.) Hence,

533. A pure circulating decimal can be expressed as a common fraction by *writing the repeating figures as the numerator and as many 9's for the denominator as there are repeating figures.* Thus, $0.\dot{6} = \frac{6}{9}$; $0.\dot{0}\dot{8} = \frac{8}{99}$; $0.\dot{2}1\dot{3} = \frac{213}{999}$.

Change to common fractions in their lowest terms

(28.) $0.\dot{4}$.	(33.) $0.\dot{8}\dot{1}$.
(29.) $0.\dot{3}\dot{6}$.	(34.) $0.\dot{1}4\dot{4}$.
(30.) $0.\dot{0}1\dot{8}$.	(35.) $0.\dot{0}00\dot{9}$.
(31.) $0.\dot{2}1\dot{6}$.	(36.) $0.\dot{0}819\dot{9}$.
(32.) $0.\dot{0}23\dot{4}$.	(37.) $0.\dot{2}4678\dot{9}$.

38. Reduce $0.2\dot{5}$ to a common fraction.

$$\begin{array}{l} 0.2\dot{5} \times 10 = 2.555... \\ \underline{0.2\dot{5} \times \ \ 1 = 0.255...} \\ 0.2\dot{5} \times \ \ 9 = 2.3 \end{array}$$

Hence, $0.2\dot{5} = \frac{2.3}{9} = \frac{23}{90}$, Ans.

If we subtract once $0.2\dot{5}$ from 10 times $0.2\dot{5}$ we have 9 times 0.25. But 10 times $0.2\dot{5}$ minus once 0.25 gives 2.3; that is, gives a number whose figures are the same as the figures of the mixed circulating decimal minus the figures that do not repeat.

39. Reduce $0.3\dot{4}\dot{7}$ to a common fraction.

$$\begin{array}{l} 0.3\dot{4}\dot{7} \times 100 = 34.7474... \\ \underline{0.3\dot{4}\dot{7} \times \ \ \ \ 1 = \ \ 0.3474...} \\ 0.3\dot{4}\dot{7} \times \ \ 99 = 34.4 \end{array}$$

Hence, $0.3\dot{4}\dot{7} = \frac{34.4}{99} = \frac{344}{990} = \frac{172}{495}$, Ans.

In this example we take once $0.3\dot{4}\dot{7}$ from 100 times $0.3\dot{4}\dot{7}$, and find, as before, that the remainder is a number whose figures are the same as the figures of the mixed circulating decimal minus the figures that do not repeat. Hence,

534. To reduce a mixed circulating decimal to a common fraction,

Rule.

For the numerator take the figures of the mixed circulating decimal minus the figures that do not repeat, and for the denominator as many 9's as there are repeating figures with as many ciphers annexed as there are decimal places before the repeating figures.

40. Reduce $0.76\dot{3}5\dot{4}$ to a common fraction.

$$0.76\dot{3}5\dot{4} = \frac{76354 - 76}{99900} = \frac{76278}{99900} = \frac{12713}{16650}, \text{ Ans.}$$

41. Reduce $2.04\dot{1}\dot{8}$ to a common fraction.

$$2.04\dot{1}\dot{8} = 2\frac{418 - 4}{9900} = 2\frac{414}{9900} = 2\frac{23}{550}, \text{ Ans.}$$

Reduce to common fractions in their lowest terms,

(42.) $0.8\dot{7}1\dot{6}$.
(43.) $3.04\dot{1}\dot{2}$.
(44.) $2.1\dot{8}7\dot{3}$.
(45.) $5.04\dot{1}\dot{5}$.
(46.) $4.2\dot{6}7\dot{4}$.
(47.) $52.8\dot{7}6\dot{2}$.

ADDITION AND SUBTRACTION OF CIRCULATING DECIMALS.

535. Circulating decimals can be written so as to have common denominators by repeating the figures to a sufficient number of places.

48. Add $0.\dot{6}$ and $0.2\dot{3}\dot{1}$.

$$\begin{array}{rl} 0.\dot{6} = & 0.6\dot{6}\dot{6} \\ & \underline{0.2\dot{3}\dot{1}} \\ & 0.8\dot{9}\dot{7}, \text{ Ans.} \end{array}$$

49. Add $2.\dot{6}7\dot{8}$ and $5.4\dot{9}\dot{2}$.

$$\begin{array}{rl} 2.\dot{6}7\dot{8} = & 2.6\dot{7}8678\dot{6} \\ 5.4\dot{9}\dot{2} = & \underline{5.4\dot{9}2929\dot{2}} \\ & 8.1\dot{7}1607\dot{9}, \text{ Ans.} \end{array}$$

In this example we increase the sum of the right-hand figures (6 + 2) by one, as there is one to be added from the left of the repeating figures (9 + 7) to the figures that do not repeat. The reason will be evident if the repeating figures are repeated still further to the right.

50. Subtract $0.\dot{2}3\dot{4}$ from $5.\dot{1}\dot{6}$.

$$\begin{array}{lll} 5.\dot{1}\dot{6} & = & 5.\dot{1}61616\dot{6} \\ 0.\dot{2}3\dot{4} & = & \underline{0.\dot{2}34234\dot{4}} \\ & & 4.\dot{9}2738\dot{1}, \text{ Ans.} \end{array}$$

In this example we lessen the last figure of the remainder (6 – 4) by one, as we were obliged to take one from the 5 at the left that does not repeat. The reason will be evident if the repeating figures are repeated still further to the right. Hence,

536. To add or subtract circulating decimals,

Rule.

Write the decimals so that the repeating figures shall begin and end at the same point. Then add or subtract as in simple decimals, placing the repetend dots on the result directly under the dots in the numbers added or subtracted, and remembering in addition to increase the right-hand figure as many units as are carried from the left of the repeating figures to the figures that do not repeat, and in subtraction to decrease the right-hand figure by one, if the left-hand figure of the repeating figures of the subtrahend is greater than the figure over it in the minuend.

(51.) $0.8\dot{4}\dot{3} + 2.\dot{7}6\dot{4} = ?$

(52.) $4.\dot{5}3\dot{2} - 3.8\dot{7}\dot{6} = ?$

(53.) $14.32\dot{5} + 0.\dot{2}6\dot{9} = ?$

(54.) $3.4\dot{7}\dot{6} + 8.\dot{7}3\dot{2} = ?$

(55.) $9.5\dot{4}\dot{8} - 2.\dot{3}7\dot{9} = ?$

(56.) $5.\dot{2}7\dot{9} + 4.\dot{6}\dot{3} + 0.\dot{6} = ?$

(57). $2.6\dot{1}\dot{7} + 0.\dot{3}45\dot{6} - 0.1\dot{8}\dot{7} = ?$

537. Multiplication and Division of circulating decimals by one another can be done best by first reducing the decimals to common fractions. If desired, the result may be reduced again to a decimal.

TABLES.

538. SURVEYORS' LINEAR MEASURE.

7.92 inches	= 1 link	(l.).
25 links	= 1 rod	(rd.).
4 rods, or 100 links	= 1 chain	(ch.).

539. SURVEYORS' SQUARE MEASURE.

10 square chains = 1 acre.

540. In Linear Measure we also have,

12 lines	= 1 inch.
18 inches	= 1 cubit.
6 feet	= 1 fathom.
40 rods	= 1 furlong.
8 furlongs	= 1 mile.
3 miles	= 1 league.

A slater's or mason's square = (10×10) sq. ft. = 100 sq. ft.
A perch of stone = $(16\frac{1}{2} \times 1\frac{1}{2} \times 1)$ cu. ft. = $24\frac{3}{4}$ cu. ft.

541. APOTHECARIES' FLUID MEASURE.

60 minims, or drops (♏)	= 1 fluid drachm	(f. ʒ).
8 fluid drachms	= 1 fluid ounce	(f. ℥).
16 fluid ounces	= 1 pint	(O.).
8 pints	= 1 gallon	(Cong.).

NOTE. O., for *octarius*, is the Latin for *one eighth*. Cong., for *congius*, is the Latin for *gallon*.

542. APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple	(sc. or ℈).
3 scruples	= 1 dram	(dr. or ʒ).
8 drams	= 1 ounce	(oz. or ℥).
12 ounces	= 1 pound	(lb. or ℔).

Apothecaries' Measures are used in mixing medicines; but medicines are bought and sold by *Avoirdupois Weight.*

543. TROY WEIGHT.

24 grains (gr.)	= 1 pennyweight	(dwt.).
20 pennyweights	= 1 ounce	(oz.).
12 ounces	= 1 pound	(lb.).

Troy Weight is used in weighing gold, silver, and precious stones.

NOTE 1. The pound, ounce, and grain, in Apothecaries' and Troy Weight are equal, but the ounce is differently subdivided.

NOTE 2. A pound Troy = 5760 grains, but a pound Avoirdupois = 7000 grains Apothecaries' or Troy Weight.

544. ENGLISH MONEY.

4 farthings (qr.)	= 1 penny	(d.).
12 pence	= 1 shilling	(s.).
20 shillings	= 1 pound	(£).

1 florin	= 2 shillings.
1 crown	= 5 shillings.
1 guinea	= 21 shillings.
1 sovereign	= 1 pound = $4.8665.

British Coins.

COPPER. The farthing, half-penny, and penny.

SILVER. The three-penny and six-penny pieces, shilling, florin, half-crown, and crown.

GOLD. The half-sovereign and sovereign.

545. FRENCH MONEY.

10 millimes (m.)	= 1 centime (ct.).
10 centimes	= 1 decime (dc.).
10 decimes	= 1 franc (fr.) = $0.193.

French Coins.

BRONZE. The 1, 2, 5, 10-centime pieces.

SILVER. The 25 and 50-centime pieces, and 1, 2, and 5-franc pieces.

GOLD. The 5, 10, 40 and 100-franc pieces.

546. GERMAN MONEY.

100 pfennige = 1 mark = $0.2385.

German Coins.

NICKEL. The 5 and 10-penny, and smaller pieces.

SILVER. The 20-penny, 1 and 2-mark pieces.

GOLD. The 5, 10, and 20-mark pieces.

547. The estimate of values contained in the following Table has been made by the Director of the Mint, and proclaimed by the Secretary of the Treasury of the United States;

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin	Silver	$0.453
Belgium	Franc	Gold and silver	0.193
Brazil	Milreis of 1000 reis	Gold	0.545
British Possessions in North America	Dollar	Gold	1.00
Denmark	Crown	Gold	0.268
France	Franc	Gold and silver	0.193
German Empire	Mark	Gold	0.238
Great Britain	Pound sterling	Gold	4.866½
Greece	Drachma	Gold and silver	0.193
India	Rupee of 16 annas	Silver	0.444
Italy	Lira	Gold and silver	0.193
Japan	Yen	Gold	0.997
Mexico	Dollar	Silver	1.015
Netherlands	Florin	Gold and silver	0.385
Norway	Crown	Gold	0.268
Peru	Dollar	Silver	0.935
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	0.748
Spain	Peseta of 100 centimes	Gold and silver	0.193
Sweden	Crown	Gold	0.268
Switzerland	Franc	Gold and silver	0.193
Turkey	Piaster	Gold	0.043

LEAP YEARS.

548. An astronomical or solar year — that is, the time it takes the earth to revolve round the sun — is 365 days, 5 hours, 48 minutes, 50 seconds, nearly. A civil year closes with an even number of days, at 12 P. M., Dec. 31.

Reckoning 365 days a year, we lose

5 h. 48 m. 50 sec.

each year, and in 4 years we lose

4 × (5 h. 48 m. 50 sec.) = 23 h. 15 m. 20 sec.

If, then, in the fourth year we add a day, or 24 hours, giving the year 366 days, we shall add too much by

24 h. − (23 h. 15 m. 20 sec.) = 44 m. 40 sec.

If we continue for 100 years, with 366 days every fourth year, and 365 in the other years, we shall have added too much by

25 × (44 m. 40 sec.) = 18 h. 36 m. 40 sec.

If now we omit the leap day in the hundredth year we shall omit too much by

24 h. − (18 h. 36 m. 40 sec.) = 5 h. 23 m. 20 sec.

If we continue in this way (having a leap year every 4 years except the hundredth years) we shall in 4 centuries omit too much by

4 × (5 h. 23 m. 20 sec.) = 21 h. 33 m. 20 sec.

Adding a day in the 400th year, we shall add too much by

24 h. − (21 h. 33 m. 20 sec.) = 2 h. 26 m. 40 sec.

Continuing in this way for 40 centuries, or 4000 years, we shall have added too much by

10 × (2 h. 26 m. 40 sec.) = 24 h. 26 m. 40 sec.

Therefore the 4000th year ought not to be a leap year. By the present method of having leap years (Art. 254), at the end of 4000 years the civil and solar years will coincide within about a day, and if the 4000th year is not made a leap year the civil and solar years will coincide within half an hour.

The calendar, as arranged by Julius Cæsar, added a leap day every 4 years. This method of reckoning, in 1582, had caused an error of

10 days. What ought to have been, for example, called March 20, was called March 10. The spring months were moving forward into the summer, the summer into the fall, and so, in 1582, Pope Gregory XIII. ordered that 10 days should be dropped from the calendar, and that the leap years should occur as stated in Art. 254.

In 1752 the change was adopted in England by act of Parliament directing what otherwise would be Sept. 3 to be Sept. 14, the error having at that time increased to 11 days.

INTEREST.

549. Legal Rates of Interest in the Several States.

When no rate is specified in a note, or document, the rate in the left-hand column is the legal rate. Any rate up to that in the right-hand column is legal if specified in writing.

State.	Rate.		State.	Rate.		State.	Rate.	
Ala.	8		Kansas...	6	10	N. C.	6	
Arizona...	7	Any	Ky.	6		Ohio	6	8
Ark.	6	10	La.	5	8	Oregon	8	10
Cal.	7	Any	Maine....	6	Any	Penn.	6	
Col.	8	Any	Md.	6		R. I.	6	Any
Conn.	6		Mass.	6	Any	S. C.	7	8
No. Dak..	7	12	Mich.	6	8	Tenn.	6	
So. Dak...	7	12	Minn.	7	10	Texas	6	10
Del.	6		Miss.	6	10	Utah...... ..	8	Any
D. C.	6	10	Mo.	6	8	Vermont....	6	
Fla.	8	10	Mont.	10	Any	Va.	6	
Ga.	7	8	Neb.	7	10	Wash.	7	12
Idaho	7	12	Nev.	7	Any	W. Va.	6	
Ill.	5	7	N. H.	6		Wis.	6	10
Ind.	6	8	N. J.	6		Wyoming...	8	12
Iowa	6	8	N. Y.	6				

PARTIAL PAYMENTS.

550. CONNECTICUT RULE.

1. *When payments are made one year or more from the time from which the interest is reckoned, or when any payment is less than the interest then due, the work of computing*

the interest is performed according to THE UNITED STATES RULE.

2. *When payments exceeding the interest then due, are made within a year from the time from which the interest is reckoned, the amount of the principal must be found for a full year, and the amount of the payment from the time of payment to the end of such full year, and this, deducted from the amount of the principal previously obtained, will form the new principal.*

3. *If the year extends beyond the time of settlement, the last computation is to the time of settlement.*

(58.)

$875. HARTFORD, CONN., Dec. 18, 1891.

On demand, I promise to pay Samuel Hopkins, or order, eight hundred seventy-five dollars, with interest. Value received.

RICHARD JAMESON.

INDORSEMENTS: Feb. 6, 1892, $250; July 12, 1892, $150; Nov. 9, 1893, $20; Jan. 21, 1894, $375; June 12, 1894, $25. What was due Sept. 5, 1894?

Am't of $875 Dec. 18, 1891, to Dec. 18, 1892		$927.50
" $250 Feb. 6, 1892, " " "	$263.00	
" $150 July 12, " " " "	153.90	416.90
New principal " "		$510.60
(Payment $20 Nov. 9, 1893, less than int. due.)		
Int. of $510.60 Dec. 18, 1892, to Jan. 21, 1894		33.44
Amount " " "		$544.04
Sum of 3d and 4th payments " " "		395.00
New principal " "		$149.04
Int. of $149.04 Jan. 21, 1894, to Sept. 5, 1894		5.59
Amount " " "		$154.63
" of $25 June 12, " " " "		25.35
" due " "	Ans.	$129.28

(59.)

$1675. SAYBROOK, CONN., July 7, 1891.

On demand, I promise to pay George A. Howe, or order, sixteen hundred seventy-five dollars, with interest. Value received.

JOSEPH A. DAVIS.

INDORSEMENTS: Dec. 8, 1891, $300; Oct. 19, 1892, $60; Aug. 15, 1893, $500. What was due Sept. 1, 1894?

ANNUAL INTEREST.

551. In some States when notes are written "with interest annually," the interest, if it is not paid at the end of the year, when it is due, draws simple interest until paid.

NOTE 1. In Massachusetts and Maine interest is not allowed on annual interest, unless after demand for its payment or by special agreement.

(60.)

$350. MANCHESTER, N. H., May 17, 1890.

On demand, I promise to pay John Cogswell, or order, three hundred fifty dollars, with interest annually. Value received.

FRANCIS WHITNEY.

What is due on this note May 17, 1894?

Int. due on principal annually	$350 × 0.06 =	$21.00
Principal		350.00
Total annual interest	$21 × 4 =	84.00
Int. for 3 yr. on the $21 due May 17, 1891		3.78
" 2 yr. " " " 1892		2.52
" 1 yr. " " " 1893		1.26
		$441.56

NOTE 2. The simple interest on the several interests due annually can be computed at once by computing the interest on the interest due annually for a number of years equal to the sum of the number of years the several annual

interests remain unpaid. Thus, in Ex. 60, the interest on the $ 21 for 3 yr. + the interest on the $ 21 for 2 yr. + the interest on the $ 21 for 1 yr. = the interest on $ 21 for 6 yr.

552. Hence, to find the amount due on a note "with interest annually," when the interest remains unpaid,

Rule.

Add to the principal the simple interest on the principal for the given time, and also the simple interest on one year's interest of the principal for a number of years equal to the sum of the number of years the several annual interests remain unpaid.

61. What would be due on a note of $ 642 "with interest annually" in 7 yr. from its date?

62. What is the excess of annual over simple interest of $ 520 for 4 yr. 6 m.?

63. What is the amount of $ 465 at annual interest for 8 yr. 6 m.?

64. What is the amount of $ 562 at annual interest for 3 yr. 3 m. 18 d.?

553. When partial payments are made on a note that is written "with interest annually" the following is the common

Rule.

Find the amount of the principal by the annual interest method, to the end of the first year in which a payment, or payments, plus the interest on the payment, or payments, not less than the interest due at the end of the year has been made. From this amount subtract the payment, or payments, plus the simple interest on the payment, or payments, to the end of the year. The remainder will be a new principal, with which proceed as before.

(65.)

$875. CONCORD, N. H., Feb. 7, 1893.

I promise to pay Charles Edwards, or order, eight hundred seventy-five dollars, with interest annually. Value received.

PORTER W. FRENCH.

INDORSEMENTS: June 7, 1893, $20; Feb. 26, 1894, $300; Jan. 20, 1896, $200. What was due April 7, 1896?

First payment June 7, 1893, is less than the interest due at the end of the year.

Interest due on principal annually..........$875 × 0.06 =		$52.50
Principal..		$875.00
Total annual interest due Feb. 7, 1895....		105.00
Interest for 1 yr. on the $52.50 due Feb. 7, 1894.......................................		3.15
Amount to Feb. 7, 1895.....................		$983.15
Payment June 7, 1893........................	$20.00	
Interest on $20 June 7, 1893, to Feb. 7, 1895.......................................	2.00	
Payment Feb. 26, 1894.....................	300.00	
Interest on $300 Feb. 26, 1894, to Feb. 7, 1895.......................................	17.10	
Total payments plus interest................		339.10
Amount due Feb. 7, 1895....................		$644.05
Interest on $644.05 for 1 yr................		38.64
Amount to Feb. 7, 1896......................		$682.69
Payment Jan. 20, 1896........................	$200.00	
Interest on $200 Jan. 20, 1896, to Feb. 7, 1896.......................................	0.60	
Payment plus interest.........................		200.60
Amount due Feb. 7, 1896.....................		$482.09
Interest on $482.09 Feb. 7, 1896, to April 7, 1896............................		4.82
Balance due April 7, 1896..................	Ans.	$486.91

(66.)

$274. PORTSMOUTH, N. H., April 7, 1893.

I promise to pay Jacob Bowen, or order, two hundred seventy-four dollars, on demand, with interest annually. Value received.

SAMUEL FRENCH.

INDORSEMENTS: March 31, 1894, $100. What is due on this note April 7, 1896?

554. Vermont Rule for notes "with interest annually."

If the payment, or sum of the payments, is not less than the interest due at the end of the year, the rule is the same as that already given in Art. 553.

If the payment, or sum of the payments, is less than the interest due at the end of the year, with these payments plus the interest on them, (1) *cancel the simple interest due on the annual interests, and* (2) *take the remainder from the sum of the annual interests due. On the excess of interest then due cast simple interest to the end of a year when the payments, or sum of the payments, exceeds the interest due.*

NOTE. If the payment is less than the simple interest due on the annual interest, no interest must be cast on the balance of simple interest still due after deducting the payment.

(67.)

$356. MONTPELIER, VT., July 10, 1891.

I promise to pay Josiah F. Monroe, or order, three hundred fifty-six dollars, on demand, with interest annually. Value received.

JABEZ WHITMAN.

INDORSEMENTS: Dec. 6, 1891, $20; March 6, 1893, $75; Oct. 11, 1894, $15. What was due Nov. 6, 1895?

First payment Dec. 6, 1891, is less than the interest due at the end of the year.

Principal		\$356.00
Annual interest due July 10, 1892	\$21.36	
Amount of \$20 Dec. 6, 1891, to July 10, 1892	20.71	
Balance interest due July 10, 1892	\$0.65	
Interest on \$0.65 July 10, 1892 to July 10, 1893	0.04	
Annual interest due July 10, 1893	21.36	22.05
Amount due July 10, 1893		\$378.05
Payment March 6, 1893	\$75.00	
Interest on \$75 March 6, 1893, to July 10, 1893	1.55	76.55
Amount due July 10, 1893		\$301.50
Annual interest on \$301.50 due July 10, 1894	\$18.09	
Interest on \$18.09 July 10, 1894, to July 10, 1895	1.085	
Annual interest on \$301.50 due July 10, 1895	18.09	
Interest due July 10, 1895	\$37.265	
Amount \$15, Oct. 11, 1894, to July 10, 1895	15.672	
Balance interest due July 10, 1895	\$21.59	
Interest on \$21.59 July 10, 1895, to Nov. 6, 1895	0.42	
Interest on \$301.50 July 10, 1895, to Nov. 6, 1895	5.88	27.89
Amount due Nov. 6, 1895	Ans.	\$329.39

555. New Hampshire Rule for notes "with interest annually," is the same as the Vermont Rule, except

When no interest is due except what is accruing during the year, and the payment, or sum of the payments, is less than the interest due at the end of the year, these payments are applied at the end of the year without interest added.

Thus, in Ex. 67, if performed by the N. H. rule, the $20 paid Dec. 6, 1891, would not draw interest from Dec. 6, 1891, to July 10, 1892, and the amount to subtract from the annual interest due July 10, 1892, would be $0.71 less. In other respects the work is precisely the same. The Ans. by the N. H. rule is $330.24, and by the rule in Art. 553, $329.43.

68. Find the amount due Sept. 17, 1895, by the general rule (Art. 553) and by the New Hampshire and Vermont rules on a note of $2567, with interest annually, dated April 18, 1892, on which are the following indorsements: Sept. 13, 1892, $120; May 19, 1893, $955; Oct. 10, 1894, $65.

FOREIGN EXCHANGE.

556. A foreign bill of exchange is an order drawn in one country and payable in another.

NOTE. An *inland bill of exchange* is an order payable in the same country where it is drawn, and is called a *draft.*

557. In making foreign bills it is customary to draw a set of two or more bills of the same tenor and date, called a "set of exchange," each containing a clause, which renders all the bills in the set worthless except the one first presented to the drawee.

These bills are sent at different times, so that, if one or more of the set is delayed or lost, there may be no unnecessary delay in obtaining the money.

558. The par of exchange in any country is the value of the standard coin of that country in the standard coin of the country in which the quotation is given.

Thus, the present English sovereign contains 123.274 grains Troy of gold, of a fineness of 916.66 to a thousand. The U. S. $5 piece contains 129 grains Troy, of a fineness of 900 to a thousand. The U. S. government has had these two coins compared, and finds the value of the sovereign, or £, to be $4.8665 in coin of the same fineness as the

U. S. half-eagle. $ 4.8665, therefore, is the par value of an English £., or the par of exchange on England is $ 4.8665.

559. Exchange is usually at a slight premium or discount from its par value, depending upon the supply and demand, the cost of remitting specie to settle balances, the time for which the bills are drawn, etc.

560. The par value of the monetary unit of different countries in U. S. gold coin is given in Art. 547, page 347.

561. To find the value of a bill of exchange, the value expressed in the given currency must be changed to an equivalent value in the required currency.

562. In a daily paper foreign exchange is quoted as follows: —

Sterling sight 4.88, 60 days 4.86; Francs sight 5.13⅛, 60 days 5.15⅝; Marks sight 96¼, 60 days 95⅛.

By these quotations 1 £ sterling = $ 4.88 if the bill of exchange is drawn payable at sight, or $ 4.86 if drawn payable in 60 days after sight; 5.13⅛ francs = $ 1.00 if drawn at sight, or 5.15⅝ francs = $ 1.00 if drawn at 60 days after sight; 4 marks = $ 0.96¼ if drawn at sight, or $ 0.95⅛ if at 60 days after sight.

Boston, Mass., July 11, 1895.

Exchange for £ 1344. 16/6 Stg.

Sixty days after sight pay this First of Exchange, second and third of the same tenor and date not paid, to the order of William M. Edgerly, the sum of thirteen hundred forty-four pounds 16/6 stg.

Value received, and charge the same to account of

Kidder, Peabody & Co

To Messrs. Baring Bros. & Co.,
London, England.

69. Find the value in Boston of this 60 days' bill of exchange for £ 1344 16 s. 6 d., at the quotation given.

£ 1344 16 s. 6 d. = £ 1344.825.

$ 4.86 × 1344.825 = $ 6535.85. Ans.

New York, July 11, 1895.

Exchange for Fcs. 35642$\frac{50}{100}$.

At sight pay this Second of Exchange, first and third of the same tenor and date not paid, to the order of James Frothingham, the sum of thirty-five thousand six hundred forty-two and $\frac{50}{100}$ francs.

Value received, and charge the same to account of

Drexel, Morgan & Co.

To John Monroe & Co.,

Paris.

70. What will this bill of exchange for 35642.5 francs, drawn on Paris, cost in New York at the quoted rate?

$$5.13\tfrac{1}{8}\,)\,\underline{35642.5}$$
$$\$\,6946.16,\ \text{Ans.}$$

71. What must be paid in Boston for a set of exchange on Hamburg for 552 marks, payable 60 days after sight, at the quoted rate?

$ 0.955 × (552 ÷ 4) = $ 131 79, Ans.

(72.)

£ 2000. BOSTON, June 19, 1895.

At sight of this first of exchange (second and third unpaid), pay to the order of James Murray, in London, two thousand pounds sterling, value received, and charge the same to my account. BLAKE BROS. & Co.

To J. S. MORGAN & Co., London.

What is the cost of this bill in United States money, at the quoted rate?

73. Cole and Singleton of New York bought of Roger H. Lyon, a set of exchange, payable at sight, for £ 975, on Rothschild & Co., London, at 4.88. What was the cost?

74. I wish to pay a debt of £ 800 in Liverpool. Which can I best afford, to buy sovereigns at $ 4.8665 and pay $\frac{3}{8}$ % for freight and insurance, or buy a set of exchange at 4.89$\frac{1}{8}$?

75. Find the value in New Orleans, at the quoted rate, of a 60 days' set of exchange on Frankfort, for 1275 marks.

76. Find the value in Philadelphia of a set of exchange, payable at sight, at the quoted rate on Marseilles for 2785 francs.

CUSTOMS.

563. Customs duties are taxes levied by the general government on imported merchandise, to support the government and to protect home industry.

564. At certain places, called ports of entry, customhouses are established by the government, with officers to compute and collect the duties.

565. Goods are said to be entered when an invoice has been properly lodged, and permission obtained to land the goods.

566. All goods brought into the United States from foreign countries must be entered at a port of entry and landed thereat, or at specified ports of delivery.

567. All duties are regulated by government, and are different at different times.

NOTE. To bring in merchandise secretly with intent to defraud the government of duties is called smuggling, and persons so engaged are liable to punishment and to have the goods forfeited.

568. Tonnage is a tax upon the vessel, without reference to its cargo, which must be paid on entry of the vessel. The amount of tonnage depends upon the size of the vessel, being a certain rate a ton of measurement. (See Art. 601.)

569. The income from duties and tonnage is the principal source of revenue to the government.

570. Duties are either *ad valorem* or *specific.*

An **ad valorem duty** is a certain percentage computed on the market value of the goods in the country from which they are imported.

A **specific duty** is a certain rate a ton, gallon, yard, etc.

NOTE. Some articles are subject to compound duties, that is, to both ad valorem and specific duties.

571. Specific duties are computed only on the actual weight or measure of merchandise; hence certain allowances are made before final adjustment of the duties.

572. **Leakage** is an allowance for actual deficiency on liquors in casks, paying duty by the gallon.

573. **Breakage** is an arbitrary allowance of a certain percentage (5 %), which is deducted from the invoice quantity on imported liquors in bottles.

574. Tare is an allowance for the weight of the box, cask, bag, etc.

NOTE. Draft or Tret, an allowance for waste or refuse, is no longer allowed by the customs law.

575. Gross weight is the weight of the article before any of these allowances are made.

576. Net weight is the weight of the merchandise after all the allowances are made. Duties are computed on *net weight.*

77. What is the duty at $2\frac{1}{2}$ cents a pound on 4796 lbs. Russia iron?

78. What is the duty at 60 % on an invoice of silks which cost $ 7560 in Lyons?

79. The duty on French broadcloth is 50 cents a pound specific and 35% ad valorem. What amount of duties do I pay on an invoice of 19 cases, averaging 328 pounds, and 375 yards to a case, and costing 4 francs a yard? How much must I pay for a bill of exchange to remit for the goods, exchange being at 5.14? How much will the broadcloth cost me a yard if the freight and other charges beside duties are $ 13.50 a case?

80. What is the duty on 9 casks of molasses invoiced at 63 gallons each at $6\frac{1}{4}$ cents a gallon, allowing a total leakage of 15 gallons?

81. What is the duty at $2\frac{1}{2}$ cents a pound on 250 boxes of figs, weighing 112 pounds each, allowing for tare 15 pounds on each box?

82. The duty on marbles is 50 %. What will be the duty on a case of 50000 marbles, invoiced at 2 marks a thousand? What will they cost me a thousand, exchange being at 0.965, and freight and charges $ 12.50?

83. What will it cost a yard to import 25 cases black cashmere, each case containing 2046 meters 120 centimeters wide, the duty being 40 % ad valorem and 8 cents a square yard, and the invoice price 2.06 francs a (linear) meter, exchange at 5.15, the freight and other charges $ 17 a case?

84. What is the duty on 75 silk umbrellas, invoiced at 8 shillings each, the duty being 60%? How much will a bill of exchange for remittance cost, exchange being at 4.88?

ROOTS.

577. Short method of squaring numbers.

$$99^2 = (99+1)(99-1)+1^2 = 100 \times 98 + 1 = 9801.$$
$$97^2 = (97+3)(97-3)+3^2 = 100 \times 94 + 9 = 9409.$$
$$994^2 = (994+6)(994-6)+6^2 = 1000 \times 988 + 36 = 988036.$$
$$48^2 = (48+2)(48-2)+2^2 = 50 \times 46 + 4 = 100 \times 23 + 4 = 2304.$$

85. Find by this method the square of 26; 34; 47; 53; 67; 78; 85; 93; 107; 147; 491; 992.

ALGEBRAIC EXPLANATION OF SQUARE ROOT AND CUBE ROOT.

578. The method of explanation of square root in Art. 441, and of cube root in Art. 447 is the geometrical method.

SQUARE ROOT.

579. To explain square root by the algebraic method we find the square of a number, for example 24, as follows:

$$\begin{array}{rcl}
24 & = & 20 + 4 \\
\underline{24} & = & \underline{20 + 4} \\
96 & = & 20 \times 4 + 4^2 \\
\underline{48} & = & \underline{20^2 + 20 \times 4} \\
576 & = & 20^2 + 2(20 \times 4) + 4^2
\end{array}$$

From this example it can be seen that the square of a number whose figures are tens and units equals

The square of the tens plus twice the product of the tens by the units plus the square of the units.

86. Find the square root of 576.

$$\begin{array}{rl} & \overset{\frown}{5\,76}\ (24,\ \text{Ans.} \\ & \underline{4} \\ 44) & \overline{176} \\ & \underline{176} \\ & \quad 0 \end{array}$$

As 576 consists of three figures, its square root has two figures (Art. 440), and the square of the tens of the root must be in the 5 (hundreds). The greatest square in 5 (hundreds) is 4 (hundreds); therefore, the tens figure of the root must be 2.

Now, as the number 576 equals the square of the tens of the root, plus twice the product of the tens by the units plus the square of the units, if we subtract from 576 the square of the tens of the root, that is 20^2, or 400, the remainder, 176, must equal twice the product of the tens of the root by the units, plus the square of the units. In order, therefore, to find the units figure of the root, we divide 176 by twice the tens, that is by 40, as a trial divisor, obtaining 4 as the (probable) units figure of the root. If now the 176 equals $2 \times (20 \times 4) + 4^2$, that is $(40 + 4) \times 4$, the 4 is the units figure of the root, and the square root of 576 is 24.

Similar reasoning applies, however many figures there are in the root.

CUBE ROOT.

580. To explain cube root by the algebraic method we find the cube of a number, for example, 45, as follows:—

$$\begin{array}{rcr}
45 & = & 40 \quad + \quad 5 \\
\underline{45} & = & \underline{40 \quad + \quad 5} \\
225 & = & 40 \times 5 \; + 5^2 \\
\underline{180} & = & \underline{40^2 + \quad 40 \times 5 \qquad} \\
2025 & = & 40^2 + 2\,(40 \times 5) + 5^2 \\
\underline{45} & = & \underline{\qquad\qquad\qquad 40 + 5} \\
10125 & = & (40^2 \times 5) + 2\,(40 \times 5^2) + 5^3 \\
\underline{8100} & = & \underline{40^3 + 2\,(40^2 \times 5) + \quad (40 \times 5^2) \qquad} \\
91125 & = & 40^3 + 3\,(40^2 \times 5) + 3\,(40 \times 5^2) + 5^3
\end{array}$$

From this example we see that the cube of a number whose figures are tens and units equals

The cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.

87. Find the cube root of 91125.

$$
\begin{array}{lr|l}
 & & 91\widehat{125}\ (45,\ \text{Ans.} \\
 & & 64 \\
\text{Trial divisor,} & 3\times 40^2 = 4800 & \overline{27125} \\
 & 3\times 40\times 5 = 600 & \\
 & 5^2 = 25 & \\
\text{True divisor,} & \overline{3\times 40^2 + 3\times 40\times 5 + 5^2 = 5425} & \underline{27125}
\end{array}
$$

As 91125 consists of five figures its cube root has two figures (Art. 446), and the cube of the tens of the root must be in the 91 (thousands). The greatest cube in 91 (thousands) is 64 (thousands), therefore the tens figure of the root must be 4.

Now as the number 91125 equals the cube of the tens of the root, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units, if we subtract from 91125 the cube of the tens of the root, that is 40^3, or 64000, the remainder, 27125, must equal three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.

In order, therefore, to find the units figure of the root we divide 27125 by three times the product of the square of the tens, that is, by 4800 as a trial divisor, obtaining 5 as the (probable) units figure of the root. As $3\,(40^2 \times 5) + 3\,(40 \times 5^2) + 5^3 = \{3\,(40^2) + 3\,(40 \times 5) + 5^2\} \times 5$, if to the trial divisor, 3×40^2, we add $3\,(40 \times 5) + 5^2$, we shall obtain the true divisor, 5425, and if 5 times the true divisor equals 27125, the 5 is the units figure of the root.

Similar reasoning applies, however many figures there are in the root.

PROGRESSION.

581. A progression is a series of numbers which increase or decrease according to a fixed law.

582. The terms of a series are the several numbers that form the series. The first and last terms are called the extremes, and the others the means.

ARITHMETICAL PROGRESSION.

583. **Arithmetical Progression** is a series of numbers increasing or decreasing by a common difference.

Thus,	2, 5, 8, 11, 14, 17,	is an ascending series,
and	35, 30, 25, 20, 15, 10,	is a descending series.

584. In Arithmetical Progression there are five elements of which any three being given, the other two can be found:—

1. The first term.
2. The last term.
3. The common difference.
4. The number of terms.
5. The sum of all the terms.

585. In an ascending series, let 6 be the first term and 5 the common difference;

then 6, 1st term.

$6 + 5 = 11$, 2d term.

$6 + 5 + 5 = 6 + 2 \times 5 = 16$, 3d term.

$6 + 5 + 5 + 5 = 6 + 3 \times 5 = 21$, 4th term.

Thus we see that, in an ascending series, the second term is found by adding the common difference once to the first term; the third

term, by adding the common difference twice to the first term, etc. A similar explanation may be given when the series is descending. Hence,

586. The first term, common difference, and number of terms given, to find the last term.

Rule.

Multiply the common difference by the number of terms less one; add the product to the first term if the series is ascending, or subtract the product from the first term if the series is descending.

88. If the first term of an ascending series is 4, the common difference 3, and the number of terms 8, what is the last term?

Ans. 25.

89. The first term of a descending series is 98, and the common difference 6; what is the 4th term?

90. What is the amount of \$ 200, at 5 %, simple interest, for 18 years?

587. The extremes and number of terms given to find the common difference.

By inspecting the formation of the series in Art. 585, it will be seen that *the difference between the extremes is equal to the common difference multiplied by one less than the number of terms.* Thus the difference between the 1st and 4th terms ($21 - 6 = 15$) is the sum of 3 *equal* additions; hence this difference divided by 3 ($15 \div 3 = 5$) gives *one* of these additions, that is the common difference. Hence,

Rule.

Divide the difference of the extremes by the number of terms less one.

91. The extremes of an arithmetical series are 4 and 55, and the number of terms is 18; what is the common difference?

Ans. 3.

92. A man has 7 children whose ages form an arithmetical series; the youngest is 3 years old and the oldest 21; what is the difference of their ages?

93. The amount of $ 300 at simple interest for 10 years is $ 450; what is the rate per cent?

588. The extremes and common difference given to find the number of terms.

By Art. 585 it is evident that the difference of the extremes is the common difference multiplied by one less than the number of terms. Hence, conversely,

Rule.

Divide the difference of the extremes by the common difference, and add one to the quotient.

94. The extremes of an arithmetical series are 15 and 57, and the common difference is 6; what is the number of terms?

Ans. 8.

95. The common difference in the ages of the children in a family is 2 years, the youngest is 5 years old, and the oldest 23; how many children in the family?

589. The extremes and number of terms given to find the sum of the series.

Let	3	5	7	9	11	13	be an arithmetical series,
and	13	11	9	7	5	3	be the series reversed.

Then $16 + 16 + 16 + 16 + 16 + 16 =$ twice the sum of the series,

or $16 \times 6 \div 2 = 48$ $=$ the sum of the series. Hence,

Rule.

Find one half of the product of the sum of the extremes multiplied by the number of terms.

96. The extremes of a series are 5 and 61, and the number of terms is 15; what is the sum of the series? Ans. 495.

97. How many strokes does a clock strike in 12 hours?

GEOMETRICAL PROGRESSION.

590. Geometrical Progression is a series of numbers increasing or decreasing by a common ratio.

Thus,	2,	6,	18,	54,	162,	is an ascending series,
and	64,	32,	16,	8,	4,	is a descending series.

In the first series 3 is the ratio, and in the second $\frac{1}{2}$.

591. In Geometrical Progression there are five elements, of which any three being given the other two can be found: —

1. The first term.
2. The last term.
3. The ratio.
4. The number of terms.
5. The sum of all the terms.

592. The first term, ratio, and number of terms given, to find the last term.

In a series, let 2 be the first term and 4 the ratio;

then 2, 1st term.

$2 \times 4 = 8$, 2d term.

$2 \times 4 \times 4 = 2 \times 4^2 = 32$, 3d term.

$2 \times 4 \times 4 \times 4 = 2 \times 4^3 = 128$, 4th term.

In this series we see that the second term is found by multiplying the first term by the ratio; the third term, by multiplying the first by the square of the ratio; the fourth, by multiplying the first by the cube of the ratio, the index of the power of the ratio always being one less than the number of the term sought. A similar explanation may be given when the series is descending. Hence,

Rule.

Multiply the first term by that power of the ratio whose index is equal to the number of terms less one.

98. The first term of a geometrical series is 5, the ratio 2, and the number of terms 8; what is the last term? Ans. 640.

99. The first term is 9, and the ratio $\frac{1}{3}$; what is the 6th term?

100. The 1st term is 4, the ratio 1.05; what is the 4th term?

101. What is the amount of \$ 15 at compound interest for 5 years at 6 %?

102. Supposing money at compound interest to double once in 12 years, to what will \$ 100 amount in 60 years?

593. The extremes and number of terms given to find the ratio.

Since the last term is obtained (Art. 592) by multiplying the first term by that power of the ratio whose index is equal to the number of terms less one, hence, conversely,

Rule.

Divide the last term by the first, and of the quotient take that root whose index is one less than the number of terms.

103. The first term in a geometrical series is 3, the last term 768, and the number of terms 5; what is the ratio? Ans. 4.

104. The extremes are 5 and 625, and the number of terms 4; what is the ratio?

105. The extremes are 8 and 1728, and the number of terms 4; what is the ratio?

594. The extremes and ratio given to find the sum of the series.

Let the series be 2, 10, 50, 250, 1250;

then $2 + 10 + 50 + 250 + 1250 \qquad = \text{sum of the series},$
and $\qquad 10 + 50 + 250 + 1250 + 6250 = 5 \text{ times sum of the series}.$
Hence, $\qquad 6250 - 2 = 4 \text{ times the sum of the series}.$

In subtracting the upper series from the lower all the terms cancel except 2 and 6250, and the remainder will be four times the sum of the series; for once a series from five times a series must leave four

times the series; hence, one fourth of this remainder must be the sum of the given series; but 4 is the ratio less 1. Hence,

Rule.

Multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio less one.

106. The extremes are 4 and 2916, and the ratio 3; what is the sum of the series? Ans. 4372.

107. The extremes are 3 and 9375, and the ratio 5; what is the sum of the series?

108. What debt will be discharged by 12 monthly payments, the first payment being \$ 1, the second \$ 2, and so on in a geometrical series?

595. The first term, ratio, and number of terms given, to find the sum of the series.

Let the first term be 3, the ratio 2, and the number of terms 6; then the series will be 3, 6, 12, 24, 48, 96.

Let 1, 2, 4, 8, 16, 32, be a series whose first term is 1, but the ratio and number of terms the same as in the first series. The sum of this series found by the Rule in Art. 594 is $\frac{64-1}{2-1}$. But the sum of the first series is evidently 3 times the sum of this second series; that is, is $\frac{64-1}{2-1} \times 3$; and the 64 is the ratio raised to a power whose index is equal to the number of terms. Hence,

Rule.

Raise the ratio to a power whose index is equal to the number of terms, subtract one, divide the remainder by the ratio less one, and multiply this quotient by the first term.

109. The first term is 5, the ratio 3, and the number of terms 7; what is the sum of the series?

ALLIGATION.

596. Alligation treats of mixing simple substances of different qualities, producing a compound of some intermediate quality. It is of two kinds, *Medial* and *Alternate.*

597. Alligation Medial is the process by which we find the price of the mixture, when the quantities and prices of the simples are given.

110. If a merchant mixes 7 pounds of nuts worth 8 cents a pound, with 6 pounds worth 9 cents, 9 pounds worth 10 cents, and 10 pounds worth 12 cents, what is the value of a pound of the mixture?

$$\begin{array}{rcrcr}
\$0.08 & \times & 7 & = & \$0.56 \\
0.09 & \times & 6 & = & 0.54 \\
0.10 & \times & 9 & = & 0.90 \\
0.12 & \times & \underline{10} & = & \underline{1.20} \\
 & & 32 &) & \underline{\$3.20} \\
 & & & & \$0.10, \text{ Ans.}
\end{array}$$

As the total value of 32 pounds is \$3.20, the value of one pound is \$3.20 ÷ 32, or \$0.10, Ans.

111. A hostler mixes 8 bushels of corn worth 60 cents a bushel with 4 bushels of rye at 75 cents a bushel, 15 bushels of oats at 36 cents a bushel, and 6 bushels of barley at 70 cents a bushel; what is the price a bushel of the mixture?

598. Alligation Alternate is the process of mixing quantities of different values so as to obtain a mixture of a required intermediate value.

NOTE. It is evident that the value of a mixture must be greater than the value of the ingredient of the least value, and less than the value of the ingredient of the greatest value, that is, must be intermediate.

112. A grocer has nuts worth, respectively, 7, 9, 12, and 14 cents a pound. How many pounds of each can he take to make a mixture worth 10 cents?

$$10\begin{cases} 7, & +3\times 2= & +6 \\ 9, & +1\times 8= & +8 \\ 12, & -2\times 1= & -2 \\ 14, & -4\times 3= & -12 \end{cases}$$

Ans. 2 lbs. of 7, 8 of 9, 1 of 12, and 3 of 14.

If a pound worth 7 is sold for 10 cents, there is a gain of 3 cents (marked +3); and if a pound worth 9 is sold for 10 cents, a gain of 1 cent; but if a pound worth 12, or 14, is sold for 10, there is a loss of 2, or 4 cents (marked − 2 and − 4). We take any convenient number of pounds of the 7, 9, and 12, respectively, as 2, 8, and 1, and find a gain of 6 + 8, and a loss of 2, leaving a balance of gain of 14 − 2, or 12, cents. In order to balance this gain of 12 cents we must take as many pounds of the 14 cent as 4 cents (the loss on a pound of the 14 cent) is contained times in 12 cents, that is, 3 pounds. With 2 pounds of the 6, 8 of the 9, 1 of the 12, and 3 of the 14 cents, sold at 10 cents a pound, the gains and losses balance. Hence,

599. To solve an example in alligation alternate,

Rule.

Take such quantities of each ingredient as to make the gains and losses equal.

NOTE 1. If there are but two ingredients the quantities taken must have the same ratio to each other; but if there are more than two ingredients the quantities taken may vary infinitely; that is, there are an infinite number of correct solutions to an example in alligation alternate when there are more than two ingredients. The only restriction is that the gains and losses should be equal.

113. A grocer has 12 pounds of tea worth \$0.50 a pound, 8 pounds worth \$0.75 a pound which he wishes to mix with teas worth, respectively, \$0.62 and \$0.70 a pound so as to get a mixture worth \$0.65 a pound. How many of each of the last two can he take?

$$65\begin{cases} 50, & +15 \times 12 = & +180 \\ 75, & -10 \times 8 = & -80 \\ 62, & +3 \times 5 = & +15 \\ 70, & -5 \times 23 = & -115 \end{cases}$$

We take 12 of the 50 cent and 8 of the 75 as is required, and take such quantities of the 62 and 70 as will make the gains and losses balance.

Ans. 5 lbs. of the 62 and 23 of the 70.

NOTE 2. A little skill is necessary in order to obtain integral answers.

NOTE 3. In working the examples, we can begin with any of the quantities. If when we come to the last quantity, we have already too much of that which is of the same nature as this last, that is, gain or loss, we must change the quantities already taken so that there will be a balance of a nature opposite to the last.

114. A farmer wishes to mix corn worth $0.70 a bushel with rye worth $0.75, barley worth $0.60, and oats worth $0.45 to make a mixture of 60 bushels worth $0.65 a bushel. How many bushels of each kind may he take?

$$65\begin{cases} 70, & -5 \times 1 = & -5 \\ 75, & -10 \times 2 = & -20 \\ 60, & +5 \times 1 = & +5 \\ 45, & +20 \times \underline{1} = & +20 \\ & \quad\quad\;\; 5 & \end{cases}$$

To make a mixture worth 65 cents, we find he can take of the corn, barley, and oats 1 bushel of each, and of the rye 2. This makes a mixture of 5 bushels; in order to have 60 bushels, therefore, he must take $\frac{1}{5}$ of 60 bushels, or 12 of corn, 12 of barley, and 12 of oats, and $\frac{2}{5}$ of 60 bushels, or 24 of rye.

Ans. 12 bushels of corn, 12 of barley, 12 of oats, and 24 of rye.

115. A farmer has cows worth $35, $42, $48, $50, and $55 a head. What number of each may he sell without loss at an average price of $45 a head?

116. How many pounds of wool worth, respectively, 30, 35, and 40 cents a pound, can be mixed with 40 pounds worth 45 cents, to make a mixture worth 38 cents a pound?

117. How many ounces of gold 700, 840, 900, and 1000 fine, respectively, can be taken to form a mass of 8 ounces 880 fine?

METRIC EQUIVALENTS.

600. The following equivalents of the metric measures and weights have been established by Congress for use in all legal proceedings.

LINEAR MEASURE.

1 centimeter = 0.3937 inch.	1 inch = 2.54 centimeters.
1 decimeter = 3.937 in. = 0.328 ft.	1 foot = 3.048 decimeters.
1 meter = 39.37 in. = 1.0936 yd.	1 yard = 0.9144 meter.
1 dekameter = 1.9884 rods.	1 rod = 0.5029 dekameter.
1 kilometer = 0.62137 mile.	1 mile = 1.6093 kilometers.

SQUARE MEASURE.

1 sq. centimeter = 0.1550 sq. in.	1 sq. in. = 6.452 sq. centimeters.
1 sq. decimeter = 0.1076 sq. ft.	1 sq. ft. = 9.2903 sq. decimeters.
1 sq. meter = 1.196 sq. yd.	1 sq. yd. = 0.8361 sq. meter.
1 ar = 3.954 sq. rd.	1 sq. rd. = 0.2529 ar.
1 hektar = 2.47 acres.	1 acre = 0.4047 hektar.
1 sq. kilometer = 0.386 sq. m.	1 sq. m. = 2.59 sq. kilometers.

MEASURES OF VOLUME.

1 cu. centimeter = 0.061 cu. in.	1 cu. in. = 16.39 cu. centimeters.
1 cu. decimeter = 0.0353 cu. ft.	1 cu. ft. = 28.317 cu. decimeters.
1 cu. meter, 1 ster = 1.308 cu. yd.	1 cu. yd. = 0.7646 cu. meter.
1 cu. meter, 1 ster = 0.2759 cd.	1 cord = 3.624 sters.
1 liter = 0.908 qt. dry.	1 qt. dry = 1.101 liters.
1 liter = 1.0567 qt. liq.	1 qt. liq. = 0.9463 liter.
1 dekaliter = 2.6417 gal.	1 gal. = 0.3785 dekaliter.
1 dekaliter = 1.135 pecks.	1 peck = 0.881 dekaliter.
1 hektoliter = 2.8375 bush.	1 bush. = 0.3524 hektoliter.

WEIGHTS.

1 gram	= 0.03527 ounce.	1 ounce	= 28.35 grams.
1 kilogram	= 2.2046 pounds.	1 pound	= 0.4536 kilogram.
1 metric ton	= 1.1023 English tons.	1 English t.	= 0.9072 metric ton.

601. APPROXIMATE METRIC EQUIVALENTS.

1 decimeter	= 4 inches.	1 liter	= { 1.06 qt. liquid. 0.9 qt. dry.
1 meter	= 1.1 yards.		
1 kilometer	= $\frac{5}{8}$ of a mile.	1 hektoliter	= $2\frac{5}{8}$ bushels.
1 hektar	= $2\frac{1}{2}$ acres.	1 kilogram	= $2\frac{1}{5}$ pounds.
1 ster, or cu. meter	= $\frac{1}{4}$ of a cord.	1 metric ton	= 2200 pounds.

602. To reduce metric weights or measures to those in customary use.

1. Reduce 127^{km} to miles.

Solution. As 1 kilometer = 0.62137 of a mile, 127^{km} will be 127×0.62137 miles = 78.914 miles, Ans. Hence,

Rule.

Multiply the value of the corresponding metric unit in the table by the given number of metric units.

2. Reduce 312^{m} to inches. Ans. 12283.44 in.
3. Reduce 12000^{K} to pounds.
4. In a cubic meter how many quarts liquid measure?

603. To reduce customary weights and measures to those of the metric system.

5. Reduce 280 inches to meters.

Solution. As 1 inch = 0.0254 meters, 280 inches = 280×0.0254 meters = 7.112 meters, Ans. Hence,

Rule.

Multiply the value of the corresponding unit in the table by the given number of units.

6. Reduce 27 tons to kilos.

7. Find the weight in kilos of 15 gallons of water.

8. Find the capacity in hektoliters of a bin 7 ft. 3 in. long, 3 ft. 10 in. wide, and 2 ft. 2 in. deep.

9. How many square meters in 16 acres?

10. How many liters in 5 bushels?

11. A grocer buys nuts at $8\frac{1}{2}$ cents a pound, and sells them for 22 cents a kilo; how much does he gain or lose on 100 pounds?

12. A man buys 65 liters of wine at $1 a liter, and sells it at $1 a quart; how much does he gain or lose?

13. A man buys 10 tons of coal for $6 a ton of 2240 pounds; what must he get a metric ton to gain $5 on the cost?

14. The latitude of New Orleans is 30°, and that of Chicago is 42°; how many kilometers north of New Orleans is Chicago?

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